Journal of Achievements in Materials and Manufacturing Engineering

VOLUME 14
ISSUE 1-2
January-February
2006

Analytical methods application to the study of tube drawing processes with fixed conical inner plug: Slab and Upper Bound Methods

E.M. Rubio *
Department of Manufacturing Engineering, National Distance University of Spain (UNED), Juan del Rosal 12, 28040, Madrid, Spain
* Corresponding author: E-mail address: erubio@ind.uned.es

Received 15.11.2005; accepted in revised form 31.12.2005

Analysis and modelling

ABSTRACT

Purpose: The paper analyses the thin-walled tube drawing processes made in conical converging dies with fixed inner plugs.
Design/methodology/approach: The analysis has been made by analytical methods. Concretely, the Slab Method (SM), with and without friction effects, and the Upper Bound Method (UBM) have been applied. In this last case, the plastic deformation zone has been modelled by Triangular Rigid Zones (TRZ). The friction between the out surface of the tube and the die, and between the inner surface of the tube and the plug has been modelled by Coulomb friction. Besides, the change in the diameter of the tube has been considered negligible during the process, then, the forming process can be assumend that it is made under a state of plane strain.
Findings: The results obtained by each method have been compared between them and, besides, with the obtained ones by the Finite Element Method (FEM) and the experimental ones extract from the literature about the theme. It has been able to be proven that all of them are reasonably close to a value for a certain set of parameters values.
Research limitations/implications: This work is a first approache to the problem. As suggestion for future researches it is possible to remark a study of the different triangular rigid zones configurations in order to identify the TRZ pattern that requires smaller energy consumption.
Practical implications: Unless it was needed to know a great number of outputs, the analytical methods can be a good option and, especially, the Upper Bound Method since, besides of completing and improving the classical analysis of other metalforming processes previously.
Originality/value: The paper is orginal since the bibliographical review has allowed testing that previous works about the tubes drawing analysis made by the Upper Bound Method under plane strain and Coulomb friction conditions did not exist until the moment.
Keywords: Machining and processing; Plastic forming; Tube drawing; Slab method; Upper bound method

1. Background

Most of metallic tubular pieces obtained by primary processes, such as casting, impact extrusion, deep drawing or ironing, require, before being used, other processes, denominated secondary, to take its diameter, the thickness of its wall or both until standardized values of provision [2,3,26,48,52].

The secondary processing of tubes can be carried out by diverse processes of metalforming. The basic process consists of making pass a tube through a conical convergent die by means of which, of controlled form, its diameter and/or its thickness are reduced. The process can require or not the use of mandrels and plugs that are located inside the tube and delimit the dimensions of their inner diameter. The main variants of this process are: *sinking*, *mandrel and plug drawing* and *ironing*.

Tubes drawing processes began to study at the beginning of the last century. It is possible to emphasize the work of Siebel and Weber that, in 1935, studied the distribution of tensions and the metal flow in this type of processes [11].

In the middle of the Fortieth, Sachs directed to a series of works about the diminution of diameter and thickness of thin-walled tubes [19-22]. Sachs and his collaborators calculated the necessary stress to draw tubes using different internal tools, mainly movable mandrels. Their studies, based on the method of stress local analysis, were tested by means of experiments in later works [19].

At the beginning of the Fiftieth, Hill analysed, from an industrial point of view, the requirements necessary to predict the drawing force and the change in the wall thickness [51] and Sachs, along with Hoffman, compiled in their book entitled *Introduction to the theory of plasticity for engineers* the main theories and conclusions published previously on the subject in scientific journals [48].

The researches of Blazynski and Cole [59], Johnson and Mellor [60] and Green [46] are in the same line of that one initiated by Sachs. Blazynski and Cole made an experimental study of the tube drawing using plugs. The most important result of that work was the semiempirical method that they proposed for the calculation of the additional or redundant power [59]. Green contributed to the study of the processes of tube drawing with his theoretical works for the calculation of the additional power applied to cases of flat deformation [46]. On the other hand, Elion and Alexander carried out an industrial research for the optimization of variables of this kind of processes [52]. The experimental and theoretical study of the reduction of tube diameter made by Moore and Wallace is also of this time [22].

Until the Ninetieth, when the first works of tubes drawing analysed by numerical methods began to appear and, in particular, by the Finite Element Method (FEM) [10][35][40-43][55], the contributions to the subject that deserve a special mention are those of Avitzur [2], who experimentally analysed the main processes related to the tubes drawing, of Collins and Williams [27], who proposed an axisimmetric slide-lines field analogous to Hill one [50] used in the bars drawing, and of Um and Lee, who gave an approximated solution to the problem of tubes drawing with fixed plugs with partial friction by means of the Limit Analysis (LA) [37].

The bibliographical review has allowed to conclude that, from the appearance of numerical methods, the analytical methods have been displaced. Not only in the study of the tube drawing but also in the rest of the metalforming processes.

Basically, due to the numerical methods allow, in general: modelling these processes in a relatively simple way, even when these involve a great amount of variables; obtaining multitude of results with a high precision; and comparing results by simple observation of the simulations offered by most of software programmes [1,5,7,9,14,15,23,28,30,42,44,47,49,58]. Nevertheless, numerical methods require great resources of calculation and a good knowledge of the used programme, which usually is translated in high economic investments, for the acquisition of the hardware and software necessary to make the calculations, and time investment, for its learning and handling. Then, unless it is needed to know a great number of outputs, the analytical methods can be a good option [4,9,17,24,29,38,39].

In this work a study of the thin-walled tubes drawing (h_i<<D_{Ii}, where h_i is the initial thickness of the tube and D_{Ii} its initial inner diameter) in conical convergent dies with an inner plug has been made. The used analytical method has been the Limit Analysis, and, more concretely, the Upper Bound Method (UBM). The plastic deformation zone has been modelled by means of Triangular Rigid Zones (TRZ). The friction in the interfaces: tube outer surface-die and tube inner surface-plug has been assumed as Coulomb type, denominating them by μ_1 and μ_2 respectively. Besides, the tube inner diameter diminution has been considered negligible ($\Delta D_{Ii}/D_{Ii}$<<1), as it usually happens in most of industrial processes. Then it is possible suppose that the inner diameter is constant and very close to the outer diameter value. In this case, like circumferential deformation does not exist, it is possible to consider that the process takes place under plane strain conditions. The results have been compared with the obtained ones by the Slab Method (SL), with and without friction, the Finite Elements Method and other ones extracted from the experimental tests found in the literature about this subject.

2. Tubes drawing in convergent dies with a fixed plug

2.1. Geometric definition

The process to analyse is the thin-walled tube drawing through conical convergent dies with an inner, conical or cylindrical, plug fixed to the draw bench where the process is made, like Figure 1 shows.

The tube inner diameter is going to be considered constant along the process $D_{Ii} \approx D_{If} \approx D$, varying only the thickness from an initial value of h_i, to a final one of h_f, as it is shown in Figure 2 in a schematic way. In order to see better the zone between the die and the plug, this one has been extended.

2.2. Process variables

The process variables related to with the geometry are:

- The *conical convergent die semiangle*, α.
- The *fixed conical plug semiangle*, β, placed inside of the die.
- The *tube cross-sectional area reduction*, r, defined by:

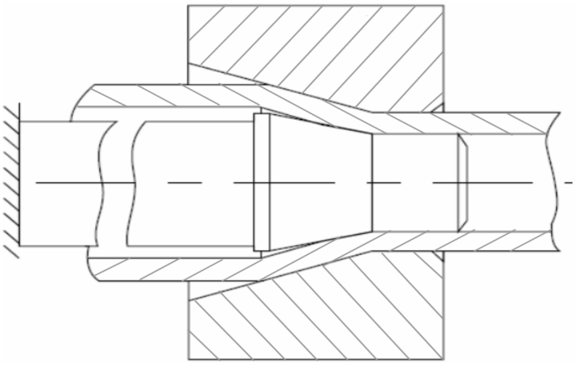


Fig. 1. Scheme of the tubes drawing process in convergent conical die with fixed conical plug.

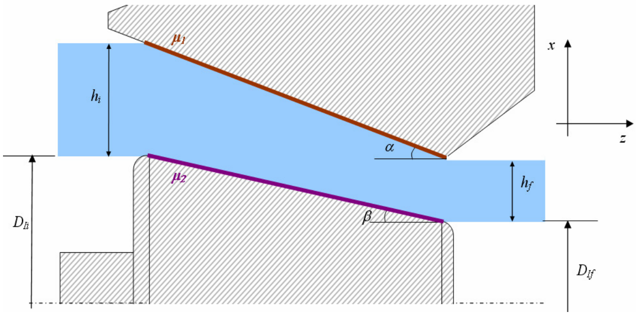


Fig. 2. Detail of the plastic deformation zone.

$$r = \frac{A_i - A_f}{A_i} \tag{1}$$

Taking into account:

$$A_i = \pi\left[\left(\frac{D_{Ei}}{2}\right)^2 - \left(\frac{D_{Ii}}{2}\right)^2\right] \tag{2}$$

$$A_f = \pi\left[\left(\frac{D_{Ef}}{2}\right)^2 - \left(\frac{D_{If}}{2}\right)^2\right] \tag{3}$$

where D_{Ei} and D_{Ef} represent the initial and final outer diameters of the tube respectively and, D_{Ii} and D_{If} the inner ones, initial and final as well. Both are related by means of the following expressions:

$$D_{Ei} = D_{Ii} + 2h_i \tag{4}$$

$$D_{Ef} = D_{If} + 2h_f \tag{5}$$

being h_i and h_f, the thickness at the entrance and at the exit of the die respectively. Introducing the values given by (4) and (5) in the expressions (2) and (3) and, carrying these into (1) it is possible to write:

$$r = \frac{D_{Ii}^2\left[\left(\frac{h_i}{D_{Ii}}\right) + \left(\frac{h_i}{D_{Ii}}\right)^2\right] - D_{If}^2\left[\left(\frac{h_f}{D_{If}}\right) + \left(\frac{h_f}{D_{If}}\right)^2\right]}{D_{Ii}^2\left[\left(\frac{h_i}{D_{Ii}}\right) + \left(\frac{h_i}{D_{Ii}}\right)^2\right]} \tag{6}$$

Taking into account that in the tube drawing $D_{Ii} \approx D_{If} \approx D$, the previous expression can be written:

$$r = \frac{\left[\left(\frac{h_i}{D}\right) + \left(\frac{h_i}{D}\right)^2\right] - \left[\left(\frac{h_f}{D}\right) + \left(\frac{h_f}{D}\right)^2\right]}{\left[\left(\frac{h_i}{D}\right) + \left(\frac{h_i}{D}\right)^2\right]} = 1 - \frac{\left[\left(\frac{h_f}{D}\right) + \left(\frac{h_f}{D}\right)^2\right]}{\left[\left(\frac{h_i}{D}\right) + \left(\frac{h_i}{D}\right)^2\right]} \tag{7}$$

Besides, $h_i << D_{Ii} \approx D_{If} \approx D$ and $h_f < h_i$, then $h_f << D_{Ii} \approx D_{If} \approx D$ and, therefore, in the numerator and the denominator can be depreciated the second order terms, being, finally, the expression:

$$r = 1 - \frac{h_f}{h_i} \tag{8}$$

If there were variation of the tube inner diameter and then $D_{Ii} > D_{If}$ was verified, the expression for the *tube cross-sectional area reduction* would have to be written in the following way:

$$r = 1 - \frac{D_{If} h_f}{D_{Ii} h_i} \tag{9}$$

2.3. Process conditions

The stress state in which the deformation along a direction is null, is known like *plane strain*. When in a thin-walled tube drawing process with fixed plug there is no appreciable variation of its inner diameter ($h_i \approx h_f << D_{Ii} \approx D_{If} \approx D$), the material placed between the die and the plug is under a stress state like the mentioned one. Then, the process is carried out under *plane strain conditions*.

In this type of processes, the metal flow is always parallel to the plane (x, z) and is restricted in the circumferential direction, by the die and the plug. Therefore, $d\varepsilon_2 = 0$ and, if there is neither change of volume nor elastic deformation (incompressible rigid-plastic materials), also $d\varepsilon_1 = - d\varepsilon_3$. According to Hill [28], these expressions represent a material under shear stress state in which the flow takes place as a result of applying k, denominated *shear yield stress*.

In addition, a superposed hydrostatic stress, σ_2, (generally of compression) can exist. This stress alters the values of σ_1 and σ_3 but does not influence on the flow. In these conditions the principal stresses can be written:

$$\sigma_1 = \sigma_2 - k \tag{10}$$

$$\sigma_3 = \sigma_2 + k \tag{11}$$

$$\sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3) \tag{12}$$

this takes to:

$$\sigma_1 - \sigma_3 = 2k \tag{13}$$

And, taking into account the relationship from von Mises, it can be written

$$k = \frac{S}{2} = \frac{Y}{\sqrt{3}} \tag{14}$$

Where: Y, it is the *uniaxial yield stress* and, S, the *yield stress under plane strain*.

The metal at the die exit is in a state of uniaxial stress rather than of plane strain, because it is free to undergo transverse or circumferential strains [25][45][51]. For that reason, some authors recommend that the plug was slightly larger than that necessary one to obtain the precise dimensions of the tube [2].

The strength that finally limits the last pass is the *uniaxial yield stress*, Y, and no the *yield stress under plane strain*, S. Although, the plane strain conditions stay in the real deformation zone. It is supposed the breakage is reached as soon as the *uniaxial yield stress*, Y, appears in the tube drawing. This condition comes given by the following expression, that can be considered like the limit of the tubes drawing.

$$\frac{\sigma_{zf}}{S} = \frac{Y}{S} = 0{,}866 \tag{15}$$

On the other hand, when there is a relative movement between two surfaces in contact a resistance to the movement denominated friction appears. The mechanisms that produce this phenomenon are complex and exact functional relationships between the friction and other variables of the process have not been, still, established.

The most common simplifications suppose that the *friction stress*, τ, between two surfaces that slide can be:

- Coulomb type, when the *shear stress*, τ, is proportional to the pressure, p, between the surfaces in contact according to the expression $\tau=\mu p$, where the proportionality coefficient, μ, is called *Coulomb friction coefficient*.
- Partial friction type, where the *shear stress*, τ, is assumed proportional to the *shear yield stress*, k, and given by $\tau=mk$, verifying the values of the *partial friction coefficient*, m, next expression $0<m<1$.

In this case, the existing friction between the interfaces: die-tube outer surface and plug-tube inner surface have been considered of Coulomb type and with values μ_1 *and* μ_2 respectively.

3. Classical analysis

3.1. Introduction

In the tubes drawing, like in most of the metalforming processes, it is important to know the energy that is required to make it, mainly, in order to select the machines and the equipment where the process will be carried out.

The total energy E_T necessary in a metalforming process can be assimilated to the work developed by a force that produce a determined displacement. Dividing the energy by the total volume of the piece, it is possible to calculate the specific energy.

On the other hand, that energy by volume unit can be obtained as well by means of:

$$u = \frac{E}{l^3} = \sigma\varepsilon \tag{16}$$

Like the deformation, ε, is adimensional, the stress, σ, has the same dimensions that the *specific energy*, u. In big deformations and for materials with hardening by deformation ($\sigma \neq cte$ for ε variable), the specific energy by volume unit is calculated by means of:

$$u = \int_0^{\varepsilon_1} \sigma d\varepsilon \tag{17}$$

Next, two basic analyses used, traditionally, in the estimation of the energy involved in the metalforming processes are going to be applied in the tube drawing case.

The first one is the homogeneous deformation method that only considers the change in the shape of the piece. The second one is the stress local analysis method that, in addition to the homogeneous deformation energy, considers the dissipated one by friction.

3.2. Homogeneous deformation method

The Homogeneous Deformation Method (HDM) is based on supposing that any cubical element of the original metal is transformed into a parallelepiped one when the piece is plastically deformed. This method, that only considers the energy necessary to change the shape of the piece, allows obtaining a small value of the necessary energy. Sometimes has been considered like the Slab Method without friction.

The expression of the specific energy used in deforming a piece homogeneously can be written in adimensional terms, dividing by the *yield stress* of the material that is being deformed. In the plane strain case that it is being analysed, this one comes given by S and, therefore, it is possible to write:

$$\frac{\sigma_{zf}}{S} = \ln\frac{1}{1-r} \tag{18}$$

Where:

- σ_{zf} is the drawing stress at the die exit.
- S, the *yield stress under plane strain.*
- r, the tube *cross-sectional area reduction*.

The expression of the energy required in the process by means of specific and adimensional terms is especially useful since, this will allow knowing the necessary energy for making a concrete piece when its material and its volume were known.

Taking into account the r value given by (8) and the expression (14) it is possible write:

$$\frac{\sigma_{zf}}{2k} = \ln\frac{h_i}{h_f} \tag{19}$$

The solution given by the Homogeneous Deformation Method is not very precise since it only considers the reduction, r, but it does not consider neither the friction between tube-die and between tube-plug (μ_1 and μ_2) nor its geometry (α and β). This can be seen in Tables 1 and 2, where the values given by the expression (19) have been calculated for different *tube cross-sectional area reductions* (r = 0,10; 0,20 and 0,30).

Concretely, in Table 1, the specific adimensional energy versus the *semiangle of the conical plug,* β, has been collected for any values of α, μ_1 and μ_2 and, in Table 2, the specific adimensional energy versus the *semiangle of the die,* α, for any values of β, μ_1 and μ_2.

Table 1.
Homogeneous deformation versus β for any values of α, μ_1 and μ_2.

α (°)	β (°)	μ_1	μ_2	HDM r =0,10	HDM r =0,20	HDM r =0,30
0-15	0-14	0-0,30	0-0,30	0,105	0,223	0,357

Table 2.
Homogeneous deformation versus α for any values of β, μ_1 and μ_2.

α (°)	β (°)	μ_1	μ_2	HDM r =0,10	HDM r =0,20	HDM r =0,30
0-15	0-14	0-0,30	0-0,30	0,105	0,223	0,357

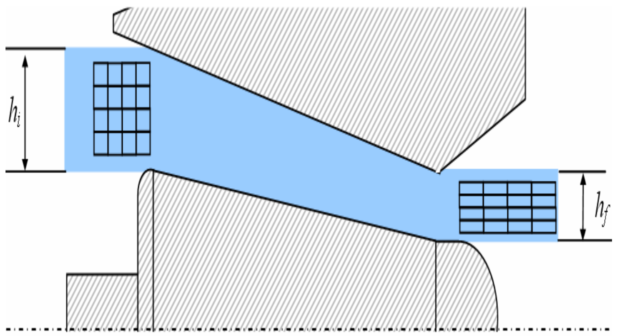


Fig. 3. Homogeneous deformation method in the tubes drawing.

3.3. Slab method

The Slab Method (SM), also known like stress local analysis method, considers the friction. Then, the results obtained by this method are, in general, a better approach to the solution of the problem.

Figure 4 presents a scheme of the process. In particular, a differential element of the tube placed between the die and the plug has been represented along with the stresses acting on it. Supposing that the process take place under plane strain conditions ($h<<D$ and $D \approx$ constant) and that the pressure on the die, p_1, is equal to the pressure on the plug, p_2, and of value, p, then, the forces balance provides the next expression:

$$\sigma_z h - (\sigma_z + d\sigma_z)(h+dh) - ptg\alpha dz + ptg\beta dz - p(\mu_1+\mu_2)dz = 0 \tag{20}$$

Taking into account that the variation of the wall thickness can be represented by:

$$dh = (tg\alpha - tg\beta)dz \tag{21}$$

the equation (21) is

$$hd\sigma_z + \left(\sigma_z + p(1+B^*)\right)dh = 0 \tag{22}$$

where, B^* comes given by:

$$B^* = \frac{\mu_1+\mu_2}{tg\alpha - tg\beta} \tag{23}$$

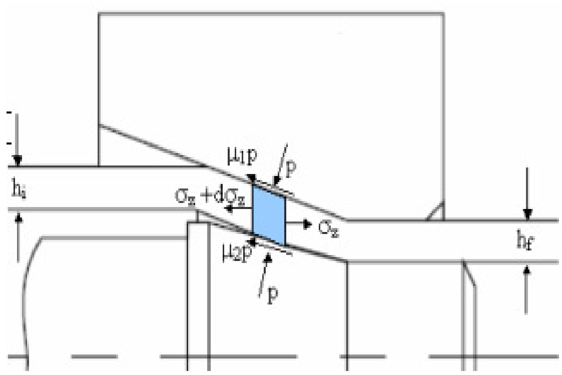


Fig. 4. Stresses acting on a thin-walled tube element.

This function is decreasing when α increases, it is creasing when β increases, and, following this general tendency, it presents higher values according to μ_1, μ_2 or both increase. This can be seen in Figures 5 and 6 where the constant B^* has been plotted versus α, for β=0°, $\mu 2$=0 and μ_1 varying from 0.01 to 0.30 (Figure 5) and, versus β, for α=20° and the same μ_2 and μ_1 values that in the previous case (Figure 6).

The forces balance in the radial direction allows supposing that the contribution of the friction to the pressure in the die is small and that the tensions can be approximated by:

$$\sigma_1 = \sigma_z \tag{24}$$

$$\sigma_2 = -p \tag{25}$$

When a closed pass is made and there is not diameter variation such as stresses can be related with the flow condition in plane strain by means of:

$$\sigma_1 - \sigma_2 = S = 1{,}155Y \tag{26}$$

$$\sigma_z + p = S \tag{27}$$

Substituting p in the expression (22) and operating:

$$\frac{d\sigma_z}{B^*\sigma_z - S(1+B^*)} = \frac{dh}{h} \tag{28}$$

This expression is valid for any B^* and S values, but the simplest solution is obtained when μ and S are constant or they take means values and the die walls and the plug are right and then, they can take constant. Integrating directly:

$$\frac{\sigma_{zf}}{S} = \frac{1+B^*}{B^*}\left(1-\left(\frac{h_f}{h_i}\right)^{B^*}\right) \tag{29}$$

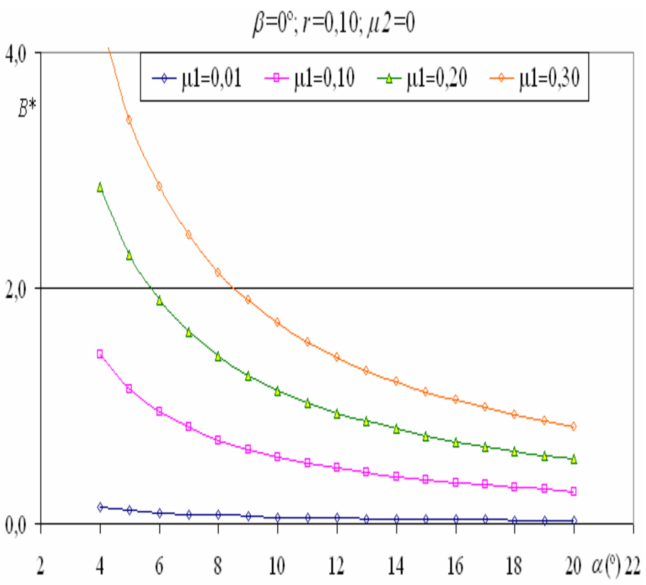


Fig. 5. Representation of the B^* function versus α.

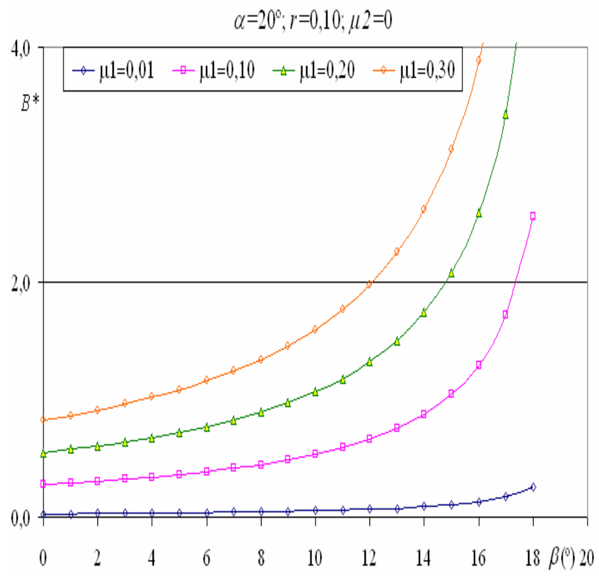


Fig. 6. Representation of the B^* function versus β.

Where the integration constant has been calculated taking into account that back pull does not exist, this is, the stress is $\sigma_z=\sigma_{zi}=0$ *for* $h=h_i$. Making the same as in the homogeneous deformation case, according to the relationship (14) S can be substituted for $2k$.

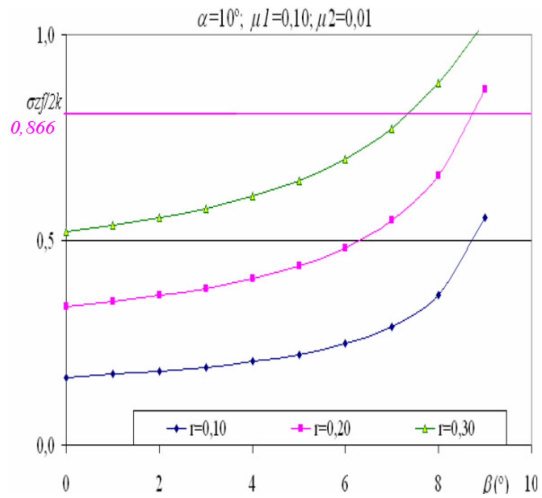
In Figure 7, the values given by the expression (29) have been represented. With the intention of seeing, at least in a qualitative way, how each one of the variables influences in the adimensional total energy, this has been calculated for different combinations of the same ones varying one of them every time.

The curves shown in Figure 7a present the adimensional total energy evaluated by means of the stress local analysis versus the β semiangle value, for α=10º, r = 0,10; 0,20 and 0,30, and $\mu_1=\mu_2$=0,01.

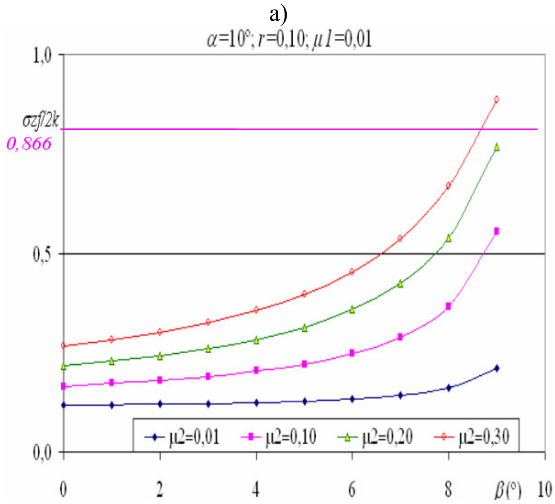
If these curves are compared with the values collected in Tables 1 and 2, it can be seen that among them certain differences exist that pick up, in fact, the influence of the geometry, through the α and β, and the friction by means of the coefficients μ_1 and μ_2 that are not kept in mind by the method of the homogeneous deformation. The curves of the Figure 7a are quite right and parallel to the axis x until values of relatively high in connection with the one rehearsed, clearly similar tendency to the one obtained in homogeneous deformation in which one does not keep in mind the friction.

Making a similar study to the shown one, but fixing the conical semiangle value of the plug, β, the graphs collected in Figure 8 have been obtained. A cylindrical plug, β=0º has been taken, since, as it can be seen in the Figure 7, this value provides the smallest values in the adimensional total energy necessary to carry out the process.

The behaviour versus the friction variations both in the die and in the plug is similar to the commented one previously. However, $\sigma_f/2k$ diminishes when the semiangle α increases. Therefore, the stress local analysis seems to indicate that, to carry out the tubes drawing under good conditions, this is, to carry out the process using the smallest possible energy, it would be necessary: high semiangles α, small semiangles β and friction coefficient μ_1 and μ_2 so small as it was possible.



a)



b)

Fig. 7. Adimensional total energy vs. β, for different reductions, r, and friction coefficient values μ_1 and μ_2.

Relationship between the pressure in the die and in the plug

Supposing that the contribution of the friction to the pressure of the die is small allows, also, justifying that the pressure in the die and in the plug can take practically same. In fact, from the forces balance provides the next relationship:

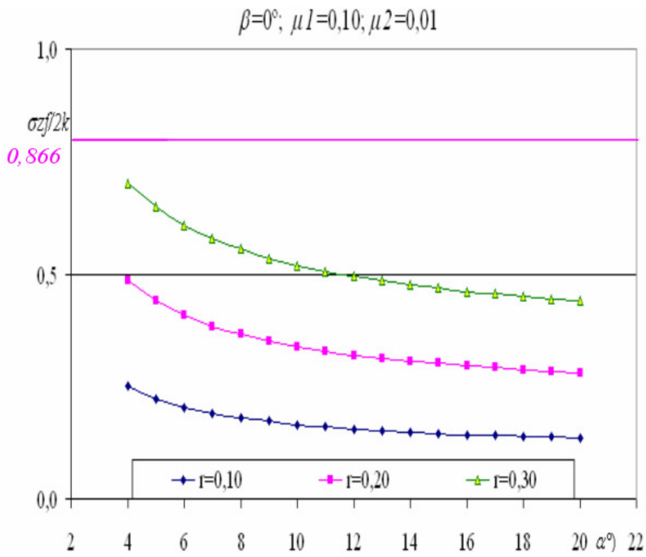
$$p_2 = p_1 \frac{1-\mu_1 tg\alpha}{1+\mu_2 tg\beta} \tag{30}$$

In order to determine the range of values that complete that the pressure in the plug, p_2, is equal to the pressure in the die, p_1, the value of the expression (30) has been calculated for values of: α between 5º and 20º; β between 0º and 15º and μ_1 and μ_2 between 0 and 0,30. In Figure 9, some of the obtained extreme values have been colleted with the purpose of determining the values that complete the supposition.

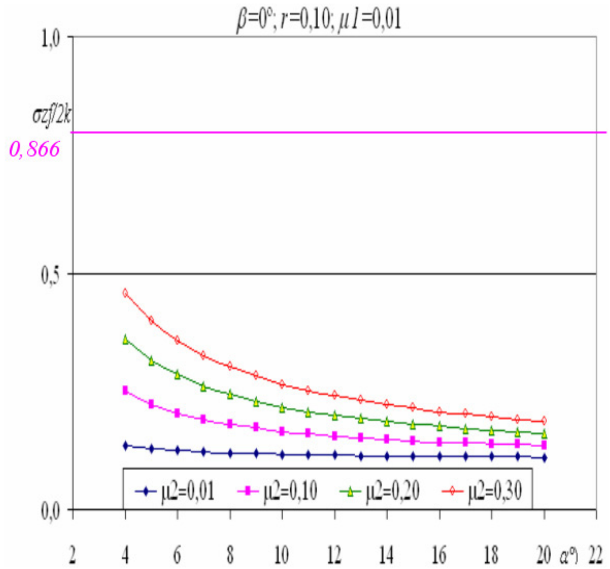
Study of the constant B^*

The constant B^* adopts, in its more general form, the expression collect in (23), however, typical cases exist, as for example cylindrical plug β=0º and with the same material that the die μ_1=μ_2, where it admits more reduced expressions as, for example, the shown ones in Table 3. It is interesting to notice that, if B^* = 0 the equation (29) adopts the following expression:

$$\frac{d\sigma_z}{S} = -\frac{dh}{h} \tag{31}$$



a)



b)

Fig. 8. Adimensional total energy vs. α, for different reductions, r, and friction coefficients values μ_1 and μ_2.

The integration of this expression taking into account that there is no back pull at the die entrance provides the expression of the adimensional stress at the die exit given by the homogeneous deformation method:

$$\frac{\sigma_{zf}}{S}=\ln\frac{h_i}{h_f} \tag{32}$$

Therefore, an approach to the adimensional stress due to the friction will be possible to have subtracting to the obtained value with the expression (29) the value provides by (32). This is:

$$\left(\frac{\sigma_{zf}}{S}\right)_T=\frac{1+B^*}{B^*}\left(1-\left(\frac{h_f}{h_i}\right)^{B^*}\right) \tag{33}$$

$$\left(\frac{\sigma_{zf}}{S}\right)_F=\frac{1+B^*}{B^*}\left(1-\left(\frac{h_f}{h_i}\right)^{B^*}\right)-\ln\frac{h_f}{h_i} \tag{34}$$

where:

- The subindex T has been used for representing the adimensional total energy and
- The F one for the energy necessary to overcoming the external friction.

The values of (32), (33) and (34) have been represented in Figure 10 versus the semiangle of the plug β, for α=10º, r=0,10 and μ_1=μ_2=0,01, in Figure 10a, and μ_1=μ_2=0,10, in the Figure 10b.

Maximum reductions

The stresses local analysis allows calculating the maximum *tube cross-sectional area reduction*, r_{max} that it is possible to reach for some certain values of the rest of the variables that intervene in the process. Indeed, combining the equations (15) and (29):

$$\frac{1+B^*}{B^*}\left(1-\left(\frac{h_f}{h_i}\right)^{B^*}\right)=0{,}866 \tag{35}$$

This expression provides the h_f/h_i value that produces the maximum reduction, as it is shown in the following expression:

$$r_{\max}=1-\frac{h_f}{h_i}=1-\left(\frac{1+0{,}133B^*}{1+B^*}\right)^{\frac{1}{B^*}} \tag{36}$$

The $\sigma_{zf}/2k$ value versus the reduction r, has been plotted in Figure 13 for different values of the constant B^*.

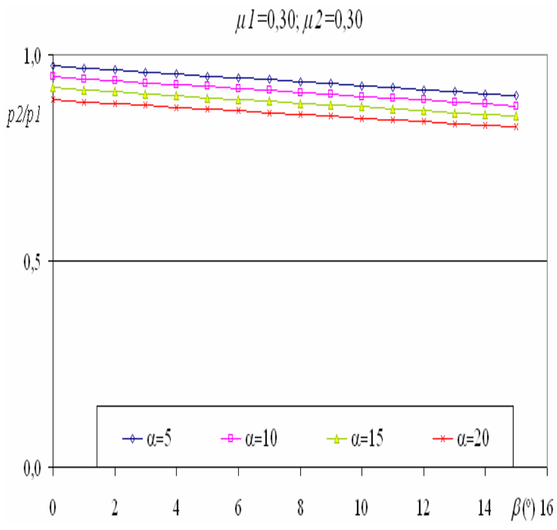
The limit imposed to the tubes drawing given by the expression (15) has been represented as well by means of a horizontal line.

The maximum reduction r_{max} is obtained by means of the intersection of each one of the curves with this limit horizontal line.

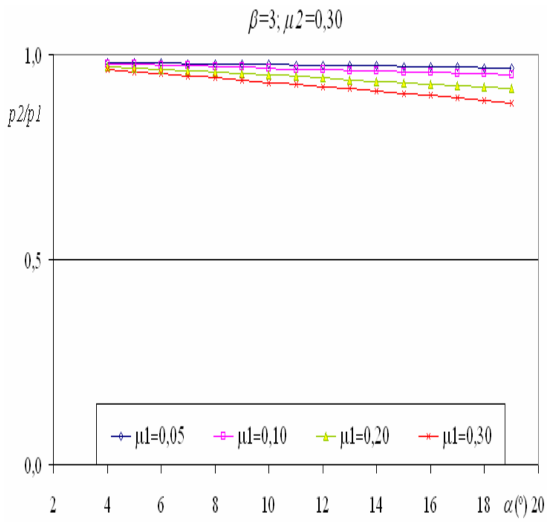
The maximum reduction, r_{max}, obtained in the drawing of tubes made of a rigid-perfectly plastic material and carried out in a die with an inner plug and without considering the friction $B^*=0$, is equal to 0,58, as it is possible to see in the Figure 11.

Table 3.
Typical expressions of the constant B^*.

Cylindrical plug β=0º	$B^*=\dfrac{\mu_1+\mu_2}{tg\alpha}$
Cylindrical plug β=0º and made up of the same material that the die μ_1=μ_2=μ	$B^*=\dfrac{2\mu}{tg\alpha}$
Plug made up the same material that the die μ_1=μ_2=μ	$B^*=\dfrac{2\mu}{tg\alpha-tg\beta}$
Conical plug with β= - α and made up of the same material that the die μ_1=μ_2=μ	$B^*=\dfrac{\mu}{tg\alpha}$

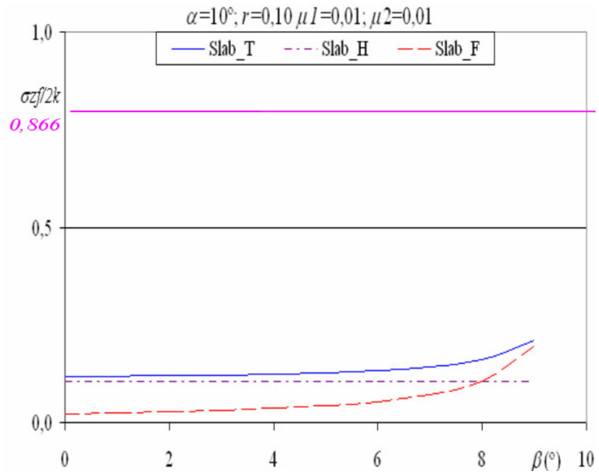


a)

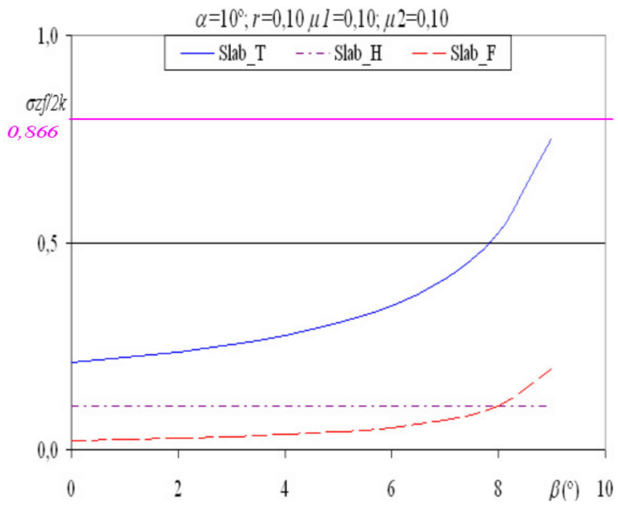


b)

Fig.9. Relationship between the pressures in the plug and in the die for some of the most unfavourable values of α, β, μ_1 and μ_2.



a)



b)

Fig. 10. Adimensional total energy (Slab_T), homogeneous energy (Slab_H) and energy necessary to overcoming the external friction (Slab_F) estimated by means of stress local analysis.

4. Analysis by the upper bound method

The Limit Analysis is an analytic tool that allows obtaining a low mark and an upper one of the exact solution. The first one by means of the Lower Bound Method (LBM) and, the second one, used in this work, by means of the Upper Bound Method (UBM).

This last one is able to provide a value, bigger than or equal to that the one looked for. Therefore, taking the solution corresponding to the equality a quite approximated estimation is obtained, in many cases, to the solution of the problem.

In this work, the processes of tubes drawing in convergent conical dies with inner plug have been analysed by means of the UBM. This has been chosen, mainly, because it allows obtaining, not only, an upper mark of the necessary energy to carry out the process but, also, because to differ among the employee in carrying out the homogeneous deformation, the necessary one to overcoming the external friction and the due one to the internal distortion. The UBM, besides of completing and improving the classical analyses, does not require big calculation resources and has been successfully used in the analysis of other metalforming processes [2][4][19] [27][29][31,32].

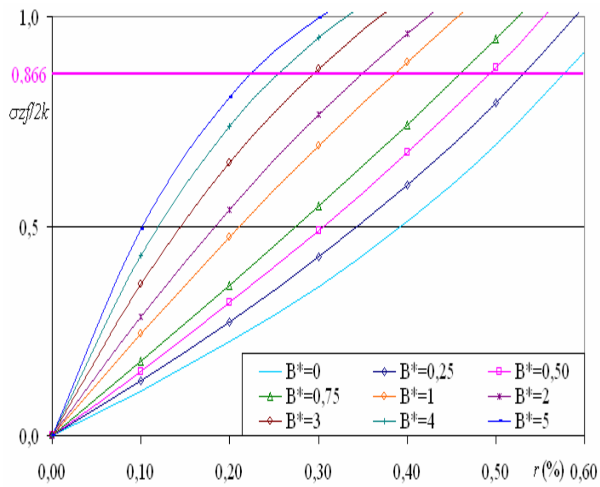


Fig. 11. Adimensional total energy in function of the reduction, r, and the constant B^*.

4.1. Deformation zone modelling

The UBM application requires the previous deformation zone modelling. In this case, the pattern of multiple Triangular Rigid Zones (TRZ) has been used. According to the literature about the topic, this type of surfaces provides quite low solutions of the UBM [2].

In a multi-triangular field of velocities the plastic deformation zone is divided into n zones. In the pattern represented in Figure 12, n=3.

It has been considered that the individual reductions obtained in each one of them are identical. Its value can be calculated through the expression:

$$r_1 = r_2 = r' = 1 - \sqrt{1-r} \tag{37}$$

4.2. Theoretical development

Taking a tube made up of rigid-perfectly plastic material, modelling the plastic deformation zone as Figure 12 shows and applying the Upper Bound Method to the drawing process it is possible to write:

$$\begin{aligned} \dot{W}_T &= \sigma_{zf}\pi D h_f v_f = 2\pi D\cdot(k\overline{OA}\Delta v_{i1} + k\overline{OB}\Delta v_{12} + k\overline{BC}\Delta v_{23} + \\ &+ k\overline{CD}\Delta v_{3f} + \mu_1 p\overline{AB}v_1 + \mu_2 p\overline{OC}v_2 + \mu_1 p\overline{BD}v_3) \end{aligned} \tag{38}$$

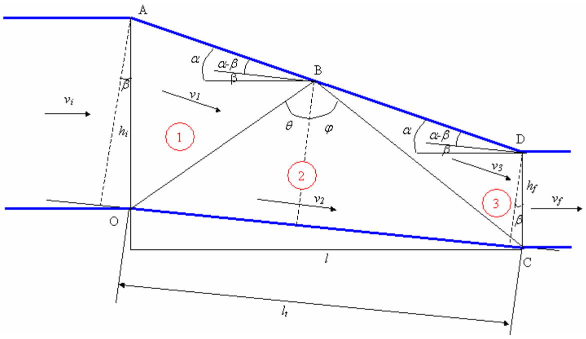


Fig. 12. Modelling of the plastic deformation zone with three Triangular Rigid Zones.

Where:

- $\dot{W}_T$ is the necessary power to carry out the process.
- σ_{zf}, the stress at the die exit.
- D and h_f, the diameter and the final thickness of the tube respectively.
- v_f, le velocity at the tube exit.
- k, the shear yield stress.
- $k\overline{OA}\Delta v_{i1}$, $k\overline{OB}\Delta v_{12}$, $k\overline{BC}\Delta v_{23}$, $k\overline{CD}\Delta v_{3f}$ the mechanic effects along the discontinuities lines $\overline{OA}$, $\overline{OB}$, $\overline{BC}$ and $\overline{CD}$ respectively.
- $\mu_1 p\overline{AB}v_1$ and $\mu_1 p\overline{BD}v_3$ the friction effect between the material in the deformation zone and the die along the $\overline{AB}$ and $\overline{BD}$, and $\mu_2 p\overline{OC}v_2$ the friction effect between the material in the deformation zone and the inner plug. Along the $\overline{OC}$. The pressure in the die has been supposed equal to the pressure in the plug and of value p. This can be justified considering, when the radial forces balance is made, that the contribution to the friction of the die is small and that the main stress can be taken as: $\sigma_1 = \sigma_z$ and $\sigma_2 = -p$; being related in a closed pass without diameter variation:

$$\frac{p}{2k} = 1 - \frac{\sigma_z}{2k} \tag{39}$$

that represents the flow condition Under plane strain.

- v_{ij}, the relative speed between the i and j blocks (the three triangular ones and the rectangular at the entrance and at the exit of the tube).

Keeping in mind the value of the pressure given by the expression (39), the symmetry of the problem and the geometric and cinematic relationships that exist between the segments and the relative velocities (Figure 12), the equation (38) can be written by means of the following expression that, like it will be seen later on, it represents the adimensional total energy necessary to carry out the process:

$$\left(\frac{\sigma_{zf}}{2k}\right)_T = \frac{\overline{OA}\Delta v_{i1} + \overline{OB}\Delta v_{12} + \overline{BC}\Delta v_{23} + \overline{CD}\, v_{3f}}{h_f v_f + 2\left[\mu_1 \overline{AB} v_1 + \mu_2 \overline{OC} v_2 + \mu_1 \overline{BD} v_3\right]} + \frac{2\left[\mu_1 \overline{AB} v_1 + \mu_2 \overline{OC} v_2 + \mu_1 \overline{BD} v_3\right]}{h_f v_f + 2\left[\mu_1 \overline{AB} v_1 + \mu_2 \overline{OC} v_2 + \mu_1 \overline{BD} v_3\right]} \tag{40}$$

In order to validate the pattern, the obtained results with the expression (40) have been compared with the values gotten by the Slab method calculated by means of the expression (29). Both have been represented in Figure 13.

Besides, they have been compared, also, with those that Yoshida and his collaborators achieved using the Finite Elements Method (FEM) and experimental tests [35]. Both have been collected in Figure 14.

5. Conclusions

In this work, the study of the thin-walled tube drawing processes has been approached for a convergent conical die using a conical inner plug fixed to the draw bench where the process is carried out.

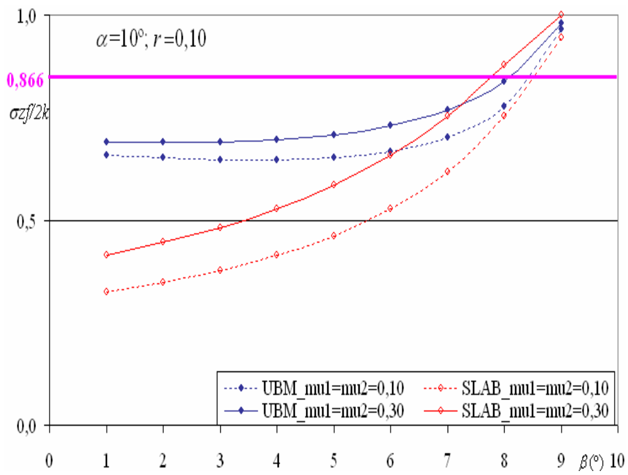


Fig. 13. Validation of the pattern for comparison with the results obtained by the Upper Bound Method and by the Slab one.

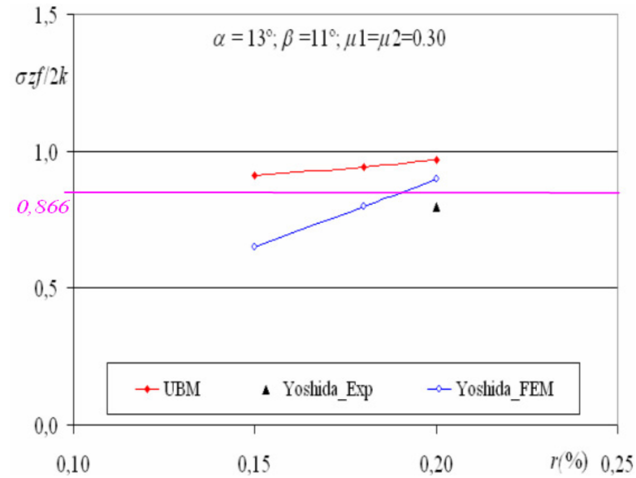


Fig. 14. Validation of the pattern for comparison with the results of Yoshida, obtained by FEM (Yoshida_FEM) and by experimental tests (Yoshida_Exp).

The process has been analyzed by means of the Upper Bound Method under plane strain and Coulomb friction condition. The deformation zone has been modelled by means of Triangular Rigid Zones (concretely three).

The expression of the adimensional total specific energy that provides this method when it is applied to the described model has been calculated.

A first validation of the results has been made comparing them with those obtained by the Slab method, the Finite Elements Method and the experimental data found in the literature about the topic.

The study began by applying the classical analysis methods, such as the Homogeneous Deformation Method and the Slab one. This has allowed to obtain a minimum mark of the necessary energy to carry out the process, a first estimation of the energy vanished by friction in the process and a range of values of the variables involved in it. Shortly, the Slab method seems to indicate that the best conditions to carry out the process are: dies with high semiangles, cylindrical plugs or with a little conicity, this is, small β-values, and friction coefficients as low as possible.

The mathematical expression that relates the pressures in the die and in the plug has been determined and the variables values that allow considering that the pressure is the same in both elements without incurring in important errors have been calculated.

Simplified expressions of the constant B^* for the most usual cases (cylindrical plug made up of equal or different material from the die) have been presented and an expression for the calculation of the maximum reductions r_{max}, in function of B^* has been found.

References

[1] A.M. Camacho, R. Domingo, E.M. Rubio and C. González, J. Mat. Proc. Tech. 164-165 2005, 1167-1174.

[2] B. Avitzur, Handbook of Metalforming Processes, John & Wiley, 1983.

[3] B. Avitzur, Metal Forming: The Application of Limit Analysis, Marcel Dekker, Inc., New York, 1980.

[4] B. Tomov and R. Radev, J. Mat. Proc. Tech. 157-158 2004, 617-619.

[5] B. Tomov, R. Minev, R. Radev and V. Gagov, J. Mat. Proc. Tech. 133 (1-2) 2003, 199-202.

[6] C.E. Snee, Iron and Steel Engr. 33 (1956) 124-133.

[7] C.J. Luis, J. León and R. Luri, J. Mat. Proc. Tech. 164-165 2005, 1218-1225.

[8] C.W. Wu, and R.Q. Hsu, Int. J. Mech. Sciences 42 (2000) 473-486.

[9] E. Bayraktar, N. Isac and G. Arnold, J. Mat. Proc. Tech. 162-163 2005, 471-476.

[10] E. Shaghouei, D.C. Webb and Kormi K., J. Mat. Proc. Tech. 55(3-4) (1995) 391-407.

[11] E. Siebel, and E. Weber, Stahl Eisen 55 (1935) 331.

[12] E.M. Rubio, A.M. Camacho, L. Sevilla and M.A. Sebastián, J. Mat. Proc. Tech. 162-163 2005, 551-557.

[13] E.P. Polushkin, Trans. Amer. Soc. Metals 22 (1934) 635-656.

[14] G. Ambrogio, L. De Napoli, L. Filice, F. Gagliardi and M. Muzzupappa, J. Mat. Proc. Tech. 162-163 2005, 156-162.

[15] G. Ambrogio, L. Filice, G. Palumbo and S. Pinto, J. Mat. Proc. Tech. 162-163 2005, 454-460.

[16] G. Espey and G. Sachs, J. Applied Mechanis 14 (1947) 81-87.

[17] G. Palumbo and L. Tricarico, J. Mat. Proc. Tech. 164-165 2005, 1089-1098.

[18] G. Sachs and G. Espey, Trans. ASME, 69 (1947) 139-143.

[19] G. Sachs and W.M. Baldwin, Trans. Am. Soc. Mech. Engrs. 68 (1946) 647-654.

[20] G. Sachs, and W.M. Baldwin, Trans. Am. Soc. Mech. Engrs. 68 (1946) 655-662.

[21] G. Sachs, J.D. Lubahn and D.P. Tracy, J. Applied Mechanis 11 (1944) 199-210.

[22] G.G. Moore and J.F. Wallace, Proc. Inst. Mech. Engrs. 182, (1967-68) 19-32.

[23] G.J. Kang, W.J. Song, J. Kim, B.S. Kang and H.J. Park, J. Mat. Proc. Tech. 164-165 2005, 1212-1217.

[24] G.P. Pires, M.H. Robert and R. Arrieux, J. Mat. Proc. Tech. 157-158 2004, 596-603.

[25] G.W. Rowe, Conformado de metales, Urmo, Bilbao, 1977.

[26] H.W. Swift, Phil. Mag. Ser., 11 (1949) 883-895.

[27] I.F. Collins and B.K. Williams, Int. J. Mech. Sci. 27(4) (1985) 225-233.

[28] J. Bartnicki and Z. Pater, J. Mat. Proc. Tech. 164-165 2005, 1154-1159.

[29] J. Gronostajski, A. Matuszak, A.Niechajowicz and Z. Zimniak, J. Mat. Proc. Tech. 157-158 2004, 502-507.

[30] J. Niziol and A. Świątoniowski, J. Mat. Proc. Tech. 162-163 2005, 546-550.

[31] J.D. Jevons, The metallurgy of deep drawing and pressing, 2ª Ed., Chapman&Hall, 1945.

[32] J.E. Flinn, Trans. Am. Soc. Mech. Engrs. (1969) 792-804.

[33] J.W. Park, Y.H. Kim and W.B. Bae, J. Mat. Proc. Tech. 72 (1) (1997) 94-101.

[34] K. Sawamiphakdi, G.D. Lahoti and P.K. Kropp, J. Mat. Proc. Tech. 27(1-3) (1991) 179-190.

[35] K. Yoshida, M. Watanabe and H. Ishikawa, J. Mat. Proc. Tech. 118 (2001) 251-255.

[36] K.K. Um and D.N. Lee, J. Mat. Proc. Tech. 63 (1997) 43-48.

[37] L. Sadok, J. Kusiak, M. Pako and M. Rumiski, J. Mat. Proc. Tech. 60 (1-4) (1996) 161-166.

[38] L. Tomesani, J. Mat. Proc. Tech. 64 (1-3) 1997, 379-386.

[39] M. Goodarzi, T. Kuboki, and M. Murata, J. Mat. Proc. Tech. 162-163 2005, 492-497.

[40] M. Li, Z. Sun, M. Li and H. Yang, J. Mat. Proc. Tech. 160 (3) (2005) 396-400.

[41] M. Murata, T. Kuboki and T. Murai, J. Mat. Proc. Tech. 162-163 2005, 540-545.

[42] M. Nastran and K. Kuzman, J. Mat. Proc. Tech. 133 (1-2) 2003, 166-173.

[43] M. Rumiski, J. Uksza, J. Kusiak and M. Pako, J. Mat. Proc. Tech. 80-81 (1998) 683-689.

[44] M.H. Robert and M. Adamiak, Preliminary studies on the suitability of rheocast Al alloys for deep drawing, J. Mat. Proc. Tech. 109 (1-2) 2001, 168-173.

[45] O. Hoffman and G. Sachs, Introduction to the theory of plasticity for engineers, Mac Graw-Hill Book Company, Inc., New Yor, Tornto, London, 1953.

[46] P. Green, Proc. Inst. Mech. Engrs. 174 (1960) 847-864.
[47] P. Vacher, R. Arrieux, L. Tabourot, J. Mat. Proc. Tech. 78 (1-3) 1998, 190-197.
[48] P.K. Kar, and N.S. Das, Int. J. Mech. Sciences 39(8) (1997) 925-934.
[49] P.K. Seo and C.G. Kang, J. Mat. Proc. Tech. 162-163 2005, 402-409
[50] R. Hill, The mathematical theory of plasticity, Oxford University Press, London, 1950.
[51] R.A.C. Slater, Engineering Plasticity: Theory and Application to Metal Formig Processes, The MacMillan Press Ltd., London, 1977, 363-375.
[52] S. Elion and J.M. Alexander, Engineer, 211 (1961) 682-695.
[53] S.K. Sahoo, P.K. Kar and K.C. Singh, J. Mat. Proc. Tech. 75(1-3) (1998) 75-80.
[54] S.K. Sahoo, P.K. Kar and K.C. Singh, J. Mat. Proc. Tech. 91(1-3) (1999) 105-110.
[55] S.W. Lee, J. Mat. Proc. Tech. 130-131 (2002), 47-53.
[56] S.Y. Chung and H.W. Swift, J. Iron Steel Inst. 170 (1952) 29-41.
[57] S.Y. Chung and H.W. Swift, Proc. Inst. Mech. Engrs. 165 (1951) 199-223.
[58] T. Oczkowicz, and S. Tkaczyk, J. Mat. Proc. Tech. 64 (1-3) 1997, 303 – 310.
[59] T.Z. Blazynski and I.M. Cole, Proc. Inst. Mech. Engrs. 174 (1960) 797-804.
[60] W. Johnson and P.B. Mellor, Engineering plasticity, John Wiley & Sons, (1983).