Hybrid graphs in modelling and analysis of discrete – continuous mechanical systems

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Abstract: The scope of the paper is a method of modelling vibrating mechanical systems with discrete-continuous distribution of parameters and couplings by means of hybrid graphs. The method is focused on broadening hybrid graphs formalism by mechanical systems and improving their description and design methods so that the mathematical formalism can reflect the essence of the problem involved in the designation of dynamic characteristics of such systems.

Keywords: Hybrid graph, Matrix flow graph

1. INTRODUCTION

In view of growing complexity in the structure and functionality of mechanical objects, classic mathematical methods applied to their analysis often encounter difficulty in creating and solving appropriate systems of differential equations of motion. An analytical solution of differential equations constituting a mathematical model, labeled as "classic methods" in literature, may turn to be time consuming or even impossible. For the construction of a physical, and, subsequently, mathematical model of the analyzed mechanical system, comprehensive knowledge about its properties and the processes and relations occurring within this system is absolutely essential. This knowledge should be substantiated by the laws of physics that describe the behavior of the system. When the analyzed system consists of many subsystems, the designation of dynamic characteristics involves many time and energy consuming activities. Also, every instance of modifying the structure of the system entails a new formulation of the differential equations of motion, or the construction of rigidity matrices and damping matrices anew. In such cases it is the network methods, labeled as "non-classic methods" in literature that turn useful, as they have a high degree of algorithmization, which facilitates their implementation to computer-aided calculation systems, especially as far as the designation of dynamic characteristics is concerned. Moreover, thanks to the network methods it is possible to represent the structure of the modeled system graphically. The graph and structural numbers methods applied in mechanics are all part the network methods. The research centre in Gliwice has contributed to the introduction and further development of the graph and structural numbers methods. The interest in applying the graph theory to the analysis of physical systems stems from the

following properties of graphs as models of physical systems: clarity, traceability of direct changes introduced to the systems without the need of formulating new differential equations of motion, visualization of the changes in the structure of the system, every transformation of the graph corresponds to applicable "visualization of the model", isomorphism between the structure of the model and its representation in the form of a graph, instant correction of errors in modelling, open nature of the analysis and synthesis of mechanical systems which facilitates the concurrent use of other non-classic or classic methods. The following classes of graphs are used in the graph methods: polar graphs, flow graphs, hybrid graphs, matrix hybrid graphs, hypergraphs [1, 2, 3, 4, 5].

2. OBJECT OF THE RESEARCH

The focus of the discussion is a phenomenological model of a discrete-continuous planar mechanical system.

To obtain the structure of a graph that represents the model of a system with discrete-continuous distribution of parameters the following transformation have been made [5]:

- origin of the inertial frame of reference 0(0,0,0) (equated to the generalized coordinate $^{1}s_{0}$) transformed into the reference vertex $^{1}x_{0}$ of the graph,

- generalized coordinates of the continuous subsystem at coincidence points with the discrete system into passive principal vertices of the graph,

- generalized coordinates of the discrete system into passive principal vertices of the graph,

- displacements generated by the source of kinematical excitations into active principal vertices of the graph,

- representation of the coupling points between elements c, b and the continuous subsystem and representation of the passive elements of the discrete system into passive principal vertices of the graph,

- Values of the displacements corresponding to force excitations into active principal vertices of the graph,

- susceptibility of the continuous subsystem at coupling points with the discrete subsystem into passive branches of the graph,

- values of the forces and values of the moments of passive forces into passive branches of the graph,

- values of the forces and torques of the elements of type c and b into passive principal branches of the graph,

- values of the forces and torques corresponding to the kinematical excitations into active branches of the graph,

- values of forces generated by the dynamic excitations into active branches of the graph,

- coupling relations of the system into the coupling edges of the graph.

In accordance with the sequence of above transformations the graph of the dynamic structure of the system is derived

$$X^S = \left\langle \left\{ {}_1 X \right\}, \left\{ {}_2 X \right\}, \left\{ {}_3 X \right\}, \left\{ F \right\} \right\rangle. \tag{1}$$

To obtain the loaded structure of a graph that represents the model of a system with discrete-continuous distribution of parameters the following transformations have been made:

- to vertex ${}_1 x_0$ – the zero value of the beginning of frame of reference,

- passive principal vertices of the graph ${}_1 x_i$ – generalized coordinates of the continuous subsystem at coincidence points with the discrete system,

- passive principal vertices of the graph – generalized coordinates of the discrete system,

- active principal vertices ${}_1 x_i$ – displacements generated by the source of kinematical excitations,

- passive principal vertices ${}_1 x_i$ – representation of the coupling points between elements c, b and the continuous subsystem and representation of the passive elements of the discrete system,

- active principal vertices – values of the displacements corresponding to force excitations,

- passive branches of the graph ${}_2 x_i$ – susceptibility of the continuous subsystem at coupling points with the discrete subsystem,

- passive branches of the graph ${}_2 x_i$ – values of the forces and values of the moments of passive forces of the discrete subsystem,

- active branches of the graph ${}_2 x_j$ – values of the forces and torques corresponding to the kinematical excitations,

- passive principal branches of the graph ${}_2 x_k$ – values of the forces and torques of the elements of type c and b,

- active branches of the graph ${}_2 x_k$ – values of forces generated by the dynamic excitations,

- coupling edges $_2 x_k$ – coupling relations.
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The hybrid graph as a model of a discrete-continuous mechanical system with couplings is obtained in the course of the above representations and transformations. The hybrid graph contains all information about the physical and geometrical parameters of the investigated system.

3. CONCLUSION

The representation of a discrete-continuous system as a hybrid graph constitutes the bases for further dynamic analysis of this system. Accordingly, on the bases of the information on the dynamic characteristics of the system for any inputs or outputs in the form of kinematical and dynamic excitations it is possible to designate the frequency characteristics for the discussed class of systems.

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