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Selected elements of dynamical model of cardiosurgical robot

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Simulative research of dynamics of surgical robot, considering the set of detailed properties, influencing precision of control, may considerably improve the operation of robot. The paper presents model assumptions, general description of mathematical model and detailed information, concerning models of selected elements of robot.

1. INTRODUCTION

Surgical robotics, enabling precise operations in the minimally invasive manner, decreasing patient's pain, convalescence time and possibility of complications, is fast developing and very promising branch of medical technology.

Formulation of mathematical model of cardio-surgical robot and elaboration of computer software, enabling simulation of displacements, forces and stresses, occurring in the elements of robot in different operational conditions, may facilitate analysis of control errors, and improve precision of robot operation, by modernisation of design and control parameters[1].

The model may serve especially for analysis of motions with considerable velocities and accelerations, such as semi-automatic sewing motion or automatic transportation movement, stabilising the relative position of tool towards the surface of beating heart.

2. ASSUMPTIONS OF PHYSICAL MODEL OF ROBOT

Basic kinematic and inertial properties, as well as the set of complex, non-linear secondary phenomena, influencing precision of control, connected with elasticity, clearances, servo-motors characteristics, friction and internal damping, are considered in the model of robot dynamics :

- Robot is treated as the spatial system of links, connected by rotary or slidable constraints, equipped with the set of remotely controlled electric servo-motors, with gear, load screw or strand drive transmission[2].
- Basic structural links of robot are treated, depending on their stiffness, or as massive solid bodies, with slightly simplified geometry, or as massive elastic beams, modelled as a series of elastic elements, twisted and two-dimensionally bend,. loaded not only by external forces, and reactions, caused by neighbouring links, but also by the gravity and inertial forces, connected with mass particles, substituting the real masses of beam segments.

Internal damping of vibrations is simulated by introduction of internal friction forces, depending on the velocity of changes of elastic forces, proportional to beam deformations.

- The models of drives consider the characteristics of control loop and electric servo-motor, binding generated moment and rotational speed with the load and with control signal, introduced by the operator or by the system, controlling semi-automatic motion.
- The models of drive transmission regard the transmission ratio, frictional resistance to the motion, elongation of the strands and necessary preliminary strain, and clearances in the gear or load screw, delaying motion transmission, when direction of motion is changing.
- The models of rotary constraints simulate the frictional moment, with stopping relative motion of links, when forcing moment is less than maximal possible friction, and clearances in bearings, with shifting the pivot in the direction of resultant reaction.

3. GENERAL DESCRIPTION OF THE MATHEMATICAL MODEL

Simulation of robot dynamics is performed in successive time steps. Every step commences by preliminary determination of the servo-motors displacements, resulting from actual control signals, and from load determined in preceding time step. Then the changed positions of links, induced by the servo-motors action, are computed. Velocities and accelerations of all nodes are calculated numerically, using differences of their positions in preceding time steps. The reactions of constraints and servo-motors loads are determined from dynamic equilibrium equations, including inertial forces, resulting from calculated accelerations. Then the distortions of elastic elements, and displacements connected with clearances etc. are computed.

When the set of links, such as parallelogram, isn't constituting the open kinematic chain, then the reactions couldn't be calculated by the summation of active forces, performed from the end of the system, but must be determined by iterative solution of the set of equations, including not only equilibrium conditions, but also requirements of equity of displacements of specified links, resulting from clearances and distortions of elastic elements.

The iterative correction of the step is performed, when servo-motors characteristics show significant impact of their load on generated displacements, or when the changes of elastic distortions, or displacements connected with clearances are significant in one step.

Simulation of dynamics is preceded by determination of the static position of robot links, with iterative calculation of reactions and distortions resulting from clearances and elasticity.

Gravity and inertial forces, reactions of constraints and positions of robot links are calculated in the global co-ordinate system, joined with the fixed basis, and in local co-ordinate systems, joined with individual links, in unformed shape (in the case of elastic links).

The design of robot, which main links have the form of beams, rotationally or slidably connected with their neighbours, enables significant simplification of description of mutual position of links. It may be admitted, that greater relative rotation of both links and their local co-ordinate systems occurs only around one axis, and other revolutions are very small.

Relative displacements of links result from geometry of constraints, with considering clearances and elasticity, and from characteristics of drives. Geometrical co-ordinates (x_i, y_i, z_i) in the system, joined with the link 'i' , are expressed in the system joined with the preceding link 'i-1' ($x_{i,i,-1}$, $y_{i,i-1}$, $z_{i,i-1}$), and in the global co-ordinate system ($x_{i,0}$ $y_{i,0}$, $z_{i,0}$), by the appropriate transient equations. Similar transfer equations bind components of velocities, accelerations and forces in different co-ordinate systems [2].

$$\begin{bmatrix} x_{i,i-1} \\ y_{i,i-1} \\ z_{i,i-1} \end{bmatrix} = \mathbf{A}_{i,i-1} \cdot \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \mathbf{B}_i \; ; \quad \begin{bmatrix} x_{i,0} \\ y_{i,0} \\ z_{i,0} \end{bmatrix} = \mathbf{A}_{1,0} \cdot \left(\mathbf{A}_{2,1} \cdot \ldots \cdot \left(\mathbf{A}_{i,i-1} \cdot \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \mathbf{B}_i \right) + \ldots + \mathbf{B}_2 \right) \tag{1}$$

Detailed form of transient matrix $\mathbf{A}_{i,k}$ and vector $\mathbf{B}_i$ depends on the way of co-operation of adjacent links. Presented exemplary relation, connecting geometric co-ordinates, is valid when origins of co-ordinate systems are placed in centres of frontal sets of bearings, longitudinal axes of links are marked as 'z', revolution of link 'i' in relation to 'i-1', forced by servo-motor, proceeds around axis x_i (φ_{serv}), and when regarded are displacements of origin and additional minor rotations of axes, described in system 'i-1', resulting from clearances in bearings ($\Delta x_{b,i-1}$, $\Delta y_{b,i-1}$, $\Delta z_{b,i-1}$, $\Delta\psi_{b,i-1}$, $\Delta\eta_{b,i-1}$), bending deflection of elastic element 'i-1' ($\Delta x_{e,i-1}$, $\Delta y_{e,i-1}$), and angles of its elastic bending and torsion ($\Delta\varphi_{e,i-1}$=-d(Δy_e)/d$z_{i-1,L}$, $\Delta\psi_{e,i-1}$ =d(Δx_e)/d$z_{i-1,L,}$, $\Delta\eta_{e,,i-1}$):

$$\mathbf{A}_{i,i-1} = \begin{bmatrix} 1 & -d_{xy} & d_{xz} \\ \Delta\eta & c_\varphi & -s_\varphi \\ -\Delta\psi & s_\varphi & c_\varphi \end{bmatrix}; \quad \mathbf{B}_i = \begin{bmatrix} \Delta x_{b,i-1} + \Delta x_{e,i-1} \\ \Delta y_{b,i-1} + \Delta y_{e,i-1} \\ \Delta z_{b,i-1} + L_{i-1} \end{bmatrix}$$

where : $c_\phi = \cos\left(\varphi_{serv} + \Delta\varphi_{e,i-1}\right)$; $d_{xy} = s_\phi \Delta\psi - c_\phi \Delta\eta$; L_{i-1} – length of link 'i – 1'

$s_\phi = \sin\left(\varphi_{serv} + \Delta\varphi_{e,i-1}\right)$; $d_{xz} = c_\phi \Delta\psi + s_\phi \Delta\eta$;

$$\Delta\psi = \Delta\psi_{b,i-1} + \Delta\psi_{e,i-1}; \quad \Delta\eta = \Delta\eta_{b,i-1} + \Delta\eta_{e,i-1}. \tag{2}$$

4. MODELS OF SELECTED ELEMENTS OF ROBOT

Consideration of phenomena, connected with characteristics of servo-motors, clearances, elasticity, friction and internal damping, requires introduction of specialised models of a set of elements of robot. Models of two selected elements are presented in further part of the paper.

The model of rotary constraint, joining the links with clearances and friction, and having the form of pivot with two bearings, is based on the assumption, that clearances have the form of radial or axial gap ($\Delta_{b,r}$, $\Delta_{b,a}$), and that the pivot is permanently shifted towards bearing raceway in the direction of resultant reaction. This shift causes some relative rotation of axes and displacement of origins of local co-ordinate systems of both co-operating links. Scheme of the set of links 'i' and 'i-1', with radial clearance, rotating around axis 'y' is presented on fig.1.

Components of relative displacements of pivot in bearings (δ_x, δ_z), displacement of origin of local co-ordinate system of link 'i' (Δx_b, Δy_b, Δz_b) and additional angles of axes rotation ($\Delta\psi_b$, $\Delta\eta_b$) are determined as functions of reaction in preceding time step, with possible iterative correction. In the co-ordinate system of link 'i-1', they are described by the formulas :

$$\delta_{x,l} = -\Delta_{b,r} \cdot R_{x,l} / \sqrt{R_{x,l}^2 + R_{z,l}^2}\,; \quad \delta_{z,l} = -\Delta_{b,r} \cdot R_{z,l} / \sqrt{R_{x,l}^2 + R_{z,l}^2}\,; \quad l = 1,2\,;$$

$$\Delta x_{b,i-1} = (\delta_{x1} + \delta_{x2}) / 2; \quad \Delta z_{b,i-1} = (\delta_{z1} + \delta_{z2}) / 2; \quad \Delta y_{b,i-1} = -\Delta_{b,a} \cdot \mathrm{sign}\,(R_y)\,;$$

$\Delta\varphi_{b,i-1} = (\delta_{z1} - \delta_{z2}) / L_p ; \quad \Delta\eta_{b,i-1} = (\delta_{x2} - \delta_{x1}) / L_{piv} ;$ (3)

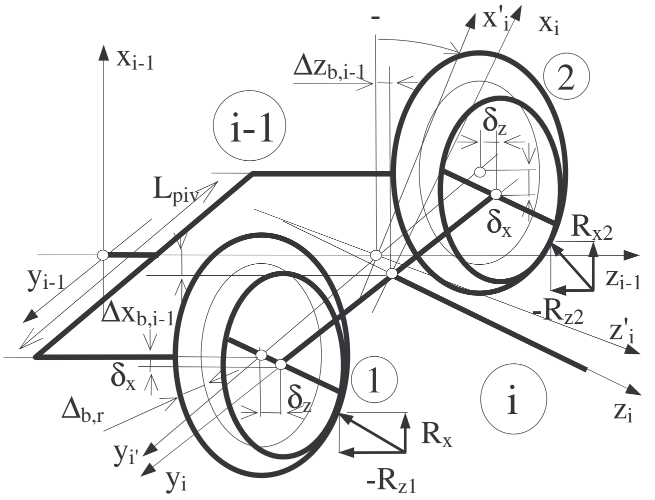

Fig.1.Scheme of radial clearances in rotational joint of links.

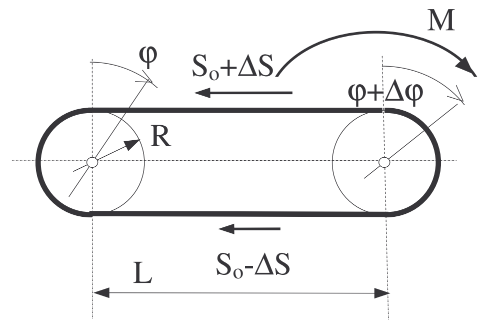

Fig.2.Scheme of deviation of rotation angles in the strand drive transmission unit.

Maximal frictional moment in the bearing, counteracting revolution, is proportional to the resulting reaction. If this value exceeds actual forcing moment, then friction is adequately decreased, and relative rotation of links is ceased.

Elongation of strands is regarded in the analysis of strand drive transmission. It causes some deviation between the angles of rotation of driving and driven pulleys, and instantaneous stay of driven pulley, when the direction of forcing moment changes, and its value is too low for overcoming frictional resistance. For the strand unit, having length L, pulley radius R, cross section of strand A and elasticity modulus E, the deviation of rotation angles $\Delta\varphi$ during transmission of moment M is obtained by analysis of strand forces S and elongations $\Delta\varphi R$:

$\Delta S/EA = \Delta\varphi R/L; \quad M = 2\Delta S R; \quad \Delta\varphi = M\, L / 2\, R^2\, E\, A$ (4)

Appropriate initial tension of strand S_o is necessary for protection of the unit against possible sliding of the strand around the pulley. Initial tension increases reactions in bearings and frictional resistance, and generates the moments bending the link, when the pulley position is asymmetric. Minimal value of initial tension depends on the coefficient of friction between strand and pulley μ, on the wrapping angle of the strand around pulley α ($\approx\pi$) and on maximal transmitted moment M_{max}. It is described by the relation, derived from Euler's equation:

$(S_o+\Delta S)/(S_o-\Delta S) > e^{\alpha\mu}; \quad S_o > (M_{max} / 2\, R) \bullet (e^{\alpha\mu} +1) / (e^{\alpha\mu} -1)$ (5)

The other prepared models of robot elements include, among others, the models of massive elastic beam, parallelogram, load screw, servo-motor with gear, and control blocks.

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