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Influence of piezoelectric on characteristics of vibrating mechatronical system

A. Buchacz*
Institute of Engineering Processes Automation and Integrated Manufacturing Systems, Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: andrzej.buchacz@polsl.pl

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Analysis and modelling

ABSTRACT

Purpose: Application of approximate method was main purpose of work to solution task of assignment of frequency-modal analysis and characteristics of mechatronical system.
Design/methodology/approach: The problem in the form of set of differential equation of motion and state equation of considered mechatronical model of object has been formulated and solved. Galerkin's method to solving has been used. The considered torsionally vibrating mechanical system is a continuous bar of circular cross-section, clamped on one end. A ring transducer, which is the integral part of mechatronical system, extorted by harmonic voltage excitation is assumed to be perfectly bonded to the bar surface.
Findings: Parameters of the transducer have important influence of values of natural frequencies and on form of characteristics of considered mechatronical system. The poles of dynamical characteristic calculated by mathematical exact method and the Galerkin's method have approximately the same values. The results of the calculations were not only presented in mathematical form but also as a transients of examined dynamical characteristic which are function of frequency of assumed excitation.
Research limitations/implications: In the paper the linear mechatronical system has been considered, but for this kind of systems the approach is sufficient.
Practical implications: In article other approach is presented, that means in domain frequency spectrum the analysis has been considered.
Originality/value: The mechatronical system created from mechanical and electrical subsystems with electromechanical bondage has been considered. This approach is other from considered so far. Using methods and obtained results can be value for designers of mechatronical systems.
Keywords: Applied mechanics; Bar; Piezotransducer; Galerkin's method; Transmittance

1. Introduction

During designing the machines the object of special interest of researchers is lifting the machine's efficiency and reliability. Many industry branches concentrate on a problem of miniaturizing the existing systems and also on minimizing their energy-consumption. During last years a lot of attention is paid to the researches connected with new construction solutions, especially as far as the technology of drives which lean on the phenomenon of piezoelectricity and electrostriction is concerned [10,14-17,19-22]. The piezoelectric elements are also used to eliminate the oscillation [18].

The first attempt at the solution to this problem, that means to determining of dynamical characteristic of continuous bar system and various class of discrete mechanical systems concerning the frequency spectrum, using graphs and structural numbers methods, has been made in the Gliwice research centre[1)] in [1-9,11-13].

[1)] Other diverse problems have been modeled by different kind of methods next they were examined and analyzed in the center for the last several years (e.g.[17,23-27]).

2. The mechatronical system with harmonical excitation

The homogeneous elastical shaft with full section, permanent on the whole length l in Fig. 1 has been considered. The shaft is made by material with transverse modulus that means Kirchoff's modulus G and the mass density ρ .

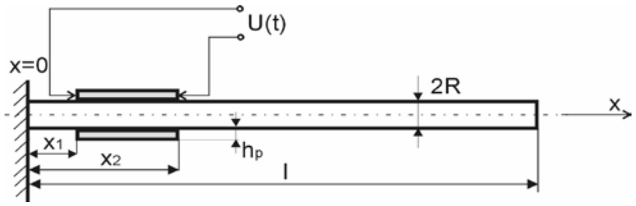

Fig. 1. The mechatronical system with electrical excitation

In this system the shaft is clamped on one of its end, the second one is free. The system is not excited by any mechanic moments or forces. To the surface of the shaft the ideal ring piezotransducer is attached. The harmonic electrical voltage which excites the system from electric side is applied to the converter clips. This voltage makes the deformation of the piezoelement, which interacts directly with the shaft.

The dynamical equation of motion of the shaft as far as the given system takes the following form [17]

$$\rho I_o \frac{\partial^2 \varphi}{\partial t^2} - G I_o \frac{\partial^2 \varphi}{\partial x^2} = \frac{-\lambda^*}{l} U\left[\delta(x-x_1) - \delta(x-x_2)\right] \tag{1}$$

or in a different way

$$\ddot{\varphi} - a^2 \varphi_{xx} = bU\left[\delta(x-x_1) - \delta(x-x_2)\right], \tag{2}$$

where: $\lambda^* = \frac{2}{3}\pi G_p\left[\left(R+h_p\right)^3 - R^3\right]\frac{d_{15}}{l_p}$, $a = \sqrt{\frac{G}{\rho}}$, $b = \frac{-\lambda^*}{I_o l \rho}$.

Similarly dynamic equation of the transducer is given in the form:

$$\dot{U} + \alpha_1 U(t) = -\alpha_2 \dot{\varphi}(l_p, t), \tag{3}$$

where: α_1 - constant measured in $\left[\frac{1}{s}\right]$, $\alpha_2 = \frac{2\pi R^2 h_p d_{15} G_p}{l_p C_p}$,

$$C_p = 2\pi R h_p \frac{e_1}{l_p}\left(1 - \frac{2 d_{15} G_p}{e_1}\right).$$

Taking into consideration the equations (1) and (3), the set of equations that will start further considerations can be written. The considered mechatronical system is described next set of equations

$$\begin{cases} \ddot{\varphi} - a^2 \varphi_{xx} = U\left[\delta(x-x_1) - \delta(x-x_2)\right], \\ \dot{U} + \alpha_1 U(t) = -\alpha_2 \dot{\varphi}(l_p, t). \end{cases} \tag{4}$$

According to the Galerkin's discretisation of searching the solutions of difference equation system with partial derivative, the solution will be searched in order to the own sum function, that means function of the time and displacement variables, which are strictly established and which realize the boundary conditions [5, 6-14,15].

It is accepted that the dislocation, that means angle of torsion of cross-section takes form:

$$\varphi(x,t) = A\sum_{n=1}^{\infty} \sin\frac{(2n-1)\pi x}{2l}\cos\omega t . \tag{5}$$

Moreover it is assumed that the system is excited with harmonic voltage as follow:

$$U(t) = U_0 \sin\omega t . \tag{6}$$

3. The dynamical characteristic for first mode vibration

For the first mode vibration, that means when n=1 angle of torsion (5) takes form:

$$\varphi(x,t) = A\sin\frac{\pi x}{2l}\cos\omega t . \tag{7}$$

The solution of the examined set of differential equations (4), resolves to putting the adequate derivatives. Putting the received derivatives (1), (2), (3) to the equation (4), it has been received :

$$\begin{cases} A\sin\frac{\pi x}{2l}\cos\omega t\left[a^2\left(\frac{\pi}{2l}\right)^2 - \omega^2\right] - Bb\left[\delta(x-x_1) - \delta(x-x_2)\right]\sin\omega t = 0, \\ B\omega\cos\omega t - \alpha_2 A\omega\sin\frac{\pi l_p}{2l}\sin\omega t = \alpha_1 U_0 \sin\omega t. \end{cases} \tag{8}$$

To nominate the dynamical characteristic from these equations the time function must be eliminated using the Euler's theorem [11, 12-21,22], and in this way is obtained

$$\begin{cases} A\sin\frac{\pi x}{2l}e^{i\omega t}\left[a^2\left(\frac{\pi}{2l}\right)^2 - \omega^2\right] - Bb\left[\delta(x-x_1) - \delta(x-x_2)\right]e^{i\left(\omega t - \frac{\pi}{2}\right)} = 0, \\ B\omega e^{i\omega t} - \alpha_2 A\omega\sin\left(\frac{\pi}{2l}l_p\right)e^{i\left(\omega t - \frac{\pi}{2}\right)} = \alpha_1 U_0 e^{i\omega t}. \end{cases} \tag{9}$$

The equations (9) as far as the matrix shape is considered is:

$$\begin{bmatrix} \sin\frac{\pi x}{2l}\left[a^2\left(\frac{\pi}{2l}\right)^2 - \omega^2\right] & -\frac{1}{e^{i\frac{\pi}{2}}}b\left[\delta(x-x_1) - \delta(x-x_2)\right] \\ -\alpha_2\frac{1}{e^{i\frac{\pi}{2}}}\omega\sin\frac{\pi l_p}{2l} & \omega \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_1 U_0 \end{bmatrix}. \tag{10}$$

From (10) the amplitude A of dynamical characteristic is calculated as:

$$A=\frac{\frac{1}{e^{i\frac{\pi}{2}}}b\left[\delta(x-x_1)-\delta(x-x_2)\right]\alpha_1}{\sin\frac{\pi x}{2l}\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\omega-\frac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}\left[\delta(x-x_1)-\delta(x-x_2)\right]\alpha_2\omega\sin\frac{\pi l_p}{2l}}U_0 . \quad (11)$$

Putting the received amplitude A (11) to (7) the angle of torsion of cross-section for first mode vibration, that means n=1 and x=l, has been established.

$$\varphi(x,t)=\frac{\frac{1}{e^{i\frac{\pi}{2}}}b'\alpha_1\sin\left(\frac{\pi}{2l}x\right)}{\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]-\frac{b'}{\left(e^{i\frac{\pi}{2}}\right)^2}\alpha_2\omega\sin\left(\frac{\pi}{2l}l_p\right)}U_0\cos\omega t \quad (12)$$

where: $b'=b\left[\delta(x-x_1)-\delta(x-x_2)\right]$.

Out of (12) the dynamical characteristic for the first mode vibration in the end of shaft, that means when x=l takes form

$$\left|Y_{li}\right|=\left|-\frac{b'\alpha_1\sin\left(\frac{\pi}{2}\right)}{\left[a^2\left(\frac{\pi}{2l}\right)^2-\omega^2\right]\omega+b'\alpha_2\omega\sin\left(\frac{\pi}{2l}l_p\right)}\right| . \quad (13)$$

In Fig. 2 the transient of dynamical characteristic has been shown.

4. Conclusions

Presented approach allows to look in a global way on the behavior of the mechatronical system. The influence change of the values b and α, which directly depend on piezoelement's sort and itsgeometrical sizes according to the characteristics, the sort of vibrations of the mechatronical system, mainly as far as the piezoelectric converter "activation" is concerned the contribution to the next researches can be considered.

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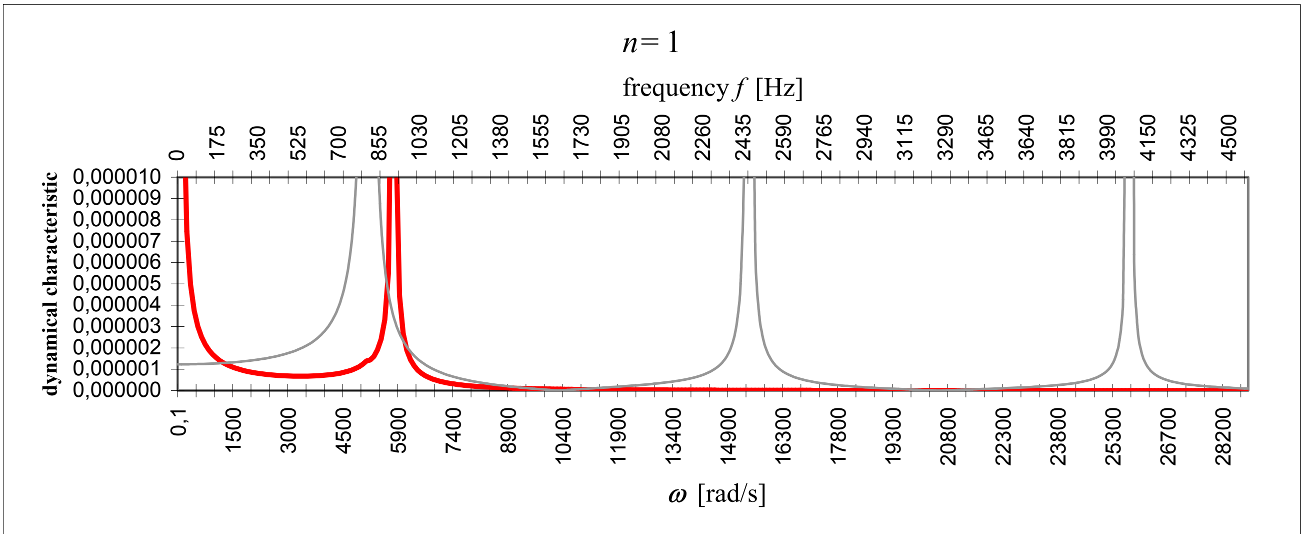

Fig. 2. Transient of dynamical characteristic for the first mode vibration

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