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FEM numerical algorithm on contact problem for non-conform elastic bodies

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This paper deals with FEM numerical algorithm for contact problem with or without friction between bodies. Numerical solution uses Updated Lagrange Formulation (UL), which is very convenient for contact between non-conform elastic bodies especially when friction is considered. Because of non-linearity of problem, that method gives a possibility to make problem linear by using load increments. Presented algorithm gives a base for making own computer programme.

1. INTRODUCTION

Contact between bodies is one of the ways of load transferring. The mechanism of that kind of load transferring depends on a nature of interaction between two or more contact surfaces. The knowledge of such mechanism has a great practical character. The direct observation of the contact phenomenon and measuring of the certain values is often impossible. Contact problem is also complicated because of fact that the behaviour of the machine elements, provoked by contact stresses, depends on property of materials, the state of surfaces, the kind and intensity of loading, the direction of loading in relation to contact surfaces and the way of elements fastening.

A great number of authors solved contact problem analytically. But contact with friction and especially non-symmetric contacts are a subject that numerical solutions must be involved.

A various kind of numerical solutions on contact problems are known nowadays. BEM [1] and FEM [1, 2, 3, 4] methods are equally represented. When contact problem with friction is considered, because of its non-linearity, FEM method with Updated Lagrange Formulation (UL) is proved to be very convenient [5].

2. STRATEGY OF CONTACT PROBLEM MODELLING

Non-conform contact problem without friction is such kind of problem where introducing of load increment is not necessary, but iteration procedure must be introduce because contact area is not known *a priory*. The problem is solved when continued distribution of normal stresses across the whole contact area is obtained. At the end of contact area the stress must be zero or very close to zero.

Non-conform contact problem with friction - because of friction presence iteration procedure must be carried out. The increments of loading must be introduced as well.

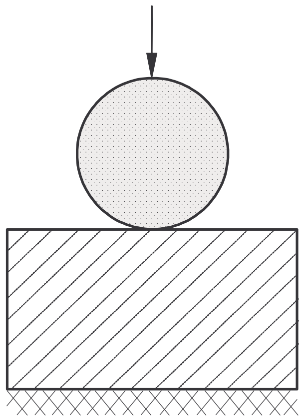

Figure 1. Non-conform contact

2.1 Fundamental problem

A general equation for total potential of N bodies in contact in time t, according to virtual work principle is [6]:

$$\sum_{L=1}^{N}\left\{\int_{{}^tV} {}^t\tau_{ij}\ \delta_t e_{ij}\, d\,{}^tV\right\} = \sum_{L=1}^{N}\left\{\int_{{}^tV}\delta u_i\ \ {}^tf^B\, d\,{}^tV + \int_{{}^tS_f}\delta u_i^S\ \ {}^tf_i^S\, d\,{}^tS\right\} + \sum_{L=1}^{N}\int_{{}^tS_c}\delta u_i^c\ \ {}^tf_i^c\, d\,{}^tS\,, \tag{1}$$

where the part in brace on the right side of equation represents usual members in virtual displacement method, while the third member is contribution of contact stresses.

3. NUMERICAL SOLUTION

A fundamental relation in non-linear FEM analysis that must be solved is:

$$\int_{{}^{t+\Delta t}V} {}^{t+\Delta t}\boldsymbol{\tau}_{ij}\ \boldsymbol{\delta}_{t+\Delta t}\boldsymbol{e}_{ij}\ \boldsymbol{d}\,{}^{t+\Delta t}\boldsymbol{V} = {}^{t+\Delta t}\Re \tag{2}$$

where on the left side of (2) is internal virtual work and:

$${}^{t+\Delta t}\Re = \int_{{}^{t+\Delta t}V} {}^{t+\Delta t}\boldsymbol{f}_i^B\ \ \boldsymbol{\delta u}_i\ \boldsymbol{d}\,{}^{t+\Delta t}\boldsymbol{V} + \int_{{}^{t+\Delta t}S_f} {}^{t+\Delta t}\boldsymbol{f}_i^S\ \ \boldsymbol{\delta u}_i^S\ \boldsymbol{d}\,{}^{t+\Delta t}\boldsymbol{S}\,. \tag{3}$$

is exterior virtual work.

The fundamental problem in general non-linear analysis is to solve equilibrium state of bodies, which corresponds to real load. If we suppose that load is a function of time, the condition of equilibrium can be represented with:

$$ {}^{t}\boldsymbol{R} - {}^{t}\boldsymbol{F} = 0 \tag{4}$$

Exterior load is:

$$ {}^{t}\boldsymbol{R} = {}^{t}\boldsymbol{R}_B + {}^{t}\boldsymbol{R}_S + {}^{t}\boldsymbol{R}_C \tag{5}$$

with volume, surface and concentrated forces.

If momentary stress is concerned like initial stress, than the node forces obtained from stress satisfy the condition:

$$ \boldsymbol{R}_I = {}^{t}\boldsymbol{F} \tag{6}$$

that is:

$$ {}^{t}\boldsymbol{F} = \sum_{m} \int_{{}^{t}V^{(m)}} {}^{t}\boldsymbol{B}^{(m)T}\, \boldsymbol{\tau}^{(m)}\, {}^{t}dV^{(m)} \tag{7}$$

for all that, the stresses and volumes of bodies in time t are unknown.

When non-linear response is supposed, then equation (4) must be satisfied through whole load history, i.e. in time t from zero to any time in which problem is concerned. In statically analysis, that contact problem is, the time is a convenient variable. The time describes various intensity of load, and also a various configurations. That is the essence of incremental method.

4. NUMERICAL ALGORITHM

4.1 Incremental-iterative method for solving the system of non-linear equations

The known values of stresses ${}^{t}\sigma$ and deformations ${}^{t}e$ in time *t*:

1. *Known*: displacement of nodes ${}^{t+\Delta t}\boldsymbol{U}^{(i-1)}$ and ${}^{t+\Delta t}\boldsymbol{e}^{(i-1)}$
2. *Solve*: stresses ${}^{t+\Delta t}\boldsymbol{\sigma}^{(i-1)}$
3. *Solve*: ${}^{t+\Delta t}\boldsymbol{F}^{(i-1)} = \int_{{}^{t+\Delta t}V} {}^{t+\Delta t}\boldsymbol{B}^{T}\ {}^{t+\Delta t}\boldsymbol{\sigma}^{(i-1)}\, dV$ to (7)
4. *Solve*: $\boldsymbol{\Delta U}^{(i)}$ using
   ${}^{t+\Delta t}\boldsymbol{K}^{(i-1)}\, \boldsymbol{\Delta U}^{(i)} = {}^{t+\Delta t}\boldsymbol{R} - {}^{t+\Delta t}\boldsymbol{F}^{(i-1)}$, and then solve ${}^{t+\Delta t}\boldsymbol{U}^{(i)} = {}^{t+\Delta t}\boldsymbol{U}^{(i-1)} + \boldsymbol{\Delta U}^{(i)}$.

Repeat steps from 1 to 4 till convergence is achieved.

4.2 Incremental-iterative method for solving contact stresses and contact status

1. *Known:* equation system ${}^{t+\Delta t}\boldsymbol{K}^{(i-1)}\, \boldsymbol{\Delta U}^{(i)} = {}^{t+\Delta t}\boldsymbol{R} - {}^{t+\Delta t}\boldsymbol{F}^{(i-1)}$
2. *Solve*: from contact status $\boldsymbol{A}\,\boldsymbol{\Delta U} - \delta \geq 0$ with linear transformation
   equation from step 1. bring to form $\boldsymbol{M}^{(i)T}\, {}^{t+\Delta t}\boldsymbol{K}^{(i-1)}\, \boldsymbol{\Delta U}^{(i)} = \boldsymbol{M}^{(i)T} \left({}^{t+\Delta t}\boldsymbol{R} - {}^{t+\Delta t}\boldsymbol{F}^{(i-1)}\right)$;
   $\boldsymbol{\Delta U}$ consists required degree of freedom potential contact nodes.

In the case of contact with friction, ${}^{t+\Delta t}\boldsymbol{F}^{(i-1)}$ is force provoked with friction in slipping zone, and is solving by means of normal contact force from previous iteration

3. *Solve*: system of equations by Gauss method
4. *Control of contact status*:

a) Potential pairs of the contact nodes outside of contact area must satisfy the condition $\boldsymbol{A}\,\boldsymbol{\Delta U} - \delta > 0$

b) Inside contact area, in pairs of the contact nodes, normal component of stress must satisfy the condition $\sigma_c < 0$

c) In the case of friction the pairs of nodes inside of the contact area must satisfy condition:

c.1) in sticking zone

$$\tau_t < \mu \left|\sigma_c\right|$$

c.2) in slipping zone

$$\tau_t = \mu \left|\sigma_c\right|.$$

5. CONCLUSION

Numerical algorithm presented above gives a base for making computer program for analysis of contact two or more non-conform elastic bodies. When contact without friction is considered only iterative method must be in use, but for contact with friction an incremental-iterative method is necessary. That is the way that non-linear problem can be made linear. So every segment is solving like system of linear equations with well-known methods. Every increment is then solving by using variable from previous equilibrium state i.e. previous increment.

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