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Investigation of Sensitivity of Continuous Vibrating Systems by Means the Polar Graphs as Models of Discrete Systems and Structural Numbers Method

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There is formulated and formalized parametrical sensitivity concept of bar system in representation of pole graphs and structural numbers as a stage of making production's decisions assist of synthesized continuous torsionally vibrating mechanical system. Such formulation makes possible to study continuous mechanical system sensibility on transformed plane of complex variable $r(=\mathrm{th}\Gamma s)$.

1. INTRODUCTION

The designed machines consist of various elements or components the properties of which are exactly specific. For those creations it makes many requirements, with regard to specific frequency spectrum, high working velocities, stability, effectiveness or first of all working reliability. Those factors cause that we form various models of designed objects, relied on continuous systems, because discrete models are too distant approximation of object properties. Mentioned dynamic elements properties we can get on design stage by structural and parametrical synthesis of requirements, with regard to designing or identifying system. Parameters of designing element or component can give in changes during operation. To make designing and then operating system characterized by demanding properties, we started working out various methods that would make possible to estimate changes of individual parameters influence on designing, identifying or researching technical mean's characteristics. Existing methods, for analysis use functions characterizing dynamic state of system. Amplitude-phase characteristic we can rate among those functions. Analysis process for such generated functions consist in comparing their forms in resonance frequency environment. In this research an attempt is made to test sensitivity of continuous system model parameters, calculated by the method of characteristic's decompose synthesis into continued fraction expansion. Then, relying on discrete quantity, elements' values of continuous system are calculated. So used transformation and retransformation system's characteristics and its elements, specific in [4-8,] is used. Formed in this way continuous and discrete model is presented in form of graphs and structural numbers. These research methods of mechanical systems allow quickly and exactly calculate the impact of individual factors on his dynamic properties.

2. FORMULATING THE SENSITIVITY OF CONTINUOUS SYSTEMS ON A TRANSFORMED SURFACE OF A CONNECTED VARIABLE *r*(=thΓ*s*) BY MEANS THE GRAPHS AND STRUCTURAL NUMBERS METHOD

Problems of examining the sensitivity in a classical approach has been introduced in many works (compare the literature memo in [1] and [13]). The extraction of the formulas in polar graphs categories and the structural numbers in a regard to discreet structures has been whereas introduced in [13]. Taking under consideration the foundation of model structure as a polar graph and its parameters on a transformed surface of a connected variable. As far as these structures are concerned we assign to the edges of the graph the dynamical stiffness: inertial or elasticity. The parametrical sensitivity in this method is examined in very direct way according to the graph edges. It is necessary to make the aggregation of a polar graph, introducing it as a graph of a higher category ${}^{k}\underset{1}{X}$ [2-12] with isolated edge ${}_{2}x_{r}$, according to which the actual sensitivity is examined. We assign to the edge indicated ${}_{2}x_{r}$, with weight a_r isolated from the block graph ${}^{k}\underset{1}{X}$,the inertial and efficient stiffness with a parameter α_r.The edge which includes variable parameter is called the sensitive edge. Through the isolation from the hypergraph ${}^{k}\underset{1}{X}$ of sensitive edge the structural number of a higher category, appears as the following [9,13]

$$ {}^{k}A = \begin{bmatrix} [a_r] & [\phi] \\ {}^{k-1}A_{a_r} & {}^{k-1}A^{a_r} \end{bmatrix} \tag{1} $$

where: ${}^{k-1}A_{a_r} = \dfrac{\partial\, {}^{k-1}A}{\partial a_r}$ is the algebraic derivative of a structural number ${}^{k-1}A$, and when the indication of the sensitive edge is concern, ${}^{k-1}A^{a_r} = \dfrac{\delta\, {}^{k-1}A}{\delta\, a_r}$ it is the anti derivative of an algebraic structural number ${}^{k-1}A$, when the sensitive edge considered.

The aim of isolation the sensitive edge is to make a special transformation of a graph ${}^{2}X$. Such process is used to join the tops of an isolated, and to remove this edge from the graph. The mentioned structural number (1) stand for determinant function

$$ \underset{Z}{\det}\,{}^{2}A = a_r \underset{Z}{\det}\,{}^{2}A_{a_r} + \underset{Z}{\det}\,{}^{2}A^{a_r}\,, \tag{2} $$

where: Z is a set of dynamical stiffness .

Taking into consideration the harmonic excitation, which affects on a k coordinate mass of the system, dynamical flexibility, of i mass can be obtained from the formula,

$$Y_{ik}(r) = \frac{\underset{Z}{\mathrm{Sim}}\left({}^2A_{a_i}, {}^2A_{a_k}\right)}{\underset{Z}{\det}{}^2 A}, \tag{3}$$

where: $\underset{Z}{\mathrm{Sim}}$ is a simultaneous function of a derivative of a graph structural number.

$$\underset{Z}{\mathrm{Sim}}\left({}^2A_{a_i}, {}^2A_{a_r}\right) = a_r \underset{Z}{\mathrm{Sim}}\left({}^2A_{a_r a_k}, {}^2A_{a_r a_i}\right) + \underset{Z}{\mathrm{Sim}}\left({}^2A^{a_r}_{a_k}, {}^2A^{a_r}_{a_i}\right) \tag{4}$$

Taking advantage of dependences (3) and (4) and the definitions of sensitivities [1] and [13], the analytical form of logarithm function, sensitivity from dynamical flexibility Y_{ik}, according to the weight of sensitive edge can be appointed in a following manner

$$S_{a_r}^{Y_{ik}(r)} = a_r \left[\frac{\underset{Z}{\mathrm{Sim}}\left({}^2A_{a_r a_k}, {}^2A_{a_r a_i}\right)}{\underset{Z}{\mathrm{Sim}}\left({}^2A_{a_i}, {}^2A_{a_k}\right)} - \frac{\underset{Z}{\det}{}^2 A_{a_r}}{\underset{Z}{\det}{}^2 A} \right] = \frac{\underset{Z}{\det}{}^2 A^{a_r}}{\underset{Z}{\det}{}^2 A} - \frac{\underset{Z}{\mathrm{Sim}}\left({}^2A^{a_r}_{a_i}, {}^2A^{a_r}_{a_i}\right)}{\underset{Z}{\mathrm{Sim}}\left({}^2A_{a_k}, {}^2A_{a_i}\right)} \tag{5}$$

In case of direct flexibility $Y_k(r)$ the formula number (5) can be wrote as

$$S_{a_r}^{Y_k(r)} = a_r \left[\frac{\underset{Z}{\det}{}^2 A_{a_r a_k}}{\underset{Z}{\det}{}^2 A_{a_k}} - \frac{\underset{Z}{\det}{}^2 A_{a_r}}{\underset{Z}{\det}{}^2 A} \right] = \frac{\underset{Z}{\det}{}^2 A^{a_r}}{\underset{Z}{\det}{}^2 A} - \frac{\underset{Z}{\det}{}^2 A^{a_r}_{a_k}}{\underset{Z}{\det}{}^2 A_{a_k}}. \tag{6}$$

The appointed function of sensitivity (6) is equal to the parametric function of sensitivity

$$S_{a_r}^{Y_{ik}(p)} = S_{S_r}^{Y_{ik}(p)},\; a_r = a_r(\alpha_r), \tag{7}$$

where: α_r is an inertial or efficient parameter.

If in result of synthesis of demands using method of dynamical characteristic into continued fraction [4,5,8] presented above, the polar graph and of parameters of its edges drawing is obtained (comp. [9]). Than functions (5) and (6) are essential for research of sensitivity of continuous bar systems represented by graphs and structural numbers as models of discrete systems.

The formulas (1÷6) and formulas defined in [3,4] are a base for the computer aided investigation of synthesized vibrating bar-systems as the task of synthesis of transformed immobility functions by the method of distribution of the dynamical characteristics into partial fraction.

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