Journal of Achievements in Materials and Manufacturing Engineering

VOLUME 17
ISSUE 1-2
July-August
2006

Thermally induced vibration of an axially-traveling strip: spectral element analysis

U. Lee*, K. Kwon
Department of Mechanical Engineering, Inha University, Incheon 402-751, South Korea
* Corresponding author: E-mail address: ulee@inha.ac.kr

Received 15.03.2006; accepted in revised form 30.04.2006

Analysis and modelling

ABSTRACT

Purpose: A spectral element model is developed for accurate prediction of the dynamic characteristics of an axially-traveling strip subjected to a sudden thermal loading.

Design/methodology/approach: The spectral element model is formulated from the frequency-dependent dynamic shape functions which satisfy the governing equations in the frequency-domain and its extremely high accuracy is evaluated by comparing with the conventional finite element model in which simple polynomials are used as the shape functions. Also some numerical studies are conducted to investigate the vibration characteristics of an example axially-traveling strip subjected to a sudden thermal loading on its upper surface.

Findings: The present spectral element model is shown to provide very accurate dynamic characteristics by treating a whole uniform strip between two boundary supports as a single finite element, regardless of its length, when compared with the conventional finite element model.

Practical implications: Numerical studies for the typical example problem show that the dynamic characteristics of an axially-traveling strip may depend on the traveling speed and the duration and frequency characteristics of the externally applied thermal loading.

Originality/value: The spectral element model presented in this paper is the first one for the axially-traveling strips subjected to thermal loadings and is applicable to the engineering problems such as the galvanized steel strip passing through a hot zinc tank, for instance.

Keywords: Applied mechanics; Spectral element model; Axially-traveling strip; Vibration; Thermal loading

1. Introduction

When a thin-walled structure is subjected to a sudden thermal loading, a very rapid thermal process may occur to induce very rapid movements in the structure, thus causing the structure to vibrate. The thermally induced vibration of a beam subjected to a suddenly applied heat flux distributed along its span was studied first by Boley [1]. Since then, numerous studies have been conducted for various thermoelastic structures [2]. The existing previous studies on the thermally induced vibration have been focused mostly on the stationary (*i.e.*, not axially-traveling) structures. To the authors' best knowledge, the dynamics of axially-traveling thermoelastic structures such as the galvanized steel strips passing through the hot zinc tank has not been investigated yet. Furthermore the spectral element method (SEM) has not been applied to such axially-traveling thin structures.

The SEM is one of element methods such as the finite element method (FEM). The key differences from FEM are as follows. In SEM, (1) the spectral element matrix (*exact* dynamic stiffness matrix), which is formulated in the frequency-domain by using the frequency-dependent dynamic shape functions, is used, and (2) the FFT algorithm is used to efficiently reconstruct the time-domain responses from the frequency-domain solutions. Because no approximation or assumption is made in the course of spectral element formulation, the SEM indeed provides exact solutions and thus it is well recognized as an *exact* solution method [3].

Thus, the purposes of this paper are: (1) to develop a spectral element model for axially-traveling thin strips which are subjected

© Copyright by International OCSCO World Press. All rights reserved. 2006

to thermal loadings and (2) to investigate the dynamic of an example axially-traveling thin strip.

2. Dynamic equations of motion

Consider a thin strip is traveling in the axial x (axial) direction at a moving speed of c. The strip has the thickness h and width b, and its material properties are given by the Young's modulus E and Poisson's ratio ν. Assume that the strip has a small amplitude vibration and its displacements don't vary along the width (y) direction. Accordingly define $w(x, t)$ and $u(x, t)$ as the transverse displacement and axial displacement, respectively, of a thin strip which is axially-traveling over two simply supports of distance L.

Based on the Kirchhoff's hypothesis, the equations of motion and relevant boundary conditions for the small amplitude vibration of a thin strip can be derived from the Hamilton's principle as

$$\overline{EA}\,u'' - \rho A\,\ddot{u} = -p_x(x,t) + \frac{1}{2}N_T'$$
$$D\,w'''' + \rho A c^2\, w'' + 2\rho A c\,\dot{w}' - \rho I\,\ddot{w}'' + \rho A\,\ddot{w} = p_z(x,t) - \frac{1}{2}M_T'' \tag{1}$$

where $p_x(x, t)$ and $p_z(x, t)$ denote the external loads in the x and z directions, M_T and N_T are the thermally induced moment and axial force, respectively, and the following definitions are used:

$$D = \frac{EI}{(1-\nu^2)}, \quad I = \frac{bh^3}{12}, \quad \overline{EA} = \frac{EA}{1-\nu^2} \tag{2}$$

Assume that the thermal load is applied only on the top or bottom surface of the strip. Because of the geometry of the thin strip, the temperature variation due to a sudden heating on a surface of the strip will be more significant in the thickness direction rather than in the in-plane directions. Accordingly, the heat conduction equation can be derived into the form as

$$kT^{\circ\circ} - \left(T_0\,\alpha^2\,E_\nu + \rho\,c_p\right)\dot{T} = 0 \tag{3}$$

where $E_\nu = (1+\nu)E/(1-2\nu)(1-\nu)$.

3. Spectral element model

By using the DFT (discrete Fourier transforms) theory [5], one can express

$$u(x,t) = \sum_{n=0}^{N-1} U_n(x)e^{i\omega_n t}, \quad w(x,t) = \sum_{n=0}^{N-1} W_n(x)e^{i\omega_n t}$$
$$p_x(x,t) = \sum_{n=0}^{N-1} P_{xn}(x)\,e^{i\omega_n t}, \quad p_z(x,t) = \sum_{n=0}^{N-1} P_{zn}(x)\,e^{i\omega_n t} \tag{4}$$
$$N_T(t) = \sum_{n=0}^{N-1} N_{Tn}e^{i\omega_n t}, \quad M_T(t) = \sum_{n=0}^{N-1} M_{Tn}e^{i\omega_n t}$$

where U_n, W_n, P_{xn}, P_{zn}, N_{Tn} and M_{Tn} (n = 0, 1,…, N-1) are the spectral components of u, w, p_x, p_y, N_T and M_T, respectively, and N is the number of samples. Substituting Eq. (4) into Eq. (1) gives

$$\overline{EA}U_n'' + \rho A\omega_n^2\,U_n = -P_{xn}$$
$$DW_n'''' + \left(\rho A c^2 + \rho I\omega_n^2\right)W_n'' + 2i\rho A c\omega_n\,W_n' - \rho A\omega_n^2\,W_n = P_{zn} \tag{5}$$

To formulate the spectral element, the general solutions of the homogeneous equations of (13) are assumed as

$$U_n(x) = A_n\,e^{\kappa_n x}, \qquad W_n(x) = B_n\,e^{\lambda_n x} \tag{6}$$

where κ_n and λ_n denote the wavenumbers for the axial and transverse wave modes, respectively. Substituting Eq. (6) into the homogeneous equations of (5) will provide two dispersion relations, from which two wavenumbers k_{nr} (r =1, 2) for the axial wave mode and four wavenumbers λ_{nr} (r =1, 2, 3, 4) for the transverse wave mode can be computed. By using these six wavenumbers, the general solutions of the homogeneous governing equations of Eq. (5) can be obtained in term of spectral nodal DOFs vector $\{\boldsymbol{d}_n\}$ as

$$U_n(x) = [\boldsymbol{E}_{Un}][\boldsymbol{X}_n]^{-1}\{\boldsymbol{d}_n\} \equiv [\boldsymbol{N}_{Un}(x;\omega_n)]\{\boldsymbol{d}_n\}$$
$$W_n(x) = [\boldsymbol{E}_{Wn}][\boldsymbol{X}_n]^{-1}\{\boldsymbol{d}_n\} \equiv [\boldsymbol{N}_{Wn}(x;\omega_n)]\{\boldsymbol{d}_n\} \tag{7}$$

where

$$\{\boldsymbol{d}_n\} = \{U_{n1}\;\; W_{n1}\;\; \Phi_{n1}\;\; U_{n2}\;\; W_{n2}\;\; \Phi_{n2}\}^{\mathrm{T}}$$
$$[\boldsymbol{X}_n] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & \lambda_{n1} & \lambda_{n2} & 0 & \lambda_{n3} & \lambda_{n4} \\ e^{k_{n1}l} & 0 & 0 & e^{k_{n2}l} & 0 & 0 \\ 0 & e^{\lambda_{n1}l} & e^{\lambda_{n2}l} & 0 & e^{\lambda_{n3}l} & e^{\lambda_{n4}l} \\ 0 & \lambda_{n1}e^{\lambda_{n1}l} & \lambda_{n2}e^{\lambda_{n2}l} & 0 & \lambda_{n3}e^{\lambda_{n3}l} & \lambda_{n4}e^{\lambda_{n4}l} \end{bmatrix} \tag{8}$$

$$[\boldsymbol{E}_{Un}(x;\omega_n)] = \begin{bmatrix} e^{k_{n1}x} & 0 & 0 & e^{k_{n2}x} & 0 & 0 \end{bmatrix}$$
$$[\boldsymbol{E}_{Wn}(x;\omega_n)] = \begin{bmatrix} 0 & e^{\lambda_{n1}x} & e^{\lambda_{n2}x} & 0 & e^{\lambda_{n3}x} & e^{\lambda_{n4}x} \end{bmatrix}$$

In Eq. (7), $[\boldsymbol{N}_{Un}]$ and $[\boldsymbol{N}_{Wn}]$ are the frequency-dependent dynamic shape function matrices.

By using the variational approach [5], the spectral element equation can be formulated from Eq. (7) as

$$[\mathbf{S}_n(\omega)]\{\mathbf{d}_n\} = \{\mathbf{f}_n\} \tag{9}$$

where

$$[\boldsymbol{S}_n(\omega)] = [\boldsymbol{X}_n^{-1}]^{\mathrm{T}}([\boldsymbol{R}_{Un}] + [\boldsymbol{R}_{Wn}])[\boldsymbol{X}_n^{-1}]$$
$$\{\boldsymbol{f}_n\} = \{\boldsymbol{f}_n\}_1 + \{\boldsymbol{f}_n\}_2 \tag{10}$$

with

$$
[\mathbf{R}_{Un}] = \begin{bmatrix} Y_{n11} & 0 & 0 & Y_{n14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ Y_{n14} & 0 & 0 & Y_{n44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
[\mathbf{R}_{Wn}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_{n11} & X_{n12} & 0 & X_{n13} & X_{n14} \\ 0 & X_{n12} & X_{n22} & 0 & X_{n23} & X_{n24} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_{n13} & X_{n23} & 0 & X_{n33} & X_{n34} \\ 0 & X_{n14} & X_{n24} & 0 & X_{n34} & X_{n44} \end{bmatrix} \tag{11}
$$

$$
Y_{nij} = \frac{e^{(k_{ni}+k_{nj})l} - 1}{k_{ni} + k_{nj}} \left(\overline{EA}\, k_{ni} k_{nj} - \rho A \omega_n^2 \right)
$$

$$
X_{nij} = \frac{e^{(\lambda_{ni}+\lambda_{nj})l} - 1}{\lambda_{ni} + \lambda_{nj}} \{ D \lambda_{ni}^2 \lambda_{nj}^2 - R_n \lambda_{ni} \lambda_{nj} + i\rho A c \omega_n (\lambda_{ni} - \lambda_{nj}) - \rho A \omega_n^2 \}
$$

$$
\{\boldsymbol{f}_n\}_1 = \{ N_{1n} \quad V_{1n} \quad M_{1n} \quad N_{2n} \quad V_{2n} \quad M_{2n} \}^{\mathrm{T}}
$$

$$
\{\boldsymbol{f}_n\}_2 = \int_0^l P_{xn}(x) [\boldsymbol{N}_{Un}]^{\mathrm{T}}\, dx + \int_0^l P_{zn}(x) [\boldsymbol{N}_{Wn}]^{\mathrm{T}}\, dx - \frac{1}{2} N_{Tn} [\boldsymbol{N}_{Un}(l) - \boldsymbol{N}_{Un}(0)]^{\mathrm{T}} + \frac{1}{2} M_{Tn} [\boldsymbol{N}'_{Wn}(l) - \boldsymbol{N}'_{Wn}(0)]^{\mathrm{T}}
$$

Spectral elements can be assembled in a completely analogous way to that used in conventional FEM. Assembling all spectral elements and then applying appropriate boundary conditions yield a global system equation. The natural frequencies can be then computed from the condition that the determinant of the global spectral stiffness matrix vanishes at natural frequencies.

The temperature field is governed by Eq. (3) and thermal boundary conditions specified on the upper and lower surfaces of strip. As done for the displacements field, the temperature field can be also represented in the spectral form and its spectral components can be obtained as

$$
T_n(z) = B_{n1} e^{-\tau_n z} + B_{n2} e^{\tau_n z} \tag{12}
$$

where

$$
\tau_n = (1+i) \sqrt{\frac{\omega_n}{2k} \left(T_0 \alpha^2 E \eta + \rho c_p \right)} \tag{13}
$$

The constants B_{n1} and B_{n2} are determined by the thermal boundary conditions on the upper and lower surfaces of strip.

4. Numerical example and discussions

A thin strip, which is axially-traveling over two simple supports of distance $L = 2$ m, is considered as an example problem. The strip (Fig. 1) has the thickness h = 5*mm*, width b = 0.5 m, Young's modulus E = 73 GPa, Poisson's ratio $\nu = 0.33$, mass density ρ = 2770 kg/m^3, thermal expansion coefficient $\alpha = 23.0\times10^{-6}$/K, thermal conductivity k = 177 W/mK, and the specific heat c_p = 875 J/kg·K. In Fig. 1, T_0 is the room temperature.

Table 1 shows that the natural frequencies obtained by SEM are identical to the exact results when c = 0 m/s, and the FEM results certainly converge to the SEM results when $c \neq 0$ m/s as the number of finite elements used in FEM is increased. This proves the extremely high accuracy of the present spectral element model.

To investigate the thermally induced vibrations of the strip, the temperature on the middle part of its upper surface is suddenly heated by ΔT = 20K. It is assumed that the strip is traveling at c = 4 m/s. Figure 2 shows the time history of the transverse vibration depending on the size of heating zone, L_2. It is found that the amplitudes of both axial and transverse vibrations become larger as L_2 is increased.

To investigate the effect of the duration of heating on the thermally induced vibration, the heating is applied on $L_2 = 0.2L$ for three different durations. As illustrated in Fig. 3 for the case of transverse vibration, in general the amplitudes of both axial and transverse vibrations become larger as the duration of heating is increased.

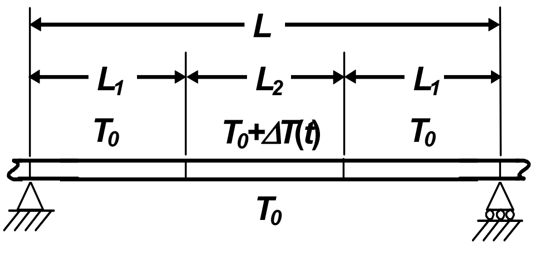

Fig. 1 An example problem

Table 1
Natural frequencies (rad/s) of a thin strip

C (*m/s*)	Method	N	ω_1	ω_3	ω_5	ω_{15}
0	Exact[6]	-	19.37	174.31	484.22	4271.2
	SEM	1	19.37	174.31	484.16	4271.2
	FEM	10	19.37	174.41	486.14	4275.6
		20	19.37	174.32	484.35	4272.3
		50	19.37	174.31	484.23	4271.4
		100	19.37	174.31	484.23	4271.3
8	SEM	1	14.12	171.39	481.60	4271.2
	FEM	10	14.12	171.51	483.69	4275.6
		20	14.12	171.41	481.80	4272.3
		50	14.12	171.40	481.67	4271.4
		100	14.12	171.40	481.66	4271.3

Note: N = used number of finite elements

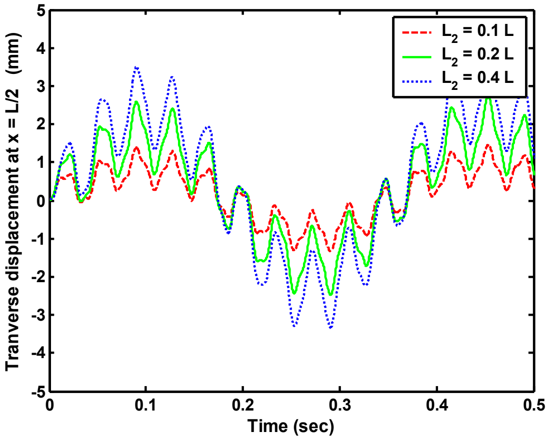

Fig. 2. Transverse vibration vs. the size of heating zone L_2

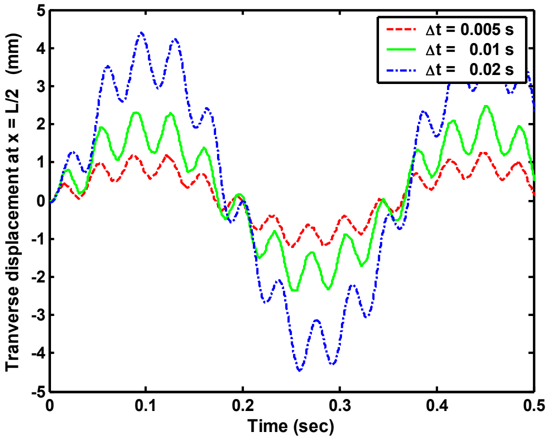

Fig. 3. Transverse vibration vs. the duration of heating Δt

Figure 4 compares the axial and transverse vibrations induced by the harmonic thermal loading defined by $\Delta T(x,t) = 10 \sin(2\pi ft)+20$ (K) when $L_2 = 0.2L$. Figure 5 shows that the resonance in transverse vibration mode occurs when the exaction frequency f is getting closer to the first transverse natural frequency (see Table 1).

5. Conclusions

This paper presents a spectral element model for the axially-traveling strip which is subjected to sudden heating on its surface. The governing equations are derived by using Hamilton's principle and the spectral element model is formulated from the frequency-dependent shape functions which are the exact frequency-domain solutions of the governing equations. The present spectral element model is then evaluated by comparing with conventional finite element solutions and some numerical studies have been conducted to investigate the thermal-induced vibrations of an example axially-traveling strip.

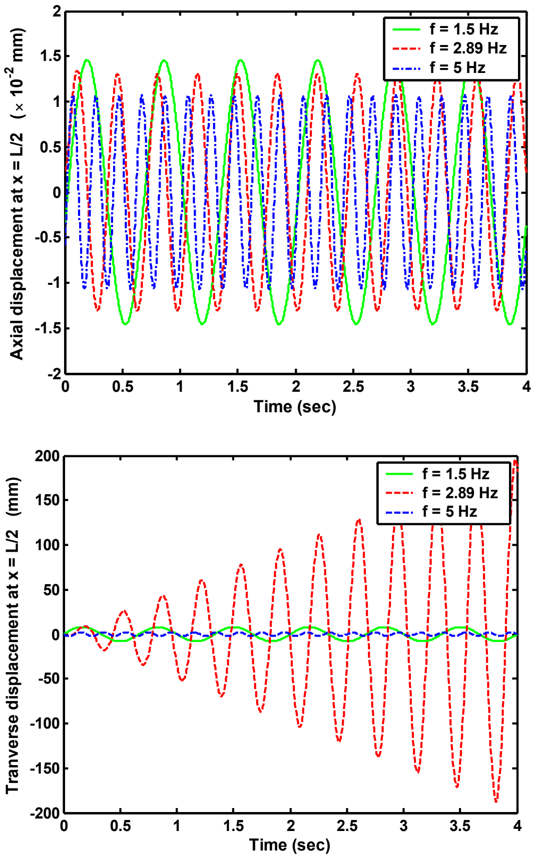

Fig. 4. Axial and transverse vibrations vs. $\Delta T(x,t) = 10 \sin(2\pi ft) +20$ (K) when $L_2 = 0.2L$.

Acknowledgements

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) (KRF-2005-041-D00042)

References

[1] B.A. Boley, Thermally induced vibrations of beams, Journal of Aeronautical Science 23 (1956) 179-181.

[2] E.A. Thornton, Thermal structures: four decades of progress, Journal of Aircarft 29 (1992) 485-498.

[3] U. Lee, Spectral Element Method in Structural Dynamics. Inha University Press, Incheon, Korea, 2004.

[4] M.N. Özisik, Heat Conduction. Second Edition, John Wiley & Sons, New York, 1993.

[5] D.E. Newland, Random Vibrations, Spectral and Wavelet Analysis. Third Edition, Longman, New York, 1993.

[6] R.D. Blevins, Formulas for Natural Frequency and Mode Shape. Van Nostrand Reinhold, New York, 1979