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# An application of arbitrary Lagrangian-Eulerian method in numerical simulation of forming processes using cap plasticity model

A.R. Khoei, M. Anahid, M. Mofid

Department of Civil Engineering, Sharif University of Technology,
P.O. Box 11365-9313, Tehran, Iran

The arbitrary Lagrangian-Eulerian method is nowdays very useful in nonlinear problems due to its capability to use advantageous of both Lagrangian and Eulerian approaches. In this paper, an ALE technique is developed based on the cone-cap plasticity model in simulation of forming processes. The ALE formulation and its FE discretization are described. Numerical example is provided to illustrate that the ALE method can be effectively used to solve problems which will encounter some difficulties in Lagrangian and Eulerian formulation.

## 1. INTRODUCTION

In numerical simulation of forming processes, there are two approaches to solve problems, Lagrangian and Eulerian methods. In Lagrangian approach, the mesh nodes are implemented to material points, therefore this formulation is well suited for simulation of problems containing materials with path dependent behavior, and free surface motion [1]. However, the method suffers from numerical difficulties in problems with very large deformation, because the mesh may undergo severe distortion, due to this fact that elements are deformed with material deformation [2]. In Eulerian approach, the mesh is fixed in space and material flows through it, which is very suitable for problems with very large material deformation such as fluid dynamics [3]. But the very important disadvantageous of this method is that it is less suited for history dependent material and problems with free boundary motion. It can be concluded that neither Lagrangian nor Eulerian formulation alone is well suited for simulation of processes involving large deformation and free boundary motion.

The basic idea of ALE technique is to combine advantageous of both Lagrangian and Eulerian approaches. In this method, the reference configuration is used for describing the motion, instead of material configuration in Lagrangian, and spatial configuration in Eulerian formulation [4]. This formulation introduces some convective terms in the finite element equations and consist of two phases. In Lagrangian phase, the mesh and material movements are identical. In Eulerian phase, it is allowed the mesh to have an arbitrary motion, independent of material motion, keeping the mesh regular. In this study, each time step is analyzed according to Lagrangian phase until required convergence is attained. Then, the Eulerian phase is applied to keep mesh configuration regular. Because of relative displacement between mesh and material, all dependent variables such as stress and strain are convected through the Eulerian phase.

## 2. ALE FORMULATION

In the ALE formulation there are three domains; material domain $\Omega_0$ , spatial domain $\Omega$, and reference domain $\hat{\Omega}$, which is also called ALE domain. These domains are mapped by transformation equations

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t) \ , \quad \mathbf{x} = \mathbf{x}(\chi, t) \tag{1}$$

which give the spatial position of material point **X** and grid point $\boldsymbol{\chi}$, respectively. The material velocity $\mathbf{v}$ and mesh velocity $\hat{\mathbf{v}}$ are time derivatives of **x**, in which **X** or $\boldsymbol{\chi}$ are fixed, respectively. Convective velocity is defined to be the relative motion between the material and mesh as

$$\mathbf{c} = \mathbf{v} - \hat{\mathbf{v}} \tag{2}$$

Similarly acceleration **a** can be expressed as

$$\mathbf{a} = \frac{\partial \mathbf{v}}{\partial t}\Big|_X = \frac{\partial \mathbf{v}}{\partial t}\Big|_\chi + \mathbf{c}\,\frac{\partial \mathbf{v}}{\partial x} \tag{3}$$

The first term in right hand side of equation (3) is the local acceleration and the second term is called convective acceleration.

### 2.1. Momentum equation

In the ALE formulation, the momentum equation is obtained by substituting **a** from equation (3) into common momentum equation in Lagrangian formulation which leads to

$$\sigma_{ji,j} + \rho b_i = \rho a_i = \rho \frac{\partial v_i}{\partial t}\Big|_\chi + \rho\, c_j\, v_{i,j} \tag{4}$$

where $\rho$ is the density, $\boldsymbol{\sigma}$ the cauchy stress and **b** the body force. In forming processes, the loads are applied slowly and inertia forces are relatively small in comparison to other terms in equation (4), thus acceleration may be omitted and the balance equation can be written as

$$\sigma_{ji,j} + \rho b_i = 0 \tag{5}$$

The above equation shows that there are no convective term in balance equation, which is common between the Lagrangian, Eulerian and ALE formulations.

### 2.2. Constitutive equations

In nonlinear solid mechanics, the material rate of stress $\frac{\partial \sigma}{\partial t}\Big|_X$ is usually related to the deformation history and current state of stress as

$$\frac{\partial \sigma}{\partial t}\Big|_X = f(\sigma, \boldsymbol{u}) \tag{6}$$

In order to obtain the constitutive equations in ALE formulation, the material rate of stress may be related to reference rate of stress by adding a convective term accounting for relative motion between material point and grid point as

$$\frac{\partial \sigma}{\partial t}\Big|_X = \frac{\partial \sigma}{\partial t}\Big|_\chi + c\,\frac{\partial \sigma}{\partial x} \tag{7}$$

## 3. MESH MOTION

In the ALE analysis, there are two sets of motion, the material motion $\boldsymbol{v}$ and mesh motion $\hat{\boldsymbol{v}}$. Although mesh motion in general is independent from material motion, the boundary of ALE and material domains should be coincide i.e. at each boundary point

$$(\boldsymbol{v} - \hat{\boldsymbol{v}})\ \mathbf{n} = 0 \tag{8}$$

where n is the normal vector of boundary points. In addition, the motion of interior grid and material points, is related by a linear function as

$$\hat{\boldsymbol{v}} = \mathbf{a} + \mathbf{B}\,\boldsymbol{v} \tag{9}$$

where a and B are the vector and matrix of constant scalars, respectively. It must be noted that the Lagrangian motion is a special case of above scheme. Considering $\mathbf{a} = \{0\}$ and $\mathbf{B} = \mathbf{I}$, with I denoting the identity matrix, it yields to $\boldsymbol{v} = \hat{\boldsymbol{v}}$. The Eulerian motion can be also obtained by assigning $\mathbf{a} = \{\mathbf{0}\}$ and $\mathbf{B} = [\mathbf{0}]$, which yields to $\hat{\boldsymbol{v}} = 0$.

## 4. TANGENTIAL STIFFNESS MATRIX

The derivation of the tangent stiffness matrix for the ALE description can be obtained by adding the transport terms to general Lagrangian tangential stiffness as

$$\mathrm{d}\mathbf{f}^{\,\mathrm{int}} = \mathbf{K}^{\,\mathrm{lag}}\ \mathbf{v} + \mathbf{K}^{\,\mathrm{ale}}\ \mathbf{c} \tag{10}$$

where $\mathrm{d}\mathbf{f}^{\,\mathrm{int}}$ is the rate of internal nodal forces and $\mathbf{K}^{\,\mathrm{lag}}$ is the stiffness matrix in Lagrangian formulation, which consists of both geometrical and material stiffness. $\mathbf{K}^{\,\mathrm{ale}}$ denotes the ALE stiffness matrix, in contrast to $\mathbf{K}^{\,\mathrm{lag}}$, is a non-symmetrical matrix. In addition, the gradients of the stress may be appeared in $\mathbf{K}^{\,\mathrm{ale}}$ whereas it doesn't exist in $\mathbf{K}^{\,\mathrm{lag}}$.

## 5. NUMERICAL SIMULATION RESULTS

In order to illustrate the applicability of the proposed model, the simulation of a coining problem is analyzed numerically. A rectangular plate with the width of 60 mm and height of 12.5 mm is deformed by a rigid frictionless punch, as shown in Figure 1. The von-Mises elasto-plastic model is employed with the Young modulus of 210 GPa, Poisson ratio of 0.3, yield stress of 240 MPa and hardening parameter of 1 GPa. A plane strain analysis is performed with applying prescribed boundary displacement at the nodes under the punch to model a 40% height reduction.

The simulation of this problem using the Lagrangian formulation will pose two difficulties. First, applying prescribed nodal displacement at the relevant nodes results to unrealistic analysis due to increase of the width of punch and thus, the lateral displacement of material points. This error may be overcome by using a contact algorithm. The second difficulty is that the Lagrangian approach leads to highly distorted mesh under the punch. The application of contact algorithm into the Lagrangian formulation causes mesh distortion and convergence problems, which yields to fluctuation in force-displacement diagram.

The ALE method will overcome the above difficulties. Figure 2 shows the deformed mesh after 5 mm reduction in height using ALE approach. Also plotted in this figure is the variation of effective plastic strain for the coining test. The predicted punch force versus displacement is presented in Figure 3. As can be observed, the fluctuation does not happen in this curve.

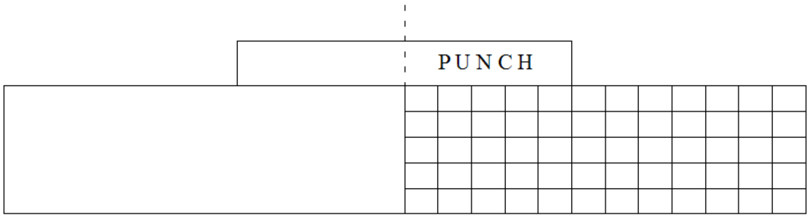
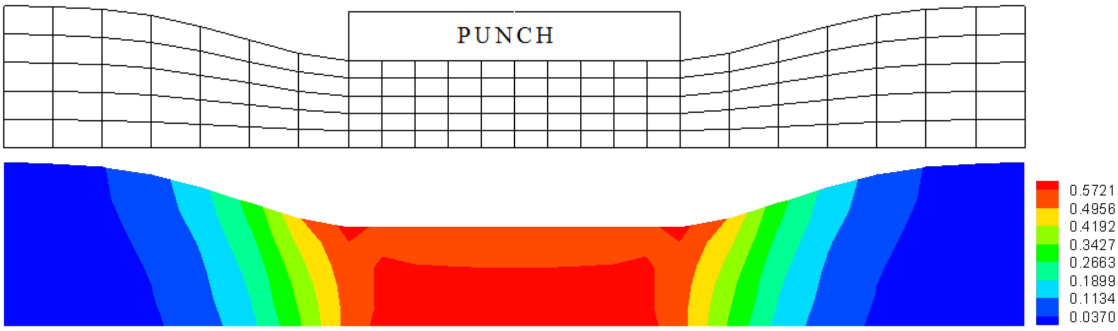



Figure 1. Coinig problem; Finite element mesh

(a)
(b)



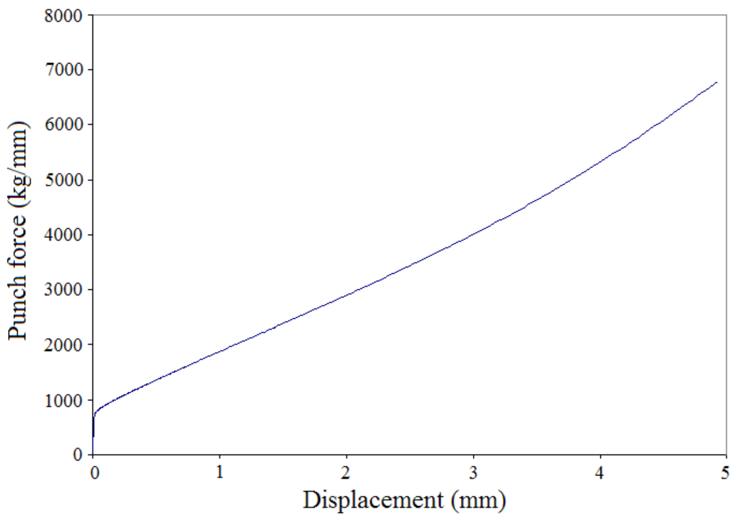




Figure 3. The punch force versus displacement curve

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