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An analytical incremental model for the analysis of the cup drawing

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Analysis and modelling

ABSTRACT

Purpose: of this paper Develop an analytical model for the cup drawing process to solve for the induced stresses and strains over the deforming sheet at any stage of deformation until a full cup is formed.
Design/methodology/approach: An analytical model is developed for the cup drawing process by determining the variation of stresses and strains over the deforming sheet. The model uses finite difference approach and numerical procedures to solve for equilibrium, continuity, and plasticity equations in an incremental fashion.
Findings: The developed analytical model results showed good correlation with experimental ones from the literature. Also, the analytical model was found to be useful in conducting parametric studies in order to determine how the different process parameters can affect the deforming cup.
Research limitations/implications: This paper includes the development of an analytical model to analyze the deep drawing of axisymmetric cups. This model is then used as the solution engine for the optimization of the blank holder force for such cups avoiding failure by wrinkling or tearing. This model also gives an insight of the modes of deformation in the deep drawing process.
Practical implications: This paper is part of a procedure that leads to the optimization of the blank holder loading scheme. The full procedure as presented in the two parts of the work may be applied in industry to minimize the maximum punch load or the work done during deep drawing process by modifying the blank holder force and at the same time avoid failures by wrinkling or tearing.
Originality/value: Developing a predictive/corrective technique for solving the unknown boundaries of the deforming sheet.
Keywords: Numerical techniques; Computational mechanics; Plastic forming; Cup drawing

1. Introduction

The cup drawing is a basic deep drawing process. Thus, understanding the mechanics of the cup drawing process helps in determining the general parameters that affect the deep drawing process. There are mainly two methods of analysis; experimental and analytical/numerical. Experimental analysis can be useful in analyzing the process to determine the process parameters that produce a defect free product. However, experimental work is usually very expensive and time consuming to perform. On the other hand, the Analytical/Numerical modeling can be used to model and analyze the process through all stages of deformation. This approach is less time consuming and more economical than experimental analysis.

There have been several efforts to solve and analyze the deep drawing problem [1-11]. Among these are the attempts to analyze the cup drawing process, which include the works of Chung and Swift [1], Woo [2-4] and Reissner and Ehrismann [5]. They developed analytical models for the cup drawing process to solve for stresses and strains over the deforming sheet metal. However, they did not explain how to determine the moving boundaries in the deforming sheet, which will be discussed later in this study.

In the present study, an incremental Analytical/Numerical modeling approach is developed. The model analyzes the stresses and strains in the cup drawing process. It is established on the solution of force equilibrium and plasticity relations using the finite difference method. The solution is obtained in an incremental fashion in order to account for the moving and variable boundary conditions that are not known a priori. More details of this technique can be found in [12].

2. Theoretical analysis and modeling of the cup drawing process

2.1. Analytical theory

Regions of the Deforming Sheet Metal

As shown in Fig. 1, the developed analytical model divides the deforming sheet into six regions from I to VI.

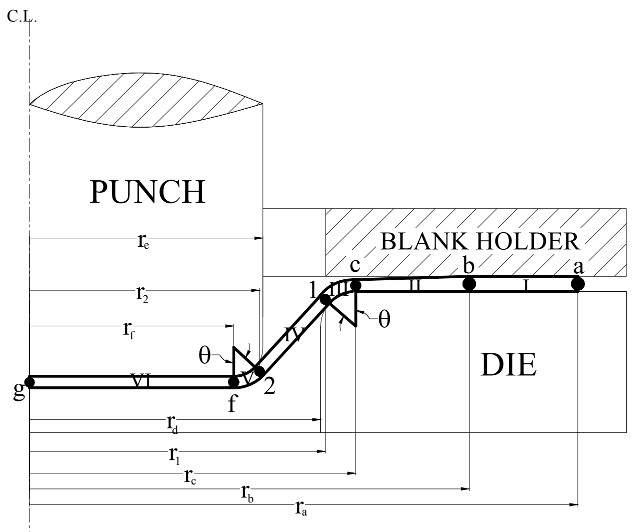

Fig. 1. Regions and boundaries in the deforming sheet

Analytical model assumptions and governing equations

The basic analytical theory used in the solution of the problem of deep-drawing a cylindrical cup is based mainly on the works of Chung and Swift [1], Woo [2-4], Reissner and Ehrismann [5], Al-Makky and Woo [6] and Kaftanoglu and Tekkaya [8]

The analytical model is established on the following assumptions: (1) elastic strains are neglected, since they are small compared with plastic strains, (2) isotropic, Von Mises material with non-linear strain-hardening is used, (3) radial (meridional), circumferential, and thickness directions are considered principal directions, (4) bending/unbending effects are neglected since their effect is negligible for a die profile radius to sheet thickness ratio greater than 6, (5) shear stress is neglected across the thickness, and (6) a straight cup wall is assumed.

The principal directions in the problem of deep-drawing a cylindrical cup are the radial (meridional), circumferential and thickness directions with r, θ, and t designations respectively.

The governing plasticity equations of the analytical model are as follows:

Effective Stress: For a material free from Bauschinger effects, Von Mises or effective stress is defined as follows:

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_t)^2 + (\sigma_t - \sigma_r)^2\right]} \tag{1}$$

Plastic Strains for the three principal directions; circumferential, thickness, and radial (meridional) directions can be expressed as:

$$\varepsilon_\theta = \ln\left(\frac{r}{R}\right) \tag{2}$$

$$\varepsilon_t = \ln\left(\frac{t}{t_o}\right) \tag{3}$$

$$\varepsilon_r = -\varepsilon_\theta - \varepsilon_t \tag{4}$$

Effective Strain: The effective incremental strain can be stated as:

$$d\bar{\varepsilon} = \sqrt{\frac{4}{3}\left[(d\varepsilon_\theta + d\varepsilon_t)^2 - d\varepsilon_\theta d\varepsilon_t\right]} \qquad \mathbf{(1)}$$

Stress-Strain Relationship: Based on Levy-Lode stress-strain relationship:

$$\sigma_\theta - \sigma_r = \frac{2}{3}\frac{\bar{\sigma}}{d\bar{\varepsilon}}(d\varepsilon_\theta - d\varepsilon_r) = \frac{2}{3}\frac{\bar{\sigma}}{d\bar{\varepsilon}}(2d\varepsilon_\theta + d\varepsilon_t) \qquad (2)$$

$$\sigma_t - \sigma_r = \frac{2}{3}\frac{\bar{\sigma}}{d\bar{\varepsilon}}(d\varepsilon_t - d\varepsilon_r) = \frac{2}{3}\frac{\bar{\sigma}}{d\bar{\varepsilon}}(2d\varepsilon_t + d\varepsilon_\theta) \qquad (3)$$

Flow Equation: The flow equation that describes the strain hardening of the material is the Ludwik-Hollomon power law which is given by:

$$\bar{\sigma} = C\bar{\varepsilon}^n \qquad (4)$$

Where, C = strength coefficient, n = strain hardening exponent.

Force equilibrium and volume constancy relationships are applied to a slab element in each of the six regions constituting the deformed sheet. The orientation, loading condition, and boundary conditions of each region is different. Thus, different equilibirum and continuity equations are deduced for each region [12].

Finite difference discretization

The objective of the analytical model is to evaluate the stresses and strains in the deforming sheet of the deep drawn cup. As indicated in

Fig. **2**, the analysis of the cup drawing process starts with a flat circular blank of initial radius R_a and thickness t_o. The finite difference solution divides the blank from rim to punch centerline into discrete number of points (*np*) designated with subscript (*j*). Each point in the initial circular blank has a radius R_j and marches in time for a finite number of stages (*ns*) designated with subscript (*i*). The time-like parameter used is governed by the incremental motion of the blank rim. At each stage of deformation, the outer rim has a radius $(r_a)_i$ and each *j* point on the sheet has a radius $r_{i,j}$ and thickness $t_{i,j}$.

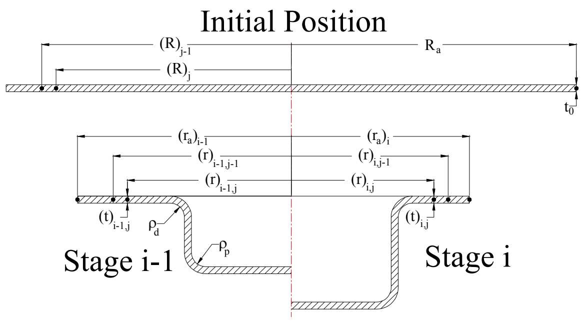

Fig. 2. Finite difference discretization of a cup

Boundary conditions

The solution for stresses and strains starts from the blank rim and ends at the punch centerline. It is important when solving for each region to determine its boundary conditions to initiate the solution. The problem of the cylindrical cup has a total of seven boundaries; four of them are moving boundaries through stages of deformation, while the other three have space fixed positions in all stages. Referring to Fig. 1, the four moving boundaries are (a), (b), (1), and (2), while the three fixed boundaries are (c), (f), and (g).

2.2. Numerical solution

The solution of the deep-drawing problem using finite difference requires iterating through different variables until convergence. This is achieved by satisfying equilibrium and continuity equations as well as the stress-strain relations subject to the given flow equation and boundary conditions.

The Numerical solution discusses the calculations of stresses and strains at each point in the different regions. Among the four moving boundaries, only two are determined in the numerical solution of region I, namely boundaries (a) and (b). However, the other two moving boundaries (1) and (2) are unknown *a priori*. Moving boundary (1) is important in order to determine the position at which calculations in region III end and those in region IV start. Also, moving boundary (2) determines the point at which calculations in region IV ends and those in region V start. The radial positions $(r_1)_i$ and $(r_2)_i$ of the moving boundaries (1) and (2) can be determined by knowing the value of the contact angle θ which is shown in Fig. 1. Since, the cup wall is assumed to be straight, it forms a tangent line between the die and punch profiles. Thus, the angle of contact between the sheet metal and the die profile or the punch profile has the same value θ.

However, there is no straightforward method for the determination of the contact angle θ [1,2,6,8]. Several attempts were carried out to develop a technique for the correct determination of the contact angle θ. This was found to be very crucial on the accuracy and convergence of the results for regions IV to VI.

In the present study, a new technique is developed for the determination of the contact angle θ at each stage of deformation. This technique follows a prediction/correction strategy.

Step 1: Prediction

In the prediction stage, a modification of Swift's approach [1] is adopted to obtain an approximate guess of the angle θ. Based on the geometry of the deforming cup at various stages, a set of three non-linear equations are solved to determine an approximate value for the angle θ.

Step 2: Correction

The experimental analysis carried out on the cup drawing process shows that circumferential and radial strains are usually equal at the punch bottom [1,13,14] . The analytical solution developed by Woo [4] suggested that for certain moving boundaries (1) and (2) locations, the monotonic solution from rim to boundary (f) is satisfactory if it gives $(\varepsilon_\theta = \varepsilon_r)_f$ at boundary (f). In the present study, the solution for $(\varepsilon_\theta = \varepsilon_r)_f$ is formulated as a minimization problem which searches for the angle θ that minimizes $(\varepsilon_\theta - \varepsilon_r)_f$. Therefore, once the initial guess of the contact angle θ is calculated from the prediction step, the following objective function is minimized by varying the value of θ through using one-dimensional search methods [15].

$$\Delta\varepsilon_f = (\varepsilon_\theta - \varepsilon_r)_f \qquad (5)$$

At each new search value for θ, evaluation of stresses and strains are performed for regions III to V to determine the value of the strains at radius r_f to satisfy equation (5). This solution approach for the determination of the contact angle was applied to several cups and was found to be successful for solving over all stages until a complete cup is formed.

2.3. Verification of the analytical model

The developed analytical model is verified by comparing its results with those of the experimental investigation carried out by Saran et al. [13], which has the folowing cup geometry, material properties, and loading conditions:

Geometry	Material: 70/30 Brass
to = 0.7*mm*	Based on the flow equation
R_a = 100*mm*	$\bar{\sigma} = C\bar{\varepsilon}^n$
ρ_d = 5*mm*	C = 895*MPa*
ρ_p = 13*mm*	n = 0.42
r_d = 51.25*mm*	E = 110*GPa*
r_e = 50*mm*	ν = 0.34
Loading	Density = 8470 Kg/m^3
F_{BH} = 100kN	
$\mu_{BH} = \mu_{DP} = \mu_{PP}$ 0.06	

The punch travel versus punch force curve, shown in fig. 3, shows very good correlation with the experimental results. A small deviation of about 5% from the experimental results is shown at the maximum punch force at a punch travel of 40mm.

The circumferential strain distributions are compared at a punch travel of 30mm. Both experimental and analytical results show good correlation and same trends as shown in fig. 4.

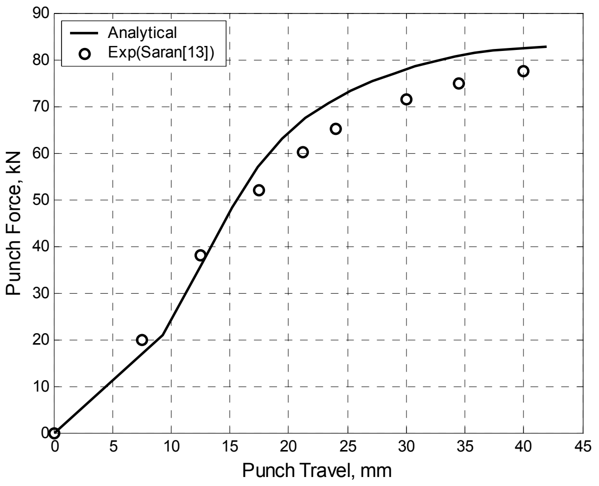

Fig. 3. Punch travel vs. punch force

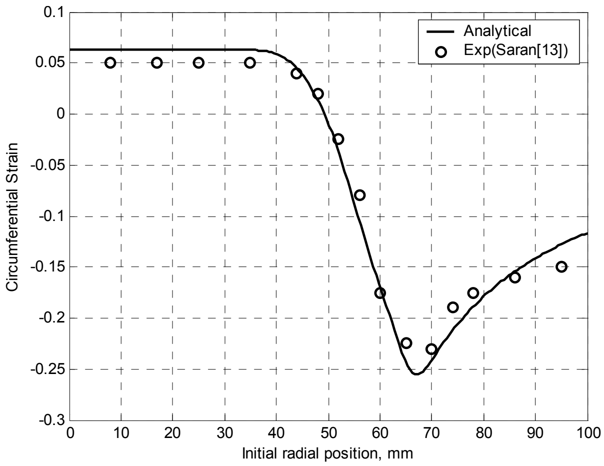

Fig. 4. Circumferential strain distribution at a punch travel of 30mm

3. Conclusions

From the results of the present study, the following conclusions can be made:

1. The results of the incremental analytical model for punch travel vs. punch force, and circumferential strains distributions show good correlation with the experimental results.
2. The present model can be useful in conducting parametric studies on the different parameters affecting the process including die design, process and material parameters.
3. The present analytical model lends itself as an analysis tool for the design of any cup drawing process. It can be used as a fast server to perform a preliminary analysis to predict the stresses and strains induced in the deforming cup and determine the suitable parameters that give the least strains. Then, a more accurate finite element analysis can be carried out for the cup with these suitable parameters. This can be followed by a full scale experimental work to verify the numerical results.

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