



## The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs

A. Buchacz

Silesian University of Technology, Mechanical Engineering Faculty,  
Department of Engineering Processes Automation and Integrated Manufacturing  
Systems, Konarskiego 18A, 44-100 Gliwice, Poland

**Abstract:** In the paper the problem of synthesis has been solved and its base has been formalized as far as the vibration discrete mechanical systems of the cascade structures graphs is concerned. In connection with this the basic field, the ideas connected with the class of considered structures and used graphs has been given. The manner of getting the “new” structures as a result of synthesis of the characteristics by means the continued fraction expansion method has been presented in farther part of this paper.

**Keywords:** Dynamical characteristic, “New” cascade structures

### 1. CONTINUED FRACTION EXPANSION METHOD OF THE SYNTHESIS OF THE CHARACTERISTICS

In [1-6] the continued fraction expansion method of chosen characteristics has been presented and which were used to continuous system, when the level of their numerator was higher than the level of denominator. However in this paper according to other cases the problem has been just signalized. In this paper the realization of the characteristics has been presented, proposed by synthesis method when the level of its denominator is higher than its level of numerator. This concern also synthesis of mobility function  $V(r)$  and immobility function  $U(r)$  (e.g. [1,3]).

Next case has been solved as a reverse of mobility, that means immobility, which is the dynamic characteristic in form

$$U(r) = \frac{d_{k-1}r^{k-1} + d_{k-3}r^{k-3} + \dots + d_1r}{c_k r^k + c_{k-2}r^{k-2} + \dots + c_0}, \quad (1)$$

where  $k$  – even natural number.

This kind of disposition is being made by the lowest cubes of variable  $r$ . In this case the nominator and the denominator has been divided into  $r$  in the highest potency

$$U(r) = \frac{d_{k-1} \frac{r^{k-1}}{r^k} + d_{k-3} \frac{r^{k-3}}{r^k} + \dots + d_1 \frac{r}{r^k}}{c_k \frac{r^k}{r^k} + c_{k-2} \frac{r^{k-2}}{r^k} + \dots + c_0 \frac{1}{r^k}}. \quad (2)$$

Transforming the equation (2) and introducing the new variable  $r' = \frac{1}{r}$ , we get

$$U(r') = \frac{d_{k-1} \frac{1}{r'} + d_{k-3} \frac{1}{r'^3} + \dots + d_1 \frac{1}{r'^{k-1}}}{c_k + c_{k-2} \frac{1}{r'^2} + \dots + c_0 \frac{1}{r'^k}}, \quad (3)$$

and at last

$$U(r') = \frac{d_{k-1} r' + d_{k-3} r'^3 + \dots + d_1 r'^{k-1}}{c_k + c_{k-2} r'^2 + \dots + c_0 r'^k}. \quad (4)$$

Considering that in (4) the level of nominator is lower than the level of denominator which means that the reverse argument and function has to be synthesized  $r'$ , that means

$$V(r') = \frac{c_k r'^k + c_{k-2} r'^{k-2} + \dots + c_0}{d_{k-1} r'^{k-1} + d_{k-3} r'^{k-3} + \dots + d_1 r'}. \quad (5)$$

Synthesizing function (5) by means the continued fraction expansion method, presented in this paper, we receive in this case

$$\begin{aligned} V(r') &= V_r^{(1)}(r') + \frac{1}{U_z^{(2)}(r') + \frac{1}{V_r^{(3)}(r') + \frac{1}{U_z^{(4)}(r') + \dots}}}} = \\ &= \frac{r'}{m_z^{(1)}} + \frac{1}{c_r^{(2)} r' + \frac{1}{\frac{r'}{m_z^{(3)}} + \frac{1}{c_z^{(4)} r' + \dots}}}} + \frac{1}{V_r^{(k-1)}(r') + \frac{1}{U_z^{(k)}(r')}}} \\ &= \frac{r'}{m_z^{(1)}} + \frac{1}{c_r^{(2)} r' + \frac{1}{\frac{r'}{m_z^{(3)}} + \frac{1}{c_z^{(4)} r' + \dots}}}} + \frac{1}{\frac{r'}{m_z^{(k-1)}} + c_r^{(k)} r'}. \end{aligned} \quad (6)$$

Making the retransformation of an argument,  $r' \rightarrow \frac{1}{r}$  we get

$$V(r) = \frac{1}{m_z^{(1)} r} + \frac{1}{\frac{c_r^{(2)}}{r} + \frac{1}{\frac{1}{m_z^{(3)} r} + \frac{1}{\frac{c_r^{(4)}}{r} + \dots} + \frac{1}{\frac{1}{m_z^{(k-1)} r} + \frac{c_r^{(k)}}{r}}}} \tag{7}$$

The form (7) corresponds with mobility function (1) in form of a polar graph  $X_{00}$  [1-7] (see Fig. 1). The mobility determined at the point indicated by the arrow is identical with (1). This graph is a model of discrete system in Fig. 2.

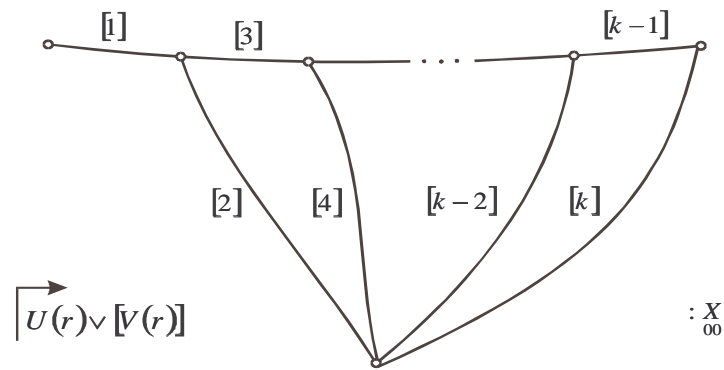


Figure 1. Graphical illustration of equation (7)

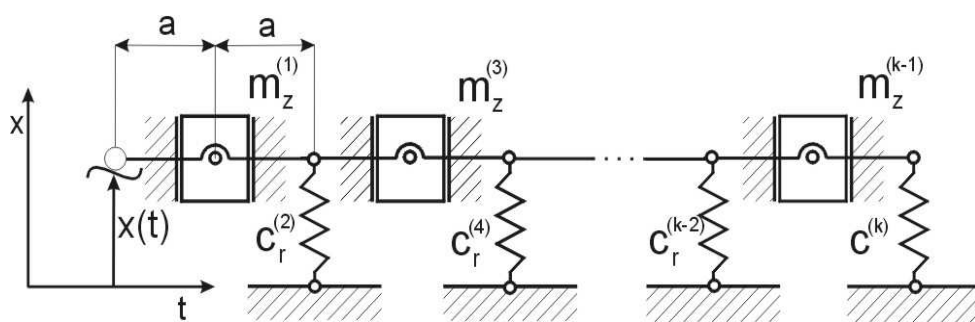


Figure 2. The model of the discrete vibration system as an implementation of the polar graph (Fig. 1)

For some edges it has to be qualify to dynamic structure along with order, which result from order of implementation the characteristic, the synthesized method of in immobility  $U(r')$  and with mobility  $V(r')$  on continued fraction.

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