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Mathematical model as a tool for evaluating the reality of Genichi Taguchi's statistical method applied to the grinding process*

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1. INTRODUCTION

Grinding is a material cutting process with undefined cutting geometry, which is unpredictable, chaotic and therefore a very demanding machining process. Traditional statistical methods often require a large number of tests that can take too much time to accomplish.

Taguchi's statistical analysis (thereinafter GT analysis) significantly lowers number of tests and is very popular, because it does not require knowledge of statistics. However, it is often inaccurate or it can even completely miss its aim. Therefore a method to test its accuracy is needed.

We would like to present a procedure that reduces the possibility of mistakes to a minimum and apply it to a real industrial case - machining by grinding.

2. THEORY

As mentioned, GT analysis dramatically reduces the number of experiments required. This is the main advantage of this method.

How did the author succeed in reducing the number of experiments? By using orthogonal arrays! Orthogonal means being balanced but not mixed. In the context of experimental matrices, orthogonal means statistically independent. If we examine a typical orthogonal array (Table 1), we will note that each level has an equal number of occurrences within each column. For each column of the array in Table 1, level 1 occurs four times, and level 2 occurs four times as well.

Table 1
Standard orthogonal array $L_8(2^7)$

No.	A 1	B 2	C 3	D 4	E 5	F 6	G 7	Test result
1	1	1	1	1	1	1	1	y_1=12
2	1	1	1	2	2	2	2	y_2=15
3	1	2	2	1	1	2	2	y_3=10
4	1	2	2	2	2	1	1	y_4=14
5	2	1	2	1	2	1	2	y_5=18
6	2	1	2	2	1	2	1	y_6=22
7	2	2	1	1	2	2	1	y_7=20
8	2	2	1	2	1	1	2	y_8=14

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Concerning statistical independence, this idea of balance goes farther than meaning simply an equal number of levels within each column. If we look at the relationship between the columns, we can notice that the same number of levels will occur within any column. We see that for factor A (column 1), level 1 occurs four times, and level 2 is repeated the same number of times. If we look at column 2 (factor B), we can see that for factor A at level 1, Factor B is at level 1 twice and at level 2 twice. The same is true for factor A at level 2. If we look at the any column, we will notice the same relationship.

The effect of a certain factor (i.e. factor A) is calculated by equation (1):

$$\text{Effect of A} = \left|\overline{\mathrm{A}_1} - \overline{\mathrm{A}_2}\right| = \left|\frac{y_1 + y_2 + y_3 + y_4}{4} - \frac{y_5 + y_6 + y_7 + y_8}{4}\right| =$$

$$= \left|\frac{12+15+10+14}{4} - \frac{18+22+20+14}{4}\right| = |12{,}75 - 18{,}50| = 5{,}75 \tag{1}$$

GT analysis can determine the effects of factors and their interactions. In this paper we will focus only on effects of factors.

Statisticians doubt in reliability of GT analysis. Is this doubt necessary? Is it necessary only in certain cases? Which ones?

In a search for answers, we have to reverse the process. We are going to elaborate a mathematical model of a process and simulate it. By understanding a model, we will know exact effect of each factor on a chosen span of factors. By comparing these exact effects to GT analysis calculated effects, one can determine the reliability and accuracy of GT analysis.

The procedure algorithm:

1) Selection of the process function - mathematical model of a process (MMPP):

$$R = R(A; B; C; D) \tag{2}$$

2) Selection of standard orthogonal array with span of observed factors and calculation of test results using mathematical model and GT analysis of these simulated results.

$$\begin{aligned} &Effect\ \ of\ \ A = GT_A \\ &Effect\ \ of\ \ B = GT_B \\ &Effect\ \ of\ \ C = GT_C \\ &Effect\ \ of\ \ D = GT_D \end{aligned} \tag{3}$$

3) Calculation of factors effects (between the levels) using the selected function of a process (MMPP):

$$\begin{aligned} &Effect\ \ of\ \ A = R(A_2,\overline{B},\overline{C},\overline{D}) - R(A_1,\overline{B},\overline{C},\overline{D}) \\ &Effect\ \ of\ \ B = R(\overline{A},B_2,\overline{C},\overline{D}) - R(\overline{A},B_1\overline{C},\overline{D}) \\ &Effect\ \ of\ \ C = R(\overline{A},\overline{B},C_2,\overline{D}) - R(\overline{A},\overline{B},C_1,\overline{D}) \\ &Effect\ \ of\ \ D = R(\overline{A},\overline{B},\overline{C},D_2) - R(\overline{A},\overline{B},\overline{C},D_!) \end{aligned} \tag{4}$$

Letters with dashes represent the average value of a factor:

$$\overline{A} = \frac{A_1 + A_2}{2}$$
$$\overline{B} = \frac{B_1 + B_2}{2}$$
$$\overline{C} = \frac{C_1 + C_2}{2}$$
$$\overline{D} = \frac{D_1 + D_2}{2} \tag{5}$$

4) With comparison of effects defined in items 2.) and 3.) we can evaluate the accuracy of GT analysis.

Table 2

Factors	Values determined by MMPP	Values determined by GT analysis
Effect of A	V_A MMPP	V_A GT
Effect of B	V_B MMPP	V_B GT
Effect of C	V_C MMPP	V_C GT
Effect of D	V_C MMPP	V_C GT

3. EXAMPLE OF REAL RESULTS PROVIDED BY GT

Selected mathematical model:

$$R = K \cdot a \cdot A \cdot b \cdot B \cdot c \cdot C \cdot d \cdot D \tag{6}$$

Table 3

Numerical values for factor levels

Numerical values		
A	50	55
B	40	44
C	30	33
D	20	21

Table 4

Values for constant **K** and factors weights

Factor weights K		
a	1,20	0,167
b	1,15	
c	1,10	
d	1,05	

Table 5

Array of factor levels and results of simulated experiments

A	B	A x B	C	A x C	B x C	D	R
50	40	1	30	1	1	20	3,188
50	40	1	33	2	2	21	3,682
50	44	2	30	1	2	21	3,682
50	44	2	33	2	1	20	3,857
55	40	2	30	2	1	21	3,682
55	40	2	33	1	2	20	3,857
55	44	1	30	2	2	20	3,857
55	44	1	33	1	1	21	4,455

Table 6
Comparison between exact values of factor effects (MMPP) and values determined by GT analysis

Factors	Values determined by MMPP	Values determined by GT analysis
Effect of A	0,3600	0,3610
Effect of B	0,3600	0,3610
Effect of C	0,3600	0,3607
Effect of D	0,1845	0,1853

The result of GT analysis matches the calculated effects of factors. Therefore, GT analysis for selected mathematical model (6) and chosen span of factors provides us with reliable information considering effects of factors.

4. EXAMPLE OF UNREAL RESULTS PROVIDED BY GT

Selected mathematical model:

$$R = \frac{K \cdot a \cdot A \cdot b \cdot B \cdot c \cdot C \cdot d \cdot D}{K_1 - A} \tag{7}$$

The value of selected model approaches infinity if we choose value of factor **A** close to constant $\mathbf{K_1}$.

Numerical values for factor levels are the same as the values defined in Table 3.

Table 7
Values for constant **K** and $\mathbf{K_1}$

Factor weights K		
a	1,20	0,167
b	1,15	K_1
c	1,10	55,5
d	1,05	

Table 8
Array of factor levels and results of simulated experiments

A	B	A x B	C	A x C	B x C	D	R
50	40	1	30	1	1	20	28,980
50	40	1	33	2	2	21	33,472
50	44	2	30	1	2	21	33,472
50	44	2	33	2	1	20	35,066
55	40	2	30	2	1	21	368,191
55	40	2	33	1	2	20	385,724
55	44	1	30	2	2	20	385,724
55	44	1	33	1	1	21	445,511

Table 9
Comparison between exact values of factor effects (MMPP; $\mathbf{K_1}$=55,5) and values determined by GT analysis

Factors	Values determined by MMPP	Values determined by GT analysis
Effect of A	363,5	363,5
Effect of B	6,004	20,850
Effect of C	6,004	20,851
Effect of D	3,075	11,288

The results presented in Table 9 shows, that GT analysis is unreal for this specific example, except for the effect of factor **A**.

Table 10
Comparison between exact values of factor effects (MMPP; $\mathbf{K_1}$=60) and values determined by GT analysis

Factors	Values determined by MMPP	Values determined by GT analysis
Effect of A	21,61	21,62
Effect of B	2,40	2,77
Effect of C	2,40	2,77
Effect of D	1,23	1,45

The results presented in Table 10 shows, that GT analysis is more accurate and reliable for this specific example. Generally the results are more realistic when values of factor **A** and $\mathbf{K_1}$ differ more.

5. BETTER USE OF GT ANALYSIS

How can we tell whether the GT analysis is accurate and reliable, if we apply it to a process with unknown mathematical model (with this being the reason for its GT analysis)?
In principle this is simple. Once we perform the GT analysis we must estimate the mathematical model of analyzed process for its confirmation or rejection. This model must satisfy two conditions:

- It must describe the process satisfactorily for a span of observed factors (the results of mathematical model must match the real results),
- There can be no peaks in function values (in mathematical model of the process) in the selected span of observed function values.

The biggest problem lies of course in estimating the mathematical model of process, which is satisfactorily accurate in the selected segment of factors. For this we need a preliminary knowledge of a process or we can try to extract the mathematical model from the GT analysis itself. If the results of simulated tests based on mathematical model match the real ones, there is a great possibility of GT analysis being accurate.

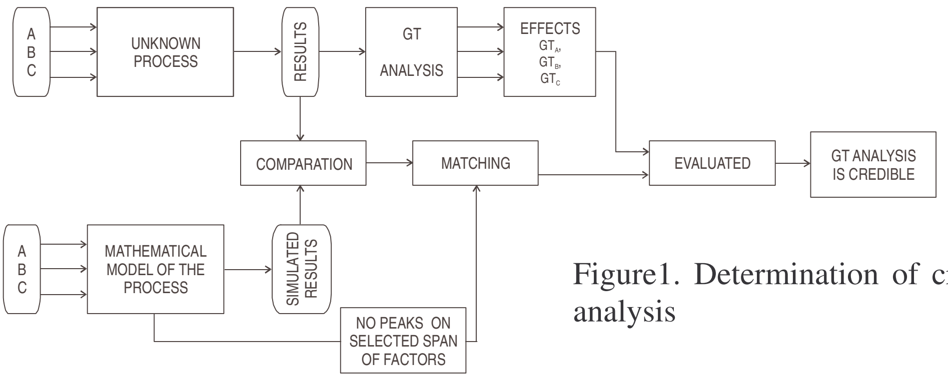

Figure1. Determination of credibility of GT analysis

6. DEMONSTRATION ON A CONCRETE MANUFACTURING EXAMPLE

Evaluation of GT analysis will be shown on a concrete manufacturing example. We would like to determine the effect of grinding segment (factor A), horizontal work speed (factor B), feed per a run (factor C) and the distance between the workpieces (factor D) on productivity of grinding.

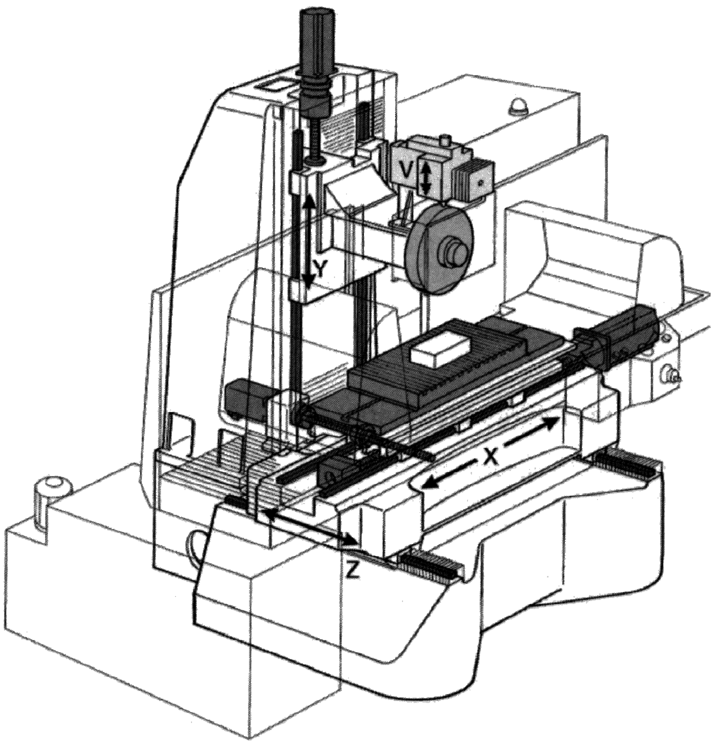

Figure1. Determination of credibility of GT analysis

The experiments were performed on a machine-tool for face grinding:
Name: Face Gockel
Power: 20 kW
Working space: 6000x700x500
Clamping: magnetic

Tool - grinding segment:
Grinding segment 1: Rappold - 8A 46 - K7/6 B14S
Grinding segment 2: Swaty - 48A 24I 10/1V 50
Number of grinding segments: 16

Machining parameters:
Horizontal work speed: 35 m/min, 40 m/min,
Circumferential speed of a grinding segment: 26 m/s
Feed per a run: 0,05 mm, 0,06 mm
Time of one feed: 3 s
Distance between the workpieces: 1 cm, 1,5 cm
The summed wear of each grinding segment was less than 1,2 mm.

The experiments were conducted with thermally untreated material UTOP MO 4, with chemical structure as shown below.

Table 11
UTOP MO 4 chemical structure

C %	Si %	Mn %	Cr %	Mo %	V %
0,50	1,00	0,30	5,00	1,40	0,90

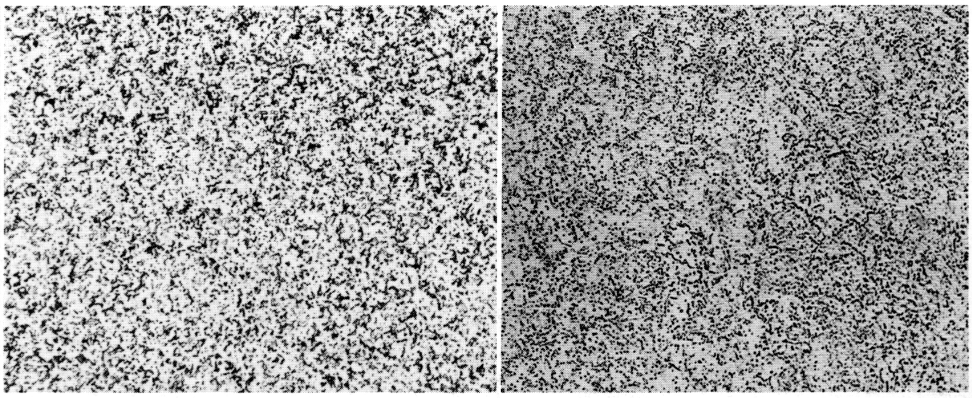

Figure3. Metallographic structure of UTOP MO 4

For our example we chose the same orthogonal arrays as in Table 1.

Table 12
Standard orthogonal array $L_8(2^7)$

Test No.	A 1	B 2	*C 3*	D 4	*E 5*	*F 6*	G 7
1	1	1	*1*	1	*1*	*1*	1
2	1	1	*1*	2	*2*	*2*	2
3	1	2	*2*	1	*1*	*2*	2
4	1	2	*2*	2	*2*	*1*	1
5	2	1	*2*	1	*2*	*1*	2
6	2	1	*2*	2	*1*	*2*	1
7	2	2	*1*	1	*2*	*2*	1
8	2	2	*1*	2	*1*	*1*	2

Columns, marked in italics can be used for studying interactions, which is not our intention.

When filling in the values for levels of factors and concluding experiments we get the results displayed in a lower table.

Table 13
Results

Test No.	A	B (m/min)	A x B	C (mm)	A x C	B x C	D (cm)	R (cm^3/min)
1	1	35	1	0,05	1	1	1	31,55
2	1	35	1	0,06	2	2	1,5	37,57
3	1	40	2	0,05	1	2	1,5	36,63
4	1	40	2	0,06	2	1	1	42,25
5	2	35	2	0,05	2	1	1,5	181,65
6	2	35	2	0,06	1	2	1	209,50
7	2	40	1	0,05	2	2	1	204,30
8	2	40	1	0,06	1	1	1,5	243,29

By applying GT analysis (using the equation (1)) we get the following results.

Table 14
GT analysis results

Influence	Value (cm^3/min)	By span of factors	
Effect of A	172,690	A_1 = gr. segm. 1	A_2 = gr. segm. 2
Effect of B	16,550	B_1 =35 m/min	B_2 =40 m/min
Effect of C	19,623	C_1 =0,05 mm	C_2 =0,06 mm
Effect of D	2,885	D_1 =1 cm	D_2 =1,5 cm

Presumed form of process function is:

$$R = A_R \cdot b \cdot B \cdot \left(0{,}163 + c \cdot C\right) \cdot \left(1 + d \cdot D\right) \tag{8}$$

$\mathbf{A_R}$ represents the average product for chosen grinding segment, **b** estimates horizontal working speed, **B** is numerical value for horizontal working speed, expression $(0{,}163 + c \cdot C)$ represents the effect of feed per a run, where **C** is numerical value for feed per a run and **c** is a weight for factor **C**. Expression $(1 + d \cdot D)$ represents the effect of distance between workpieces. When numerical value of distance is zero, there is no effect on process. **d** is a weight for factor **D**.

Expression $(0{,}163 + c \cdot C)$ is formed on assumption that the real effect of factor **C** is a regressive growing function, and that $(0{,}163 + c \cdot C)$ is only linear approximation for span of factor C (C_1=0,05mm and C_2=0,06mm). The value 0,163 was determined from experience. Effect is well shown in a diagram 1.

Expression $(1 + d \cdot D)$ is also a linear approximation of a real function (diagram 2).
The average productivity of grinding segments is known and is 30 (cm^3/min) for grinding segment 1 and 170 (cm^3/min) for grinding segment 2.

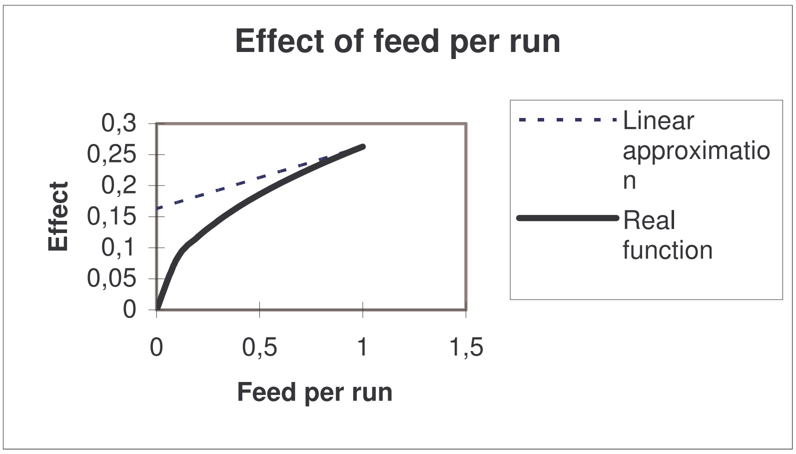

Diagram 1. Effect of feed per run

Now we must solve the equation system with three variables (b,c,d).

1.) $A_{R1} \cdot b \cdot B_1 \cdot \left(0{,}163 + c \cdot C_1\right) \cdot \left(1 + d \cdot D_1\right) = R_1$

$$2.)\ A_{R1} \cdot b \cdot B_1 \cdot (0{,}163 + c \cdot C_2) \cdot (1 + d \cdot D_2) = R_2$$

$$3.)\ A_{R1} \cdot b \cdot B_2 \cdot (0{,}163 + c \cdot C_1) \cdot (1 + d \cdot D_2) = R_3$$

By dividing the first equation by third we get value of **d**:

$$\frac{A_{R1} \cdot b \cdot B_1 \cdot (0{,}163 + c \cdot C_1) \cdot (1 + d \cdot D_1)}{A_{R1} \cdot b \cdot B_2 \cdot (0{,}163 + c \cdot C_1) \cdot (1 + d \cdot D_2)} = \frac{R_1}{R_3} \Rightarrow \frac{(1 + d \cdot D_1)}{(1 + d \cdot D_2)} = \frac{R_1 \cdot B_2}{R_3 \cdot B_1} \Rightarrow$$

$$\Rightarrow d = \frac{\frac{R_1 \cdot B_2}{R_3 \cdot B_1} - 1}{D_1 - \frac{R_1 \cdot B_2}{R_3 \cdot B_1} \cdot D_2} = \frac{\frac{31{,}55 \cdot 40}{36{,}63 \cdot 35} - 1}{1 - \frac{31{,}55 \cdot 40}{36{,}63 \cdot 35} \cdot 1{,}5} = 0{,}033$$

By dividing the second equation by third we get value of **c**:

$$\frac{A_{R1} \cdot b \cdot B_1 \cdot (0{,}163 + c \cdot C_2) \cdot (1 + d \cdot D_2)}{A_{R1} \cdot b \cdot B_2 \cdot (0{,}163 + c \cdot C_1) \cdot (1 + d \cdot D_2)} = \frac{R_2}{R_3} \Rightarrow \frac{(0{,}163 + c \cdot C_2)}{(0{,}163 + c \cdot C_1)} = \frac{R_2 \cdot B_2}{R_3 \cdot B_1} \Rightarrow$$

$$\Rightarrow c = \frac{0{,}163\left(\frac{R_2 \cdot B_2}{R_3 \cdot B_1} - 1\right)}{C_2 - \frac{R_2 \cdot B_2}{R_3 \cdot B_1} \cdot C_1} = \frac{0{,}163\left(\frac{37{,}57 \cdot 40}{36{,}63 \cdot 35} - 1\right)}{0{,}06 - \frac{37{,}57 \cdot 40}{36{,}63 \cdot 35} \cdot 0{,}05} = 20{,}18$$

From the fourth equation we get the value of **b**:

$$b = \frac{R_1}{A_{R1} \cdot B_1 \cdot (0{,}163 + c \cdot C) \cdot (1 + d \cdot D_1)} = \frac{31{,}55}{30 \cdot 35 \cdot (0{,}163 + 20{,}18 * 0{,}05) \cdot (1 + 0{,}033 \cdot 1)} = 0{,}026$$

Mathematical function, which should describe the process, is:

$$R = A_{Ri} \cdot 0{,}026 \cdot B_i \cdot (0{,}163 + 20{,}180 \cdot C_i) \cdot (1 + 0{,}033 \cdot D_i)$$

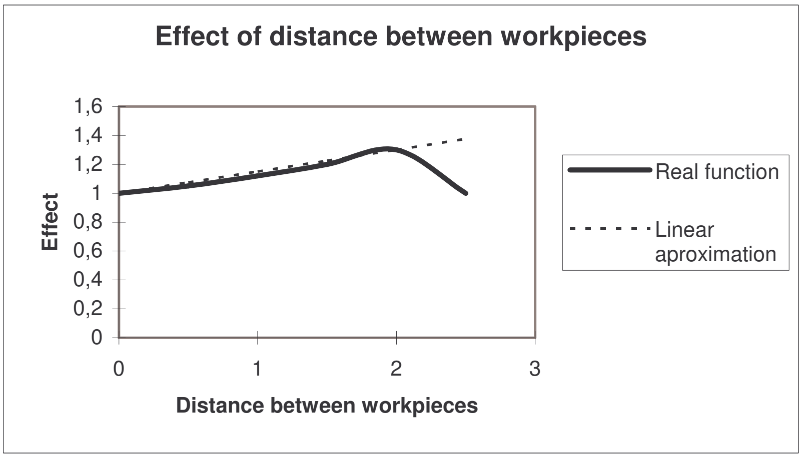

Diagram 2. Effect of distance between workpieces

Due to absence of poles on the chosen spans of factors, this function only need to match with real results and the credibility of GT analysis will be confirmed.

Table 15 shows that the last assumption is true; the calculated and real results match satisfactorily.

Table 15
Comparison between real and calculated results

Test no.	A gr. segm. type	B (m/min)	A x B	C (mm)	A x C	B x C	D (cm)	R-reality (cm^3/min)	R-Mathematical model (cm^3/min)
1	1	35	1	0,05	1	1	1	31,55	32,96
2	1	35	1	0,06	2	2	1,5	37,57	39,19
3	1	40	2	0,05	1	2	1,5	36,63	38,21
4	1	40	2	0,06	2	1	1	42,25	44,15
5	2	35	2	0,05	2	1	1,5	181,65	189,47
6	2	35	2	0,06	1	2	1	209,50	218,90
7	2	40	1	0,05	2	2	1	204,30	213,43
8	2	40	1	0,06	1	1	1,5	243,29	253,82

Because of matching of calculated and real results and the absence of poles on chosen span of factors, we can conclude, that GT analysis in this manufacturing example gives us reliable information about effects of observed factors on productivity.

7. CONCLUSIONS

For determination of accuracy of GT analysis in a machining process (e.g. grinding) one needs to elaborate a mathematical model, which is accurate enough. Once the criterion of repeatability is assured, we can calculate the results for a span of influential factors we are interested in. The calculated results have to match the real results. If the model displays no peaks in values of observed property in the selected span, the GT analysis is accurate.
Comparison of real and calculated results besides no peaks in the observed segment of factors show that for discussed manufacturing example of grinding the GT method is accurate enough and appropriate for practical use.

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