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On the identification of composite beam dynamics based upon experimental data

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Analysis and modelling

ABSTRACT

Purpose: Article describes kinds and use procedures of mathematical parametric models describing dynamics of the systems based on excitation and vibration response signals.

Design/methodology/approach: As a sample of identification of mathematical parametric models and estimation their parameters was a composite beam investigated under a white noise excitation force activity.

Findings: Model based identification leads to finitely parameterised models described by differential equations.

Research limitations/implications: Such models provide important features, in comparison with non-parametric systems: direct relationship with differential equation or physically significant modal representations used in engineering analysis, improved accuracy and frequency resolution, compactness/parsimony of representation.

Practical implications: Ability to provide complete system characterisation by relatively few parameters, suitability for analysis, prediction, fault detection and control.

Originality/value: Article is valuable for persons, that are interesting for identification of mathematical parametric models and vibration systems.

Keywords: Computational material science and mechanics; Numerical techniques; Parametric identification; Model estimation; Vibration;

1. Introduction

A parametric identification of vibrating systems is the process of finding mathematical and parameterised models for system, which is based on measured excitation and/or response signals. In normal cases, the excitation is the force, the response signal – the vibration displacement, velocity or acceleration.

A typical experiment of the system identification is being described in Fig. 1:

The measurable excitation force is vector $\{\mathbf{x}(t)\}$. The vibration response vector is $\{\mathbf{y}(t)\}$. It's described as forced (if $\mathbf{x}\neq0$), or as free (if $\mathbf{x}\equiv0$) and is corrupted by stochastic zero-mean noise $\{e(t)\}$. The transfer matrix $\mathbf{G}(s)$ represents the structural dynamics of the examined system, where the variable t indicating continuous time and s indicates The Laplace transform variable.

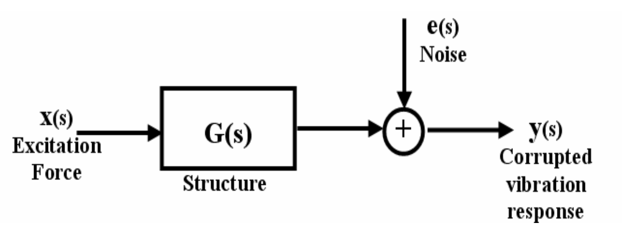


Fig. 1. Typical identification experiments.

The model based identification (also called the parametric identification) leads to finitely parameterised models described by differential equations. Such models provide important features (in comparison with non-parametric systems):

- The direct relationship with differential equation or physically significant modal representations used in engineering analysis,
- Improved accuracy and frequency resolution,
- Compactness/parsimony of representation, that is the ability to provide a complete system characterization with relatively few parameters,
- Their suitability for analysis, prediction, fault detection and control.

The prejudice of that model is the increased identification complexity and dependence of the results on the assumed model form and the estimation criterion.

2. The algorithm of a parametric identification

The five main elements of parametric identification method include:

1. The data set,
2. The selected model class,
3. The estimation criterion,
4. The model validation procedure,
5. The modal parameter extraction procedure.

The general identification procedure, based upon sampled signals is outlined in Fig. 2.

The data set consists of two signals. One of them is the excitation, the other – the response. The class of the model is a selected family of models parameterised in terms of an unknown parameter **θ**, witch is the model criterion. In most cases **θ** is the least squares criterion. The model validation procedure attempts to accept or reject the estimated model. Modal parameter extraction refers to the determination of the modal parameters of the estimated model.

A variety of model structures is available to assist in modelling a system. The choice of model structure is based upon an understanding of the system identification method and on insight into the system of undergoing identification. Even then, it is often beneficial to test a number of structures to determine the best one.

3. Parametric model structuers

Th pearametric models describe systems in terms of differential equations and transfer functions. They give insights into the system of physics and compact model structures.

Generally, one can describe the system using the following equation, which is known as the general-linear polynomial model or the general-linear model.

$$y(s) = q^{-1}\mathbf{G}(q^{-1},\theta)u(s) + \mathbf{H}(q^{-1},\theta)e(s) \qquad (1)$$

$u(n)$ and $y(n)$ are the input and output of the system, $e(n)$ is zero-mean noise, $\mathbf{G}(q^{-1},\theta)$ is the transfer function of the deterministic part of the system and $\mathbf{H}(q^{-1},\theta)$ is the transfer function of the stochastic part of the system.

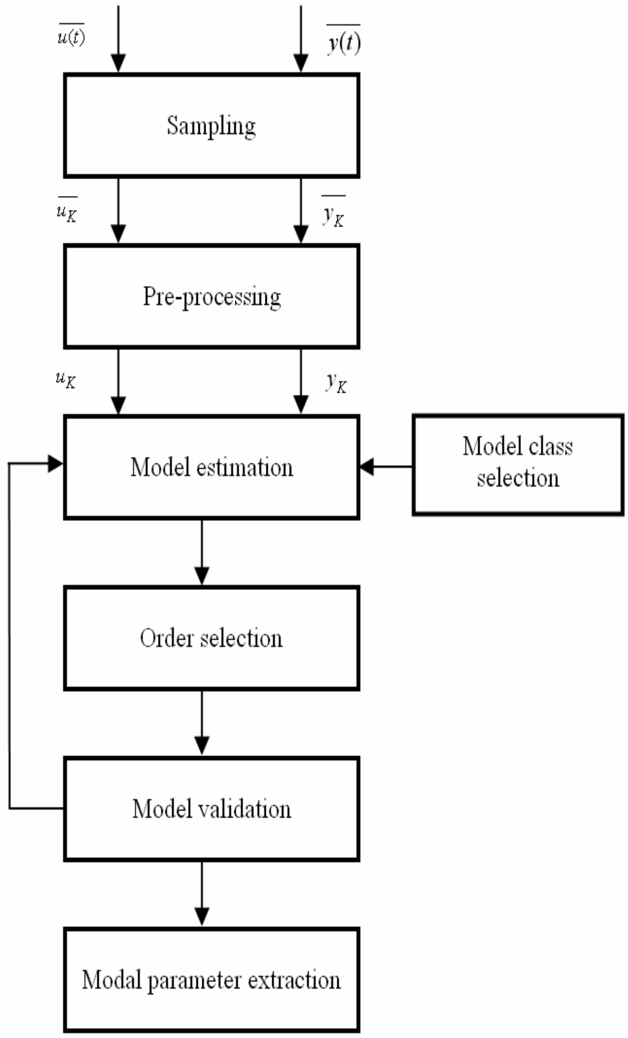


Fig. 2. The general identification procedure.

The general-linear model structure, shown in Fig. 3, provides flexibility for both, the system dynamics and stochastic dynamics. However, a nonlinear optimization method computes the estimation of the general-linear model. This method requires an intensive computation with no guarantee of global convergence.

The simpler models that are subsets of the general linear model structures are possible. By setting one or more of A(q), B(q), C(q) or D(q) polynomials equal to 1, it is possible to create these simpler models such as AR, ARX, ARMAX, Box-Jenkins, and output-error structures. Each of these methods has their own advantages and disadvantages and is commonly used in real-world applications.

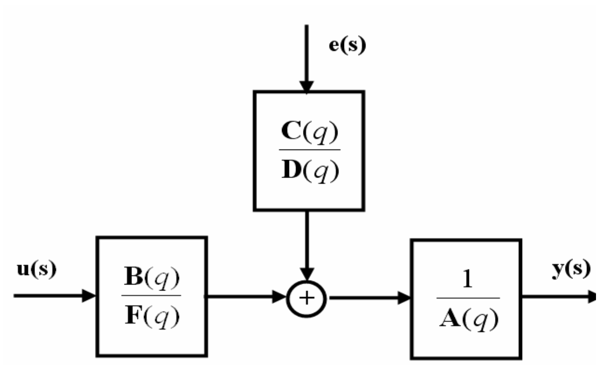


Fig. 3. The general, linear model structure.

4. Modelling

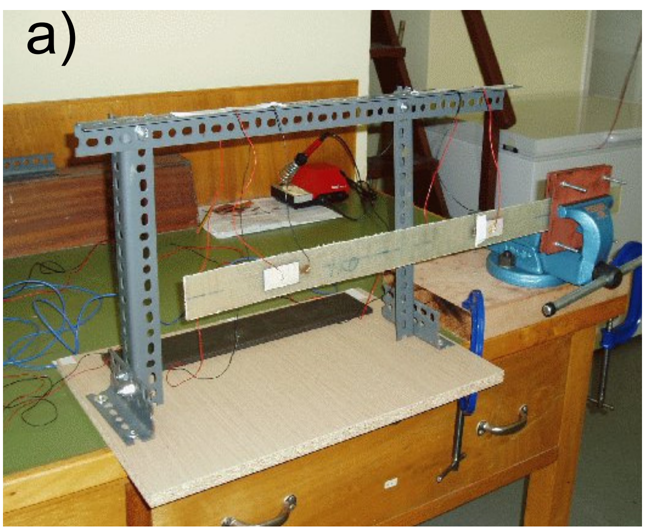
The introduced experiment depends on the identification of system and the estimation of parametric model of the composite rectangular beam. (Shown on Fig. 4a). Chemical compositions as well as the technology of the production are secret, because from these material elements of stabilisers for Greek army jet fighters are produced.

The measuring signals obtained from sensors installed on examined beam (Fig. 4b) effect modelling. As a result two signals has been obtained, both signals possessing 10240 of samples, registered with frequency 100Hz.(Fig. 5).

„Input" - is the input signal (extorting, white zero-mean noise),

„Output" - is the output signal, (answer to extortion).

All the computations are process in Matlab .It is an interactive system for numerical computation. A numerical analyst Cleve Moler wrote the initial Fortran version of MATLAB in the late 1970s as a teaching aid. It became popular for both teaching and research and evolved into a commercial software package written in C. For many years now, MATLAB has been widely used in universities and industry.



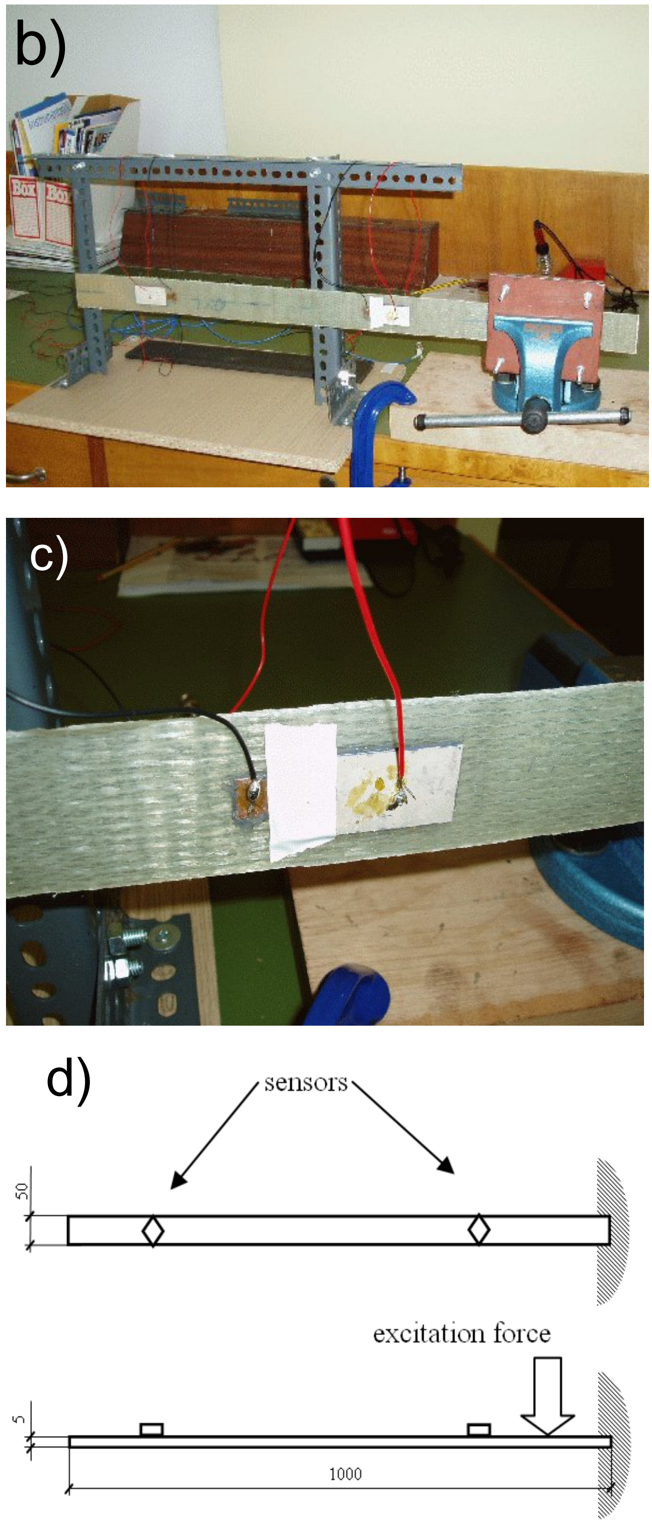


Fig. 4. Exanimate composite beam a) b) View of measuring device, b) Sensor installed on examined beam, c) Beam model.

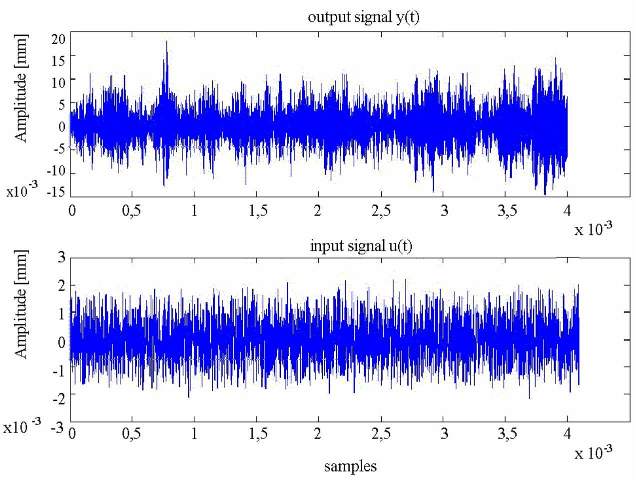


Fig. 5. Measured signals obtained by sensors, y(t) – output signal, u(t) – input signal, samples range – 0-4000.

For further use the mean values must be removed from the signal. After that, the "cleaned" signal is divided on two parts:
Z1 - samples from range 2000–6000, (used to model estimation),
Z2 - samples from range 6000–10000, (used to model validation).

To accelerate modelling additional m-files have been created to automate the process of finding the best fit of the estimated model.

These files make it possible to find such an order of model, that the comparison of the model and the real data gives the best conformity results (best fit).

$$fit = \frac{norm(y_h - y)}{\sqrt{length(y)}} \tag{2}$$

y_h is the modelled output; y is the measured output. Matlab's *norm* function returns the largest singular value of (y_h-y); *length(y)* returns the length of vector y.

If the data described in the model are ideal, fit would carry out 100%. It's not possible in real world.

It's very important NOT TO USE the same set of data for the estimation and validation. For model estimation we use signal Z1 (samples 2000-6000), for validation – Z2 (samples 6000-10000).

The last part of the modelling procedure is the verification. The common techniques are the frequency response and Cross Correlation Function.

The frequency response of a linear system is the Fourier transform of its impulse response. This description of the system gives considerable engineering insight into its properties. The relation between input and output is often written:

$$y(t) = \mathbf{G}(z)u(t) + v(t) \tag{3}$$

G is the transfer function and v is the additive disturbance. The function.

$$y(t) = G(e^{i\omega T})u(t) + v(t) \tag{4}$$

as a function of (angular) frequency ω is then the frequency response or frequency function. T is the sampling interval.

The model frequency response should be a "smooth" copy of the frequency taken from base signal.

You should require of a good model, that the cross correlation function between residuals and input does not go significantly outside the confidence region. This region corresponds to standard deviations.

A clear peak at lag *k* shows that the effect from input *u(t-k)* on *y(t)* is not properly described. A rule of thumb is that a slowly varying cross correlation function outside the confidence region is an indication of too few poles, while sharper peaks indicate too few zeros or wrong delays.

5. ARX model (Auto Regressive with eXogenous excitation)

The ARX model, shown in Fig. 6, is the simplest model incorporating the stimulus signal. The estimation of the ARX model is the most efficient of the polynomial estimation methods because it is the result of solving linear regression equations in analytic form. Moreover, the solution is unique. In other words, the solution always satisfies the global minimum of the loss function. The ARX model therefore is preferable, especially when the model order is high.

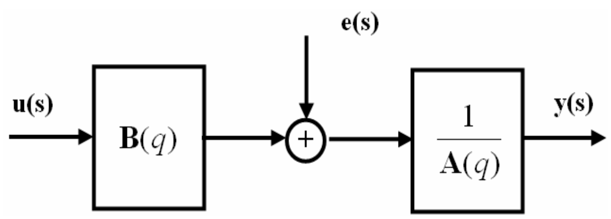


Fig. 6. The ARX model structure.

The structure is thus entirely defined by the three integers na, nb, and nk. na is equal to the number of poles and nb-1 is the number of zeros, while nk is the pure time delay (the dead time) in the system. For a system under sampled-data control, typically nk is equal to 1 if there is no dead time.

$$\begin{aligned} &y(t) + a_1 y(t-1) + ... + a_{na} y(t-na) = \\ &b_1 u(t-nk) + ... + b_{nb} u(t-nk-nb+1) \end{aligned} \tag{5}$$

The poles of a system are the roots of the denominator of the transfer function G(z), while the zeros are the roots of the numerator. In particular the poles have a direct influence on the dynamic properties of the system.

We seek the best model from ranges:
na = [1:30]
nb = [1:30]
nk = 1(for nk>1 all models was unstable).

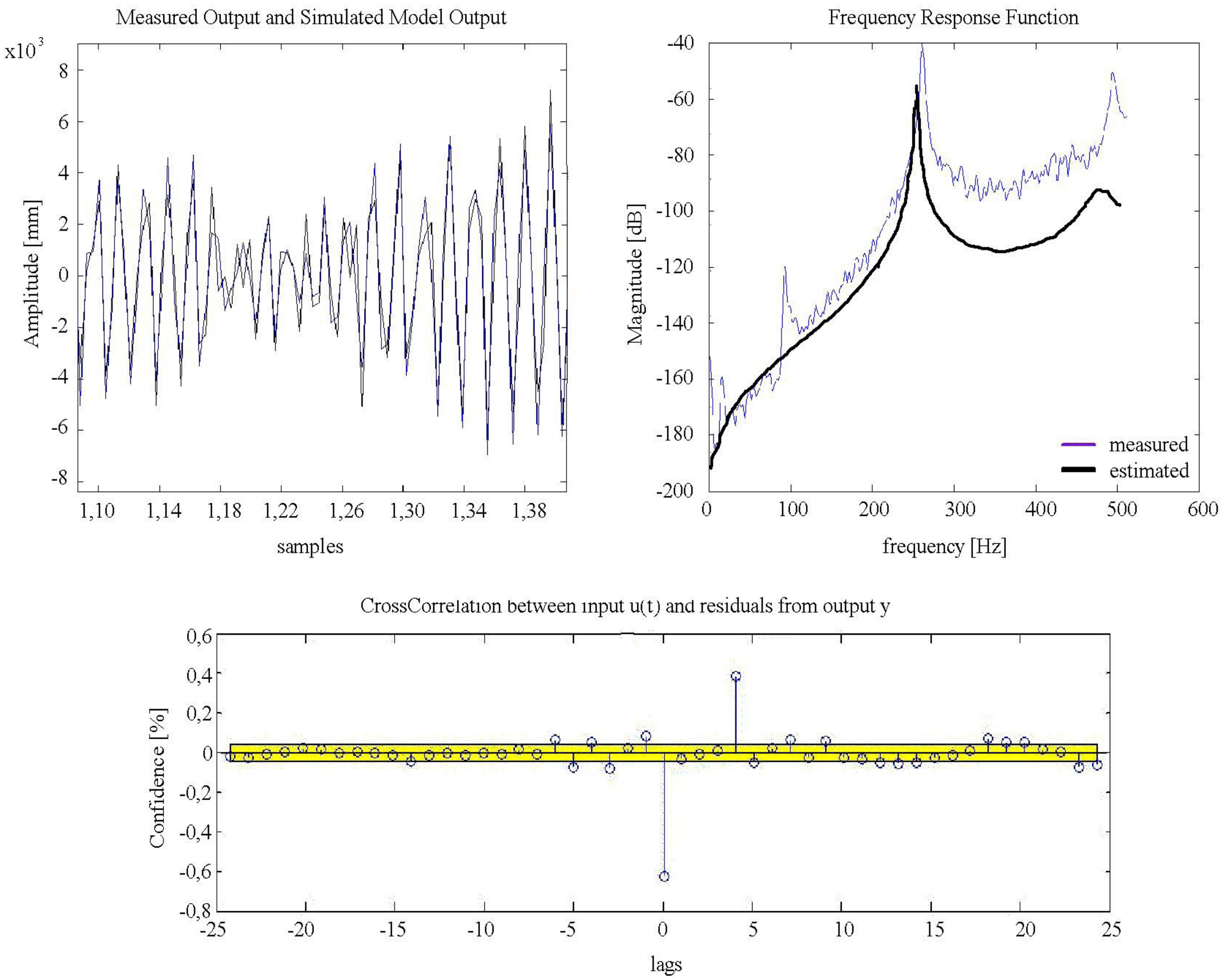


Fig. 7. Comparison between measured signals and estimated ARX model: a) fit, b) frequency response, c) cross correlation.

Syntax of the used function is following:

[M, F] = h_arx(ze,zv,[na1: na2], [nb1: nb2], nk, x)

M	-	model order [na nb nk]
F	-	fit of the model in comparison with validation data (in percentage). If F=100% the model is ideal.
ze	-	real data used to estimate model
zv	-	real data used to validate model
[na1: na2]	-	Range of parameter na model ARX (na1>na2)
[nb1: nb2]	-	Range of parameter nb model ARX (nb1>nb2)
nk	-	Parameter nc model ARX
x	-	Additional parameter. If x='pisz' then function gives values M and F for all models from given range

Best fit was found by
na=4, nb=3, nk=1, Fit = 54,99 %

6. ARMAX model (auto regressive moving average eXogenous excitation)

Unlike the ARX model, the ARMAX model structure includes disturbance dynamics. ARMAX models are useful when you have dominating disturbances that enter early in the process, such as at the input. For example, a wind gust affecting an aircraft is a dominating disturbance early in the process. The ARMAX model has more flexibility in the handling of disturbance modeling than the ARX model.

The ARMAX model in longhand would be:

$$y(t)+a_1y(t-1)+...+a_{na}y(t-na)=b_1u(t-nk)+...+ \\ b_{nb}u(t-nk-nb-1)+e(t)+c_1e(t-1)+...+c_{nc}e(t-nc) \qquad (6)$$

We also used h_armax.m file to find the best fit. Listing of this file is very similar to h_arx.m file.

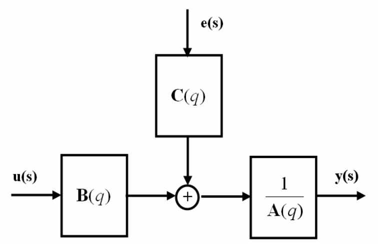


Fig. 8. The ARMAX model structure.

Syntax of function is following:
[M, F] = h_armax(ze,zv,[na1:na2], [nb1:nb2],[nc1:nc2],nk, x)

The parameters na, nb, and nc are the orders of the ARMAX model, and nk is the delay.

We seek the best model from ranges:
na = [1:30],
nb = [1:30],
nc = [1:30],
nk = 1,3 (for nk>3 all models was unstable).

Best fit for modeled data was found by:
na=4, nb=1, nc=6, nk=1, Fit = 60,98 %:

7. BJ Model (Box-Jenkins method)

The Box-Jenkins (BJ) structure provides a complete model with disturbance properties modeled separately from system dynamics.

The parameters nb, nc, nd, and nf are the orders of the Box-Jenkins model and nk is the delay.

The Box-Jenkins model is useful when you have disturbances that enter late in the process. For example, measurement of noise on the output is a late disturbance in the process.

$$y(t) = \frac{\mathbf{B}(q)}{\mathbf{F}(q)} u(t-nk) + \frac{\mathbf{C}(q)}{\mathbf{D}(q)} e(t) \tag{7}$$

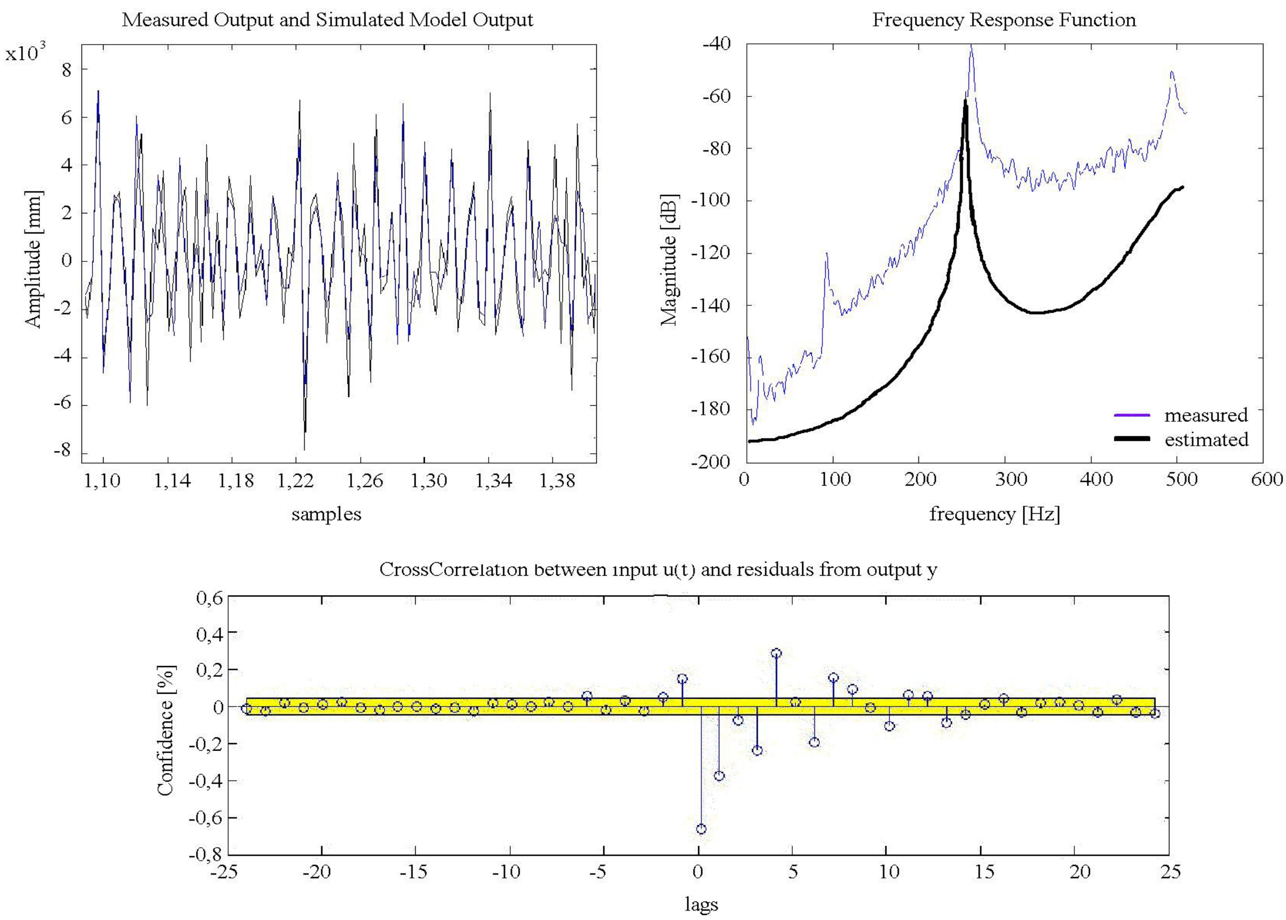


Fig. 9. Comparison between measured signals and estimated ARMAX model: a) fit, b) frequency response, c) cross correlation..

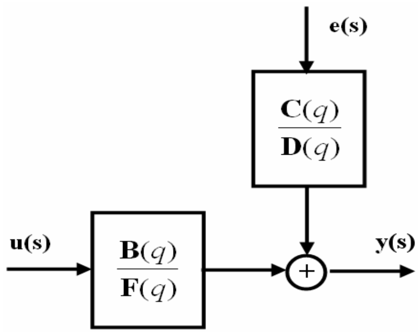


Fig. 10. The BJ model structure.

Specifically:

$$\begin{aligned} nf &: \mathbf{F}(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{nf} \\ nb &: \mathbf{B}(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1} \\ nc &: \mathbf{C}(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc} \\ nd &: \mathbf{D}(q) = 1 + d_1 q^{-1} + \ldots + d_{nd} q^{nd} \end{aligned} \tag{8}$$

Syntax of function is following:
[M, F]=h_bj(ze,zv,[nb1:nb2], [nc1:nc2],[nd1:nd2],[nf1:nf2],nk, x)

We seek the best model from ranges:
nb = [1:30],
nc = [1:30],
nd = [1:30],
nf = [1:30]
nk = 1,3 (for nk>3 all models was unstable).

Best fit for modeled data was found by:
nb=9, nc=2, nd=6, nf=4, nk=1, Fit = 66,31 %

8. OE model (Output Error)

The Output-Error (OE) model structure describes the system dynamics separately. No parameters are used for modeling the disturbance characteristics.

$$y(t) = \frac{\mathbf{B}(q)}{\mathbf{F}(q)} u(t - nk) + e(t) \tag{9}$$

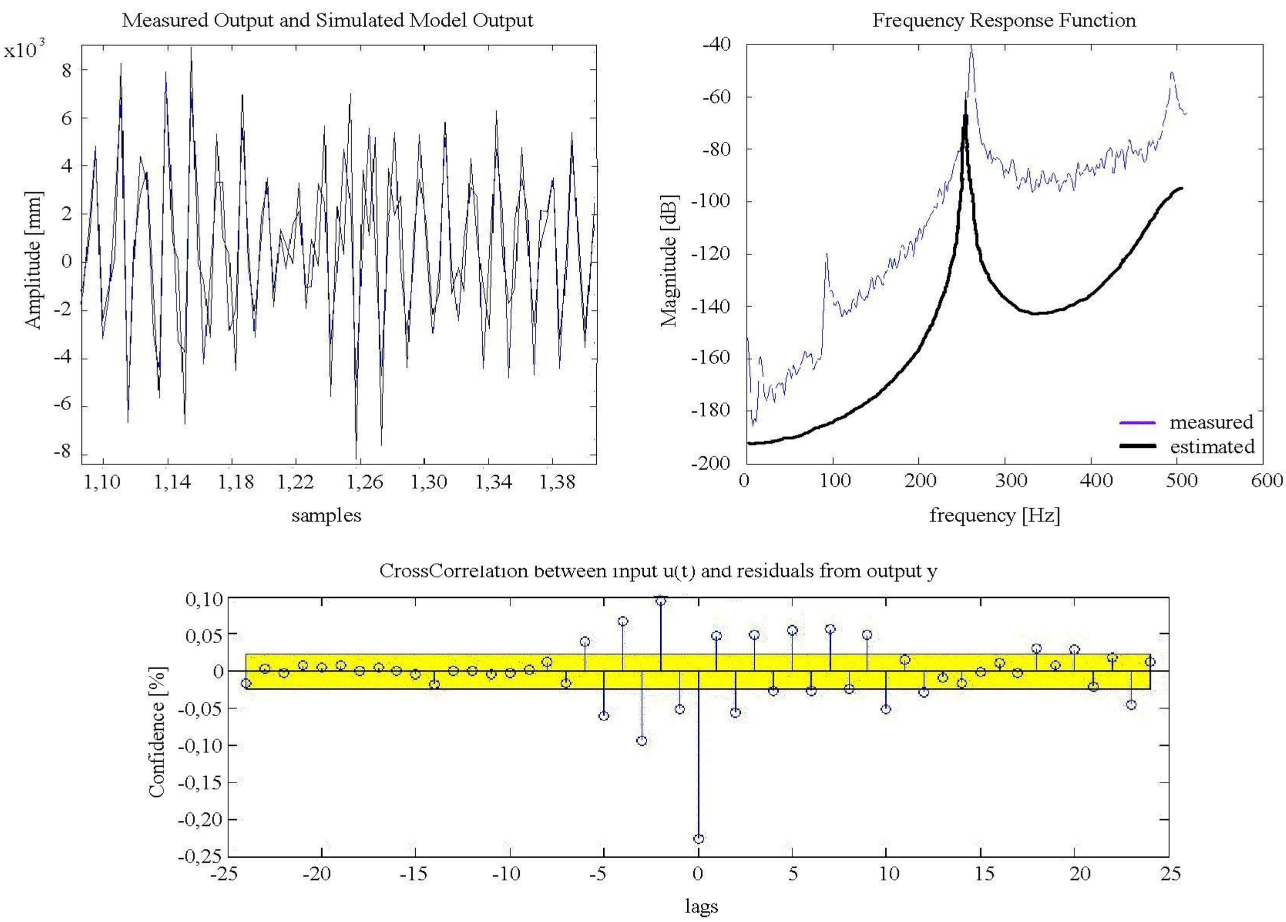


Fig. 11. Comparison between measured signals and estimated Box-Jenkins model: a) fit, b) frequency response, c) cross correlation.

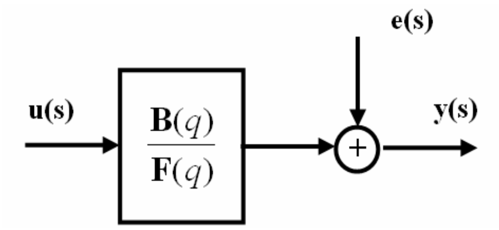


Fig. 12. The OE model structure.

Specifically

$$\begin{aligned} nf &: \mathbf{F}(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{nf} \\ nb &: \mathbf{B}(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1} \end{aligned} \tag{10}$$

Syntax of function is following:
[M, F]=h_bj(ze,zv,[nb1:nb2],,[nf1:nf2],nk, x)

The parameters nb, nf, nk, and nf are the orders of the Output Error model and nk is the delay.

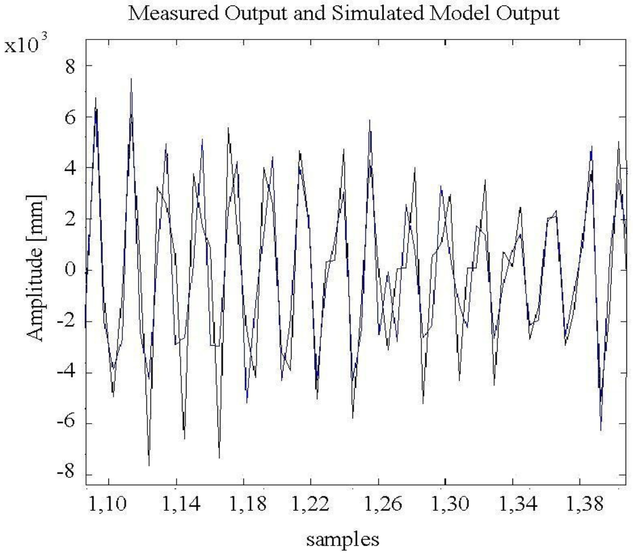
We seek the best model from ranges:
nb = [1:30]
nf = [1:30]
nk = 1 (for nk>1 all models was unstable).

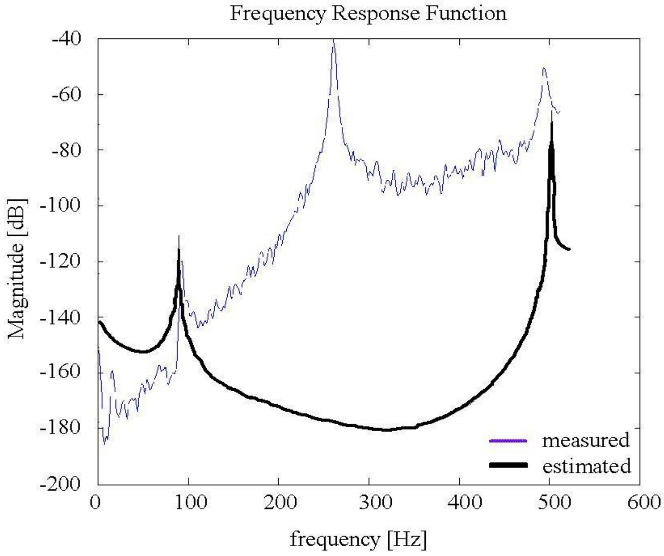
Best fit for modeled data was found by:
nb=15, nf=10, nk=1, Fit = 43,45 %

9. AR model (Auto Regressive)

Models operating on input and output data did not give any satisfactory results. We reject the input signal, and we build a model basing oneself only on given output datas. Last used model was AR.

The AR model structure is a process model used in the generation of models where outputs are only dependent on previous outputs. No system inputs or disturbances are used in the modeling. This is a very simple model that is limited in the class of problems it can solve. Strictly speaking this means that the AR model structure is the model for a signal, not a system. Time





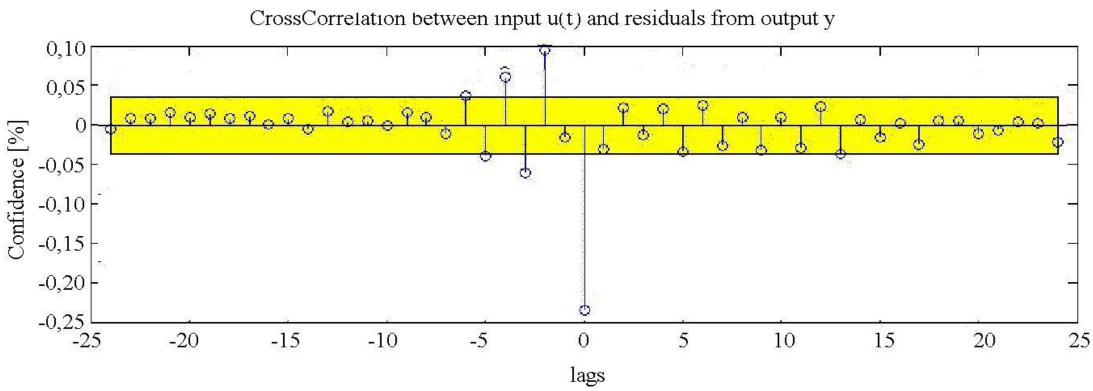


Fig. 13. Comparison between measured signals and estimated Output Error model: a) fit, b) frequency response, c) cross correlation.

series analyses, such as linear prediction coding commonly use the AR model.

Fit of AR model change linearly together with parameter A describing the model. Now our aim is not finding the best fit, only choosing such order of model, so the function of correlation will not exceeded 5% threshold with simultaneous maintenance of possibly high fit and low model order.

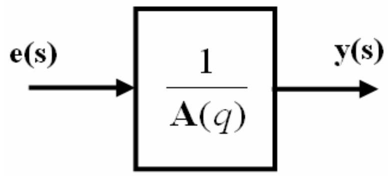


Fig. 14. The AR model structure.

We seek the best model from ranges:
na = [1:60].

Best model was found by:
na=29, Fit = 85,49 %

10. Model parameter Excitation

Once an estimated model has been validated its structural transfer function is used for extraction of modal parameters:
ω_{nl} is the *l*-th natural frequency,
ζ_l is corresponding damping factor,
λ_l and λ_l^* are the *l*-th discrete complex conjugate pair.
dt is the sampling period.

$$\omega_{nl} = \frac{1}{dt}\sqrt{\left(\frac{\ln(\lambda_l\lambda_l^*)}{2}\right)^2 + \left(\cos^{-1}\left(\frac{\lambda_l+\lambda_l^*}{2\sqrt{\lambda_l\lambda_l^*}}\right)\right)^2} \qquad (11)$$

$$\zeta_l = \sqrt{\frac{\left(\ln\left(\lambda_l\lambda_l^*\right)\right)^2}{\left(\ln\left(\lambda_l\lambda_l^*\right)\right)^2 + 4\left(\cos^{-1}\left(\frac{\lambda_l+\lambda_l^*}{2\sqrt{\lambda_l\lambda_l^*}}\right)\right)^2}} \qquad (12)$$

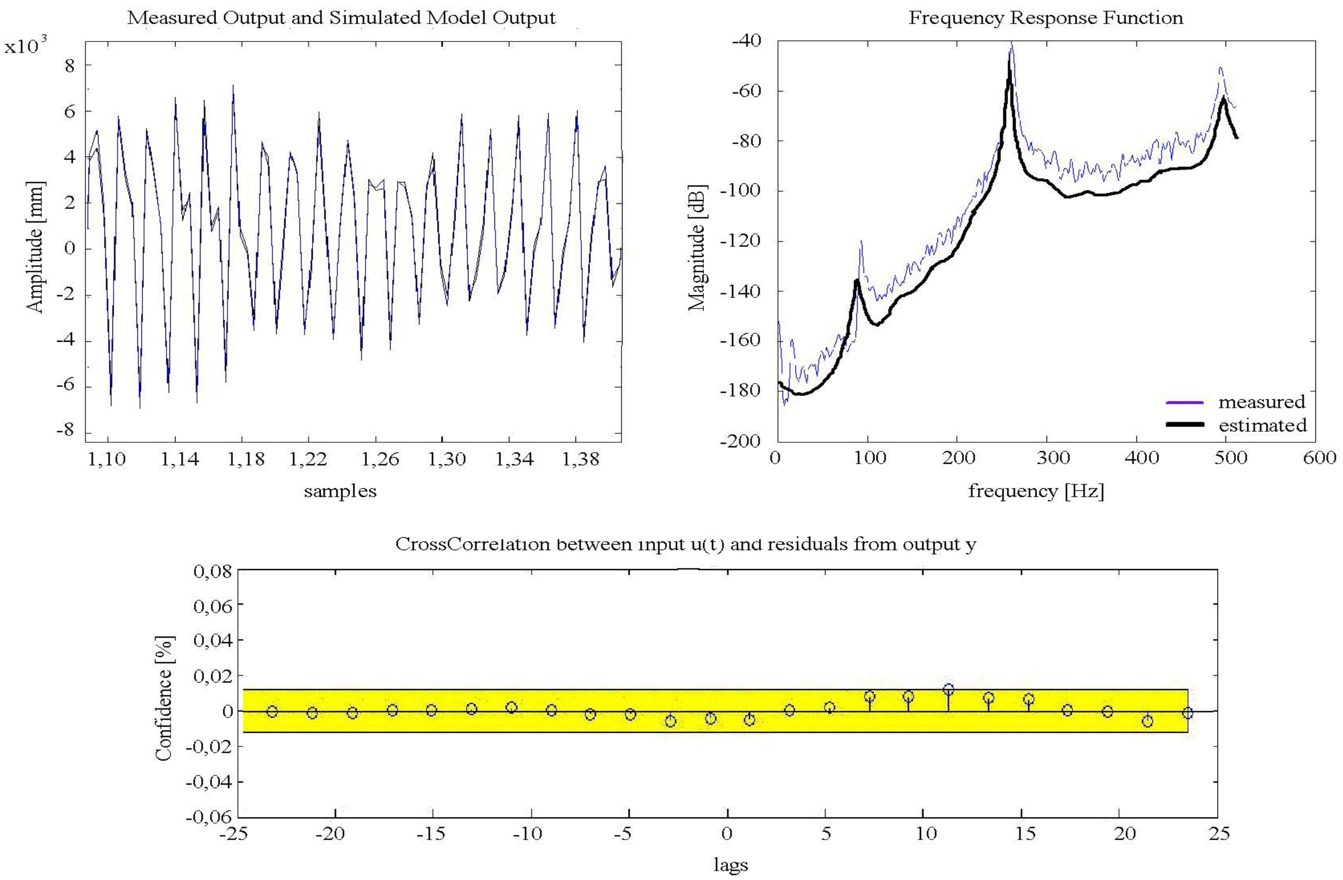


Fig. 15. Comparison between measured signals and estimated AR model: a) fit, b) frequency response, c) cross correlation.

Table 1.
Comparison of natural frequencies (ω_n) and modal damping factors (ζ_n) for all models.

Measured output		ARX(4,3,1)		ARMAX(4,1,6,1)		BJ(9,2,6,4,1)		OE(15,10,1)		AR(29)	
Fit:		54,99%		60,98%		66,31%		43,45%		85,49%	
ω_{nl}[Hz]	ζ_l [%]	ω_{nl}[Hz]	ζ_l [%]	ω_{nl}[Hz]	ζ_l [%]	ω_{nl}[Hz]	ζ_l [%]	ω_{nl}[Hz]	ζ_l [%]	ω_{nl}[Hz]	ζ_l [%]
6,52				6.54	1.0000			6.86	1.0000		
7,73		7,08	1.000								
15,7											
20,07	NOT MESURABLE					19.60	1.0000				
23,26											
43,63											
55,16								56.16	0.0043		
65,58								60.56	0.9832		
77,52										78.53	0.0612
91,56										89.73	0.0612
101,92											
103,16				102,79	0.0500						
104,61											
109,84											
110,68		110,75	1.0000					110.86	0.2957		
124,7											
132,22											
154,11								159.39	0.2415	157.07	0.0612
176,7											
179,98											
186,25											
204,43											
235,29								230.64	0.2320	235.61	0.1608
272,63										277.54	0.1608
298,4											
309,56				301.04	0.0010			301.73	0.0940		
314,76		314.44	1.0000					314.32	0.0321	314.15	0.1608
391,21								388.55	0.0001		

11. Conclusions

1. Models operating on input and output signals did not give satisfactory results (function of correlation goes outside of the admissible range),
2. Rejection of input data permitted on considerable improvement of fit,
3. Correct function of correlation succeeded to obtain only in AR model,

Use automatized procedures of finding best agreements have given considerable shortening of work time.

Acknowledgements

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