

Optimisation of welding parameters for weld penetration in FCAW

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Received 15.11.2005; accepted in revised form 15.04.2006

Analysis and modelling

ABSTRACT

Purpose: This paper describes prediction of weld penetration as influenced by FCAW process parameters of welding current , arc voltage , nozzle-to-plate distance, electrode-to - work angle and welding speed . Optimization of these parameters to maximize weld penetration is also investigated

Design/methodology/approach: It deals with the statistical technique of central composite rotatable design to develop a mathematical model for predicting weld penetration as a function of welding process parameters. The constrained optimization method is then applied to this model to optimize process parameters for maximizing weld penetration

Findings: The result obtained from the developed model indicates that the model predicts the weld penetration adequately. The optimization result also shows that weld penetration attains its maximum value when welding current, arc voltage , nozzle-to-plate distance and electrode-to-work angle are maximum and welding speed is minimum

Research limitations/implications: The statistical technique of developing a model for prediction of weld penetration is valid only within the specified limits of welding process parameters and hence maximization of penetration is also valid within these limits. This technique can be modified to include other parameters such as plate thickness affecting penetration.

Practical implications: The statistical method for modeling and the optimization method used have found increasing applications in a variety of fields for investigation because through these methods research becomes economical , fast and versatile.

Originality/value: The methods described in this paper for weld penetration prediction and optimization can eliminate the need for performing experiments on the basis of the conventional trial and error method which is time consuming and economically not justifiable.

Keywords: Manufacturing and processing; Welding; Weld penetration modelling; Optimisation; Sequential quadratic programming

1. Introduction

Inadequate weld bead dimensions such as shallow depth of penetration may contribute to failure of a welded structure since penetration determines the stress carrying capacity of a welded joint [1]. To avoid such occurrences the input or welding process variables which influence the weld bead penetration must therefore be properly selected and optimized to obtain an acceptable weld bead penetration and hence a high quality joint [2]. To predict the effect of welding process variables on weld

bead geometry and hence quality researchers have employed different techniques [2-17].

This investigation is based on a previous work [3] carried out by one of the authors in which the central composite rotatable design technique had been used to develop a mathematical model correlating the five welding process variables of welding current (I), welding speed (S), arc voltage (V), nozzle-to-plate distance (N), and electrode to work angle (Φ) to the weld bead penetration in flux cored arc welding (FCAW) of a grade of high strength low alloy (HSLA) steels. This is a useful technique which has been used by some researches [2-4,7,9,13-17]. This technique

was able to predict the linear, quadratic and two factor interactive effects [18-21] of process parameters on weld penetration.

Using this model or equation the Sequential Quadratic Programming (SQP) which is an optimization method for constrained optimization problems [22] was used to maximize weld bead penetration thus achieving a high quality joint.

2. Experiments details

2.1. Materials

A grade of HSLA steel of 12 mm thickness had been used as the plate material. Flux cored wire of 1.6 mm diameter (AWS classification E70T5) with 100% CO₂ shielding gas had also been employed on a semi automatic flux cored arc welding (FCAW) machine.

2.2. Experimental design

The required design matrix (table 1) with a total of 32 experimental runs had been developed for the five welding factors selected. The upper (+2) and lower (-2) levels of all the five variables as shown in table 2 had been established by trial runs prior to the actual welding to ensure deposition of an acceptable weld bead. The intermediate levels of -1, 0, +1 of all the variables had been calculated by interpolation.

2.3. Welding procedure and penetration measurement

Bead-on-plate welds were deposited randomly according to the levels of parameters in the design matrix. Specimens were cut off from these plates and depth of penetrations was measured using a profile projector at a magnification of 10.

3. Selection and development of the model

A model relating the depth of penetration to the levels of one or more welding factors is an indispensable aid in the interpretation of results from an experimental design. Wide experience had shown that quadratic relationships are usually adequate and so most surface designs in the literature are for fitting a quadratic equation [18,20]. Based on these considerations the following second degree polynomial relating the response or penetration to all the five welding variables was assumed.

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + \dots \quad (1)$$

where

y, Response variable or yield, e.g. penetration.

x₁,...,x₅, Independent welding factors such as welding current (I), welding speed (s), etc.

b₀, b₁,..., b₅, b₁₂, b₁₃,..., b₄₅, b₁₁,..., b₅₅,

Regression coefficients.

Table 1
Central composite rotatable design matrix for five factors, k=5

S.No.	Factors					y = x ₁ x ₂ x ₃ x ₄ x ₅
	x ₁	x ₂	x ₃	x ₄	x ₅	
1.	1	-1	-1	-1	-1	1
2.	1	1	-1	-1	-1	-1
3.	1	-1	1	-1	-1	-1
4.	1	1	1	-1	-1	1
5.	1	-1	-1	1	-1	-1
6.	1	1	-1	1	-1	1
7.	1	-1	1	1	-1	1
8.	1	1	1	-1	-1	-1
9.	1	-1	-1	1	1	-1
10.	1	1	-1	1	1	1
11.	1	-1	1	1	1	1
12.	1	1	1	1	1	-1
13.	1	-1	-1	1	1	1
14.	1	1	-1	1	1	-1
15.	1	-1	1	1	1	-1
16.	1	1	1	1	1	1
17.	1	-2	0	0	0	0
18.	1	2	0	0	0	0
19.	1	0	-2	0	0	0
20.	1	0	2	0	0	0
21.	1	0	0	-2	0	0
22.	1	0	0	2	0	0
23.	1	0	0	0	-2	0
24.	1	0	0	0	2	0
25.	1	0	0	0	0	-2
26.	1	0	0	0	0	2
27.	1	0	0	0	0	0
28.	1	0	0	0	0	0
29.	1	0	0	0	0	0
30.	1	0	0	0	0	0
31.	1	0	0	0	0	0
32.	1	0	0	0	0	0

x₀, A dummy variable

x₁, Welding current (I)

x₂, Welding speed (S)

x₃, Arc voltage (V)

x₄, Nozzle-to-plate distance (N)

x₅, Electrode-to-work angle (Φ)

Table 2
Factors and their levels

No	Factors	Unit	Notation	Level				
				-2	-1	0	+1	+2
1	Welding Current	Amp	I	250	275	300	325	350
2	Welding speed	Cm/min	S	35	40	45	50	55
3	Arc voltage	Volt	V	27	29	31	33	35
4	Nozzle to plate distance	mm	N	15	17	20	22	25
5	Electrode to work angle	degree	Φ	90	97	105	112	120

Using a computer program all the regression coefficients of the model were computed for the response or depth of penetration.

Table 3
Significant coefficients for penetration

Coefficient	Penetration
b ₀	3.531
b ₁	0.333
b ₂	-0.116
b ₃	0.066*
b ₄	-0.058*
b ₅	0.200
b ₁₂	0.112
b ₁₃	---
b ₁₄	---
b ₁₅	---
b ₂₃	-0.175
b ₂₄	---
b ₂₅	---
b ₃₄	0.162
b ₃₅	---
b ₄₅	---
b ₁₁	---
b ₂₂	---
b ₃₃	---
b ₄₄	---
b ₅₅	---

- Insignificant coefficients

The developed model was tested for adequacy by applying the analysis of variance (ANOVA) and analysis of regression techniques. Similarly to test the significance of regression coefficients the Student's test was employed to find the insignificant terms for 95% confidence level. Once the insignificant terms were identified they were dropped from the model without sacrificing the adequacy of the model. The significant coefficients are shown in table 3.

Using the significant regression coefficients the final model for penetration could be constructed as given below:

$$P = 3.531 + 0.333 * I - 0.116 * S + 0.2 * \Phi + 0.112 * I * S - 0.175 * S * V + 0.162 * V * N + 0.066 * V - 0.058 * N \tag{2}$$

The model so developed could be utilized in predicting the value of depth of penetration for each given set of welding variables by inserting the coded values of the variables in the above equation.

In addition to the adequacy test performed, the validity of the results was also tested with the help of scatter diagram Fig. 1. As is evident from this figure there is a fairly good correlation between the observed and predicted value of the bead penetration.

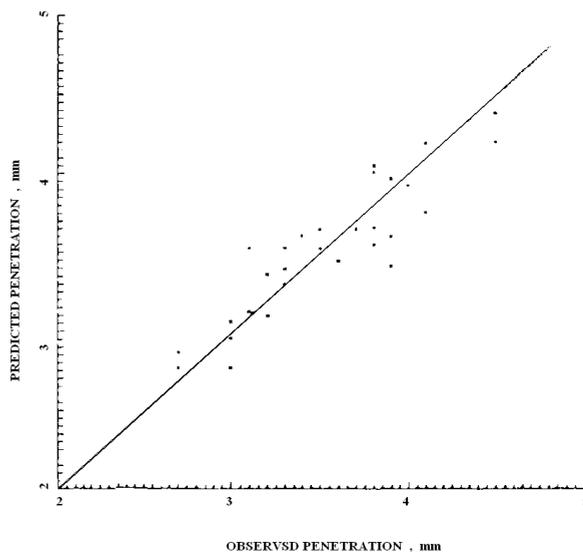


Fig. 1. Scatter diagram showing observed vs. predicted penetration

4. Definition of optimization problem

An optimization or a mathematical programming problem can be stated as follows:

$$\text{Find } X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \text{ which minimizes } f(x)$$

Subject to constraints

$$G_i(x) = 0, \quad i = 1, \dots, m_e$$

$$G_i(x) \leq 0, \quad i = m_e + 1, \dots, m \tag{3}$$

where x is the vector of length n design parameters, $f(x)$ is the objective function, which returns a scalar value, and the vector

function $G(x)$ returns a vector of length m containing the values of the equality and inequality constraints evaluated at x .

An efficient and accurate solution to this problem depends not only on the size of the problem in terms of the number of constraints and design variables but also on characteristics of the objective function and constraints. When both the objective function and the constraints are linear functions of the design variable, the problem is known as a Linear Programming (LP) problem. Quadratic Programming (QP) concerns the minimization or maximization of a quadratic objective function that is linearly constrained. For both the LP and QP problems, reliable solution procedures are readily available. More difficult to solve is the Nonlinear Programming (NP) problem in which the objective function and constraints can be nonlinear functions of the design variables. A solution of the NP problem generally requires an iterative procedure to establish a direction of search at each major iteration. This is usually achieved by the solution of an LP, a QP, or an unconstrained subproblem.

4.1. Constrained optimisation

In constrained optimization, the general aim is to transform the problem into an easier subproblem that can then be solved and used as the basis of an iterative process. A characteristic of a large class of early methods is the translation of the constrained problem to a basic unconstrained problem by using a penalty function for constraints that are near or beyond the constraint boundary. In this way the constrained problem is solved using a sequence of parameterized unconstrained optimizations, which in the limit (of the sequence) converge to the constrained problem. These methods are now considered relatively inefficient and have been replaced by methods that have focused on the solution of the Kuhn-Tucker (KT) equations. The KT equations are necessary conditions for optimality for a constrained optimization problem.

If the problem is a so-called convex programming problem, that is, $f(x)$ and $G_i(x)$, $i=1, \dots, m$ are convex functions, then the KT equations are both necessary and sufficient for a global solution point.

A function $f(x)$ is said to be convex if for any pair of points X_1 and X_2 and all λ , $0 \leq \lambda \leq 1$

$$f[\lambda X_2 + (1-\lambda)X_1] \leq \lambda f(X_2) + (1-\lambda)f(X_1)$$

that is, if the segment joining the two points lies entirely above or on the graph of $f(X)$.

Referring to the definition of optimization problem (Eq. 3) the Kuhn-Tucker equations can be stated as

$$\begin{aligned} \nabla f(X^*) + \sum_{i=1}^m \lambda_i^* \nabla G_i(X^*) &= 0 \\ \lambda_i^* G_i(X^*) &= 0 \quad i = 1, \dots, m \\ \lambda_i^* &\geq 0 \quad i = m_e + 1, \dots, m \end{aligned} \quad (4)$$

The first equation describes a canceling of the gradients between the objective function and the active constraints at the solution point. For the gradients to be canceled, Lagrange

multipliers (λ_i , $i = 1, \dots, m$) are necessary to balance the deviations in magnitude of the objective function and constraint gradients.

Because only active constraints are included in this canceling operation, constraints that are not active must not be included in this operation and so are given Lagrange multipliers equal to zero. This is stated implicitly in the last two equations of Eq. 3. The solution of the KT equations forms the basis to many nonlinear programming algorithms. These algorithms attempt to compute the Lagrange multipliers directly. Constrained quasi-Newton methods guarantee superlinear convergence by accumulating second order information regarding the KT equations using a quasi-Newton updating procedure. These methods are commonly referred to as Sequential Quadratic Programming (SQP) methods, since a QP subproblem is solved at each major iteration (also known as Iterative Quadratic Programming, Recursive Quadratic Programming, and Constrained Variable Metric methods).

4.2. Sequential Quadratic Programming (SQP)

SQP methods represent the state of the art in nonlinear programming methods. Schittkowski [23], for example, has implemented and tested a version that outperforms every other tested method in terms of efficiency, accuracy, and percentage of successful solutions, over a large number of test problems. Based on the work of Biggs [24], Han [25], and Powell ([26,27]), the method allows you to closely mimic Newton's method for constrained optimization just as is done for unconstrained optimization. At each major iteration, an approximation is made of the Hessian of the Lagrangian function using a quasi-Newton updating method. This is then used to generate a QP subproblem whose solution is used to form a search direction for a line search procedure. An overview of SQP is found in Fletcher [28], Gill et al. [29], Powell [30], and Schittkowski [31]. The general method, however, is stated here.

Given the definition of optimization problem (Eq. 3) the principal idea is the formulation of a QP subproblem based on a quadratic approximation of the Lagrangian function.

$$L(X, \lambda) = f(X) + \sum_{i=1}^m \lambda_i g_i(X) \quad (5)$$

Here Eq. 3 is simplified by assuming that bound constraints have been expressed as inequality constraints. You obtain the QP subproblem by linearizing the nonlinear constraints.

4.3. Quadratic Problem (QP) Subproblem

The quadratic problem is as follows:

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{2} d^T H_k d + \nabla f(X_k)^T d \\ \nabla g_i(X_k)^T d + g_i(X_k) &= 0 \quad i = 1, \dots, m_e \end{aligned}$$

$$\nabla g_i(X_k)^T d + g_i(X_k) \leq 0 \quad i = m_e + 1, \dots, m \quad (6)$$

This subproblem can be solved using any QP algorithm. The solution is used to form a new iterate of the following form

$$X_{k+1} = X_k + \alpha_k d_k$$

The step length parameter is determined by an appropriate line search procedure so that a sufficient decrease in a merit function is obtained. The matrix H_k is a positive definite approximation of the Hessian matrix of the Lagrangian function (Eq. 5). H_k can be updated by any of the quasi-Newton methods, although the BFGS method appears to be the most popular.

A nonlinearly constrained problem can often be solved in less iteration than an unconstrained problem using SQP. One of the reasons for this is that, because of limits on the feasible area, the optimizer can make informed decisions regarding directions of search and step length.

4.3. SQP implementation

The SQP implementation consists of three main stages:

- Updating of the Hessian matrix of the Lagrangian function
- Quadratic programming problem solution
- Line search and merit function calculation

These stages are explained briefly

Updating the Hessian matrix

At each major iteration a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function, H , is calculated using the BFGS method, where λ_i , ($i = 1, \dots, m$) is an estimate of the Lagrange multipliers.

$$H_k = H_k + \frac{q_k q_k^T}{q_k^T s_k} - \frac{H_k^T H_k}{s_k^T H_k s_k}$$

where

$$s_k = X_{k+1} - X_k$$

$$q_k = \nabla f(x_{k+1}) + \sum_{i=1}^n \lambda_i * \nabla g_i(X_{k+1}) -$$

$$\left[\nabla f(X_k) + \sum_{i=1}^n \lambda_i * \nabla g_i(X_k) \right]$$

Powell [27] recommends keeping the Hessian positive definite even though it might be positive indefinite at the solution point. A positive definite Hessian is maintained providing $q_k^T s_k$ is positive at each update and that H is initialized with a positive

definite matrix. When $q_k^T s_k$ is not positive, q_k is modified on an element-by-element basis so that $q_k^T s_k > 0$. The general aim of this modification is to distort the elements of q_k , which contribute to a positive definite update, as little as possible. Therefore, in the initial phase of the modification, the most negative element of $q_k * s_k$ is repeatedly halved. This procedure is continued until $q_k^T s_k$ is greater than or equal to $1e-5$. If, after this procedure, $q_k^T s_k$ is still not positive, modify q_k by adding a vector v multiplied by a constant scalar w , that is,

$$q_k = q_k + wv$$

where

$$v_i = \nabla g_i(x_{k+1}) \cdot g_i(x_{k+1}) - \nabla g_i(x_k) \cdot g_i(x_k)$$

if $(q_k)_i \cdot w < 0$ and $(q_k)_i \cdot (s_k)_i < 0$ ($i = 1, \dots, m$)

$$v_i = 0 \quad \text{otherwise.}$$

and increase w systematically until $q_k^T s_k$ becomes positive.

Quadratic Programming problem solution

At each major iteration of the SQP method, a QP problem of the following form is solved, where A_i refers to the i th row of the m -by- n matrix A .

$$\text{Minimize } q(d) = \frac{1}{2} d^T H d + c^T d$$

$$A_i d = b_i \quad i = 1, \dots, m_e$$

$$A_i d \leq b_i \quad i = m_e + 1, \dots, m$$

The method used in the Matlab Optimization Toolbox is an active set strategy (also known as a projection method) similar to that of Gill et al., described in [32] and [33]. It has been modified for both Linear Programming (LP) and Quadratic Programming (QP) problems.

The solution procedure involves two phases. The first phase involves the calculation of a feasible point (if one exists). The second phase involves the generation of an iterative sequence of feasible points that converge to the solution. In this method an active set, \overline{A}_k is maintained that is an estimate of the active constraints (i.e., those that are on the constraint boundaries) at the solution point. Virtually all QP algorithms are active set methods. This point is emphasized because there exist many different methods that are very similar in structure but that are described in widely different terms.

Line search and merit function calculation

The solution to the QP subproblem produces a vector d_k , which is used to form a new iterate of the following form:

$$X_{k+1} = X_k + \alpha_k d_k$$

The step length parameter is determined in order to produce a sufficient decrease in a merit function. The merit function used by Han [25] and Powell [27] is used in Matlab Optimization toolbox.

5. Weld penetration optimization problem formulation

The objective function for weld penetration which must be maximized was derived in section 3 of the paper as Eq 2. The constraints of the welding parameters are given in table 2. Here we restate these equations.

Maximize:

$$P = 3.531 + 0.333 * I - 0.116 * S + 0.2 * \Phi + 0.112 * I * S - 0.175 * S * V + 0.162 * V * N + 0.066 * V - 0.058 * N \quad (2)$$

Subject to constraint:

$$\begin{aligned} 250 &\leq I \leq 350 \\ 27 &\leq V \leq 35 \\ 15 &\leq N \leq 25 \\ 90 &\leq \Phi \leq 120 \\ 35 &\leq S \leq 55 \end{aligned}$$

Matlab Optimization toolbox and the function `fmincon` was used for this optimization problem. The `fmincon` function uses SQP method for optimization.

Two m-file was written namely `objfun` and `confun`. In the `objfun` the objective function was defined and in the `confun` the constraints were set.

Optimization result shows in order to attain the maximum weld penetration the first four factors must be at their maximum value and the last one must be at its minimum value. So the maximum weld penetration will be attained when the following values are chosen for the five factors:

$$\begin{aligned} I &= 350 \text{ amp} \\ V &= 35 \text{ volt} \\ N &= 25 \text{ mm} \\ \Phi &= 120 \text{ degree} \\ S &= 35 \text{ cm/min} \end{aligned}$$

Using these values the weld penetration (P) becomes 5.745 mm.

6. Results and discussions

In addition to the adequacy test performed, the validity of the results of mathematical modeling for prediction of depth of

penetration was also tested with the help of a scatter diagram (Fig. 1). As is evident from this figure there is a fairly good correlation between the observed and predicted value of weld bead penetration.

The optimization result shows penetration will be maximum when welding current, arc voltage, nozzle-to-plate distance and electrode-to-work angle are at their maximum possible value and welding speed is at its minimum value. The physical reasons for the above results are discussed below.

Increase in welding current (I) increases the depth of penetration (P). It is known that molten metal droplets transferring from the electrode to the plate are strongly overheated. It can be reasonably assumed that this extra heat contributes to more melting of the work piece. As current increases the temperature of the droplets and hence the heat content of the droplets increases which results in more heat being transferred to the base plate. Increase in current reduces the size but increases the momentum of the droplets which on striking the weld pool causes a deeper penetration or indentation. The increase in penetration as current increased could also be attributed to the fact that enhanced arc force and heat input per unit length of the weld bead resulted in higher current density that caused melting a larger volume of the base metal and hence deeper penetration.

Increase in welding (S) causes a decrease in depth of penetration (P). This may be attributed to lesser heat input at higher speeds per unit length of the weld bead which caused a smaller weld pool and decreased depth of penetration.

Increase in arc welding voltage (V) resulted in an increase in depth of penetration (P), because of increase in heat inputs reported by McGlone and Chadwick [34].

Increase in electrode-to-work angle from 90° to 120° (i.e. for normal to backhand) had resulted in increase of depth of penetration. The backhand welding technique has been reported [35] to provide the best penetration, as the arc force keeps the slag from running in front of the liquid pool.

Increase in nozzle-to-plate distance (N) also causes an increase in depth of penetration (P) which may be due to higher temperature of the droplets impinging on the weld pool because of more resistance heating of the wire at higher nozzle-to-plate distances.

Based on this investigation it can be concluded that the developed model can be used to predict adequately the weld bead penetration within the specified limits of the process parameters. The optimization method can also be used to find optimum welding conditions for maximum weld bead penetration.

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