Analysis of several methods for the data conversion and fitting of the Garofalo equation applied to an ultrahigh carbon steel

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Analysis and modelling

ABSTRACT

Purpose: To determine the most suitable reduction data method and the optimal fitting method for the Garofalo equation. Two fitting methods were applied. The input data for this fitting are the sets of forming variables $\{T,\sigma,\varepsilon'\}$ which have been obtained by using four different reduction methods. This procedure is applied to an ultrahigh carbon steel (UHCS).

Design/methodology/approach: High temperature torsion tests have been carried out on the UHCS. A wide range of forming variables has been used. A numerical method has been implemented for the experimental data reduction. The fitting of the Garofalo equation has been carried out by means of two numerical methods. An integral method in stages, called RCR method, and a method based on Matlab algorithms called NLD. A comparative analysis of the parameters of the Garofalo equation has been conducted.

Findings: The results show that the n and Q parameters are not dependent of the conversion method that has been used, Von Mises, Tresca or Eichinger. However, the α and A parameters seem to depend on the reduction method. Regarding the fitting, the RCR method is quick and efficient and its results, at the first stage, are close to the ones obtained by the NLD method. The evolution of the fitting parameters with strain for each conversion and fitting method has been determined.

Research limitations/implications: The evolution of the parameters of the Garofalo equation are influenced by the adiabatic heating that occurs during the torsion testing. It is necessary a correct experimental design to obtain a suitable grid of data which allows an accurate determination of the strain rate sensitivity and the strain hardening coefficient.

Practical implications: The fitting of the Garofalo equation for different strain values can provide information on the microscopic processes that take place during deformation.

Originality/value: This work is a comparative study of the usual reduction methods and shows the utility of a method for the fitting of the Garofalo equation. This method is convergent, quick and accurate.

Keywords: Numerical techniques; Garofalo equation; Reduction method; Nonlinear fitting

1. Introduction

The torsion tests allow the attainment of flow curves of metals at large strains, and high strain rates and temperatures. The deformation is carried out by means of shear stresses, allowing high deformations because no plastic instability is involved. For this reason, these tests are suitable to simulate and optimize many of the industrial metal-forming processes.

The experimental data obtained from torsion tests are the torque and the number of turns. They are used for the conversion process to obtain the equivalent physical variables, strain, strain rate and stress.

The variables, stress, σ, as a function of strain rate, $\dot{\varepsilon}$, usually are fitted by means of the Garofalo equation. This equation is given as follows:

$$\dot{\varepsilon} = A e^{-\frac{Q}{RT}} \left[\sinh(\alpha \cdot \sigma) \right]^n \tag{1}$$

where $\dot{\varepsilon}$ is the strain rate, T is the absolute temperature, σ is the stress, Q is the activation energy for deformation, R is the gas constant, and α, n and A are material constants. This equation allows extrapolating the results in order to approach the industrial conditions.

Usually, the Garofalo equation is fitted at the maximum of the flow curve and at a strain where it is considered that the steady state is reached. This allows the study of the variation of the parameters of the equation as a function of strain.

The aim of this work is twofold: a) the study of the usual reduction methods and their influence on the parameters of the Garofalo equation and b) the fitting of this equation by means of two numerical methods. This procedure has been applied to an ultrahigh carbon steel (UHCS) containing 1.3%C.

2. Material and experimental method

The UHCS studied in this investigation has the following composition: 1.3%C, 0.5% Mn, 0.6% Si, 0.18% Cr and balance Fe [1]. The manganese was added to neutralize the deleterious effects of sulfur and phosphorus. The steel was obtained at Sidenor industry as a cast of 8 liters by means of an induction furnace. The as-cast ingot was initially soaked at 1050ºC and forged into a bar of 60 mm x 55 mm cross section.

Simulation of the forming process of forged parts was carried out by means of torsion tests. An induction furnace heats the test sample and the temperature is continuously measured by means of a two-color pyrometer. A silica tube with argon atmosphere ensures protection against oxidation and minimum adiabatic heating. A helium atmosphere is used to obtain, after testing, a cooling rate of 325 K/s.

The torsion samples have an effective gage length of 17 mm and a radius of 3 mm.

Strain rates in the range of 2 to 26 s^{-1} were used. The temperature range is of 900 to 1200 ºC.

The samples were deformed in a SETARAM torsion machine at CENIM (National Center for Metallurgical Research), Madrid, Spain.

3. Conversion of torsion data

The experimental data have to be transformed into equivalent physical variables in order to be able to study the constitutive differential equations for the plastic flow and specifically the Garofalo equation.

The conversion process can be divided in two main stages: the attainment of the true stress from the torque data and the attainment of the true strain from the number of turns data.

For every tested sample at constant temperature, and constant rotation speed, $\dot{N}$, the input data for the conversion process are the torque, Γ, and the number of turns, N. The reduction process act on these experimental data and transform this data into equivalent physical variables, i.e. the output data will be the true stress, the true strain, and the true strain rate.

A proper conversion process needs an enough number of tests over a wide range of the forming variables to ensure an accurate grid over the design domain [2-3].

3.1. Phenomenological function for the fitting of {Γ,N} data

The first step to make the reduction process is to fit the {Γ, N} experimental values. This can be done by using a phenomenological function based on the macroscopic behavior of a polycrystalline metallic material under plastic flow. The expression for this fitting is as follow:

$$\Gamma(N) = \left(A \cdot N^2 + B \cdot N \right) \cdot e^{-C \cdot N} + \frac{D \cdot N^2 + G \cdot N}{E \cdot N^2 + F^2} \cdot e^{-\left(\frac{N-H}{K} \right)^2} \tag{2}$$

The first term, models the elastic part of the deformation and the zone that goes from the yield to the maximum of the curve, if exist, or to the beginning of the steady state, if not.

The second term models the steady state regime. The rational part take into account the asymptotic trend of the Γ during creep, and the exponential part represent the sharp decrease of the torque previous to rupture.

We can fit all kind of {Γ, N} set of values obtaining R^2 values of about 0.98.

Fig. 1 shows various fitting curves for the UHCS at various temperatures. The strain rate is 10 s^{-1}. The points are the experimental data {Γ$_i$, N$_i$} and the lines represent the fitted function.

The selection of this model takes into account the continuity of the deformation process and the derivability conditions over the significant zones of the flow curve. This corresponds to the zone of the stress peak and the possible steady state zone [4].

The problem is to find the A, B, C, D, E, F, G and H parameters in such a way that Eq. (2) fit the {Γ$_i$, N$_i$} data as good as possible. Therefore, we have a minimization problem and its objective function can be defined as:

$$\min_{A,\,B,\,C,\,D,\,E,\,F,\,G,\,H} \quad \frac{1}{S}\sum_{i=1}^{S}\left(\Gamma_i - \Gamma(N_i)\right)^2 \tag{3}$$

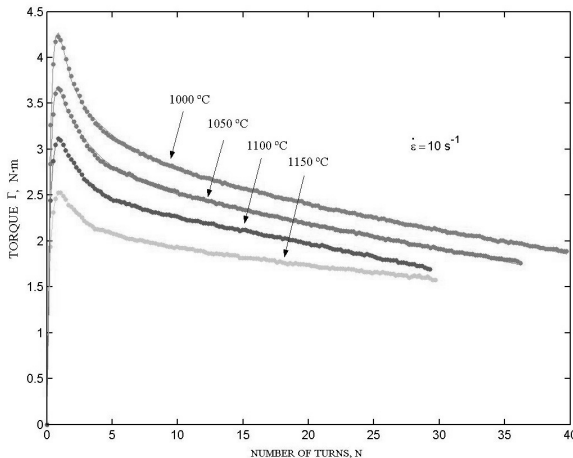where S is the number of obtained measurements for the torsion test under consideration.

Fig. 1. Torque versus number of turns for the UHCS-1.3%C, at $\dot{\varepsilon} = 10\ s^{-1}$ and at various temperatures.

3.2. True stress values

The Fields – Backofen method [5] has been employed to obtain the shear stress values, τ. The basic expression is the following:

$$\tau\,(r=R) = \frac{\Gamma}{2\cdot\pi\cdot R^3}\cdot(3+\theta+m) \tag{4}$$

where, τ is the shear stress at the outer radius, Γ is the applied torque to the cylindrical specimen, R is the radius of the specimen, θ is the strain hardening coefficient and m is the strain rate sensitivity. These last two coefficients are defined as follow:

$$\theta = \left.\frac{\partial\ln(\Gamma)}{\partial\ln(N)}\right|_{T,\dot{N}cte} = \left.\frac{N}{\Gamma}\frac{\partial\Gamma}{\partial N}\right|_{T,\dot{N}cte} \tag{5}$$

$$m = \left.\frac{\partial\ln(\Gamma)}{\partial\ln(\dot{N})}\right|_{T,Ncte} = \left.\frac{\dot{N}}{\Gamma}\frac{\partial\Gamma}{\partial\dot{N}}\right|_{T,Ncte} \tag{6}$$

The yield criteria by von Mises and Tresca are used to calculate the true stress values σ. The difference between these criteria mainly leads to a difference in the absolute heights of the flow stress curves whilst their relative shapes and strain rate sensitivity are not strongly affected [6]. Both criteria are also based on the assumption that the stress is independent of the strain rate, which implies a contradiction [6] since torsion test are often carried out on strain rate sensitive materials.

3.3. True strain and strain rate values

The strain and strain values have been calculated by two methods. The following expressions can be obtained according to the von Mises method [6]:

$$\varepsilon = \frac{2\cdot\pi\cdot R}{\sqrt{3}\cdot L}\cdot N \tag{7}$$

$$\dot{\varepsilon} = \frac{2\cdot\pi\cdot R}{\sqrt{3}\cdot L}\cdot\dot{N} \tag{8}$$

where L is the gage length.

According to Eichinger [7], the von Mises expressions are not correct because the strain values are excessively high and have no physical meaning. The Eichinger equations for the strain, ε^*, and the strain rate, $\dot{\varepsilon}^*$ are the following:

$$\varepsilon^* = \frac{2}{\sqrt{3}}\cdot\ln\left(a\cdot N + \sqrt{1+a^2\cdot N^2}\right) \tag{9}$$

$$\dot{\varepsilon}^* = \frac{2}{\sqrt{3}}\cdot\frac{a\cdot\dot{N}}{\sqrt{1+a^2\cdot N^2}} = \frac{2}{\sqrt{3}}F \tag{10}$$

where $a = \dfrac{\pi\cdot R}{L}$.

The Eichinger expressions are usually recommended to calculate large strains in torsion tests [8]. This conversion process involves that at a constant rotation speed $\dot{N}$ of the torsion machine, the true strain rate decreases with increasing strain. This can be seen in figure 2.

According to the work of Canova [9] to carry out a test at constant $\dot{\varepsilon}$, the rotation speed should be raised to have a constant F factor in Eq. (10).

3.4. Calculation of θ and m parameters

Calculation of the strain-hardening coefficient

To obtain the θ parameter it is necessary to calculate the derivate $\left.\dfrac{\partial\Gamma}{\partial N}\right|_{T,\dot{N}cte}$ and to evaluate it for every N_i measurement. The expression for the derivate was calculated analytically by means of Eq. (2). The evaluation of the derivate on every N_i gives a θ value on every measurement. This procedure to obtain the strain-hardening coefficient ensures an accurate determination [4].
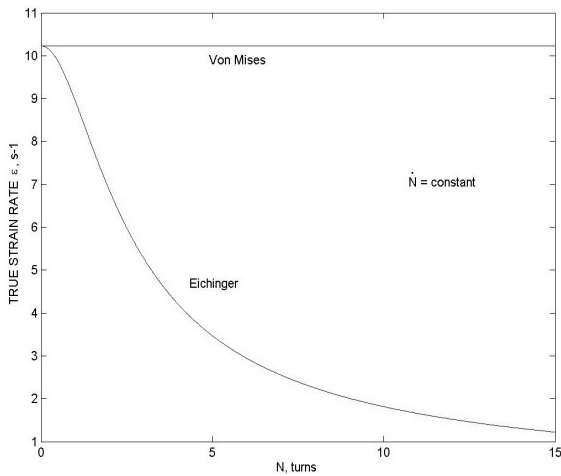
Fig. 2. Evolution of True Strain Rate with the number of turns as Von Mises and Eichinger.

Calculation of the strain rate sensitivity

To calculate the m parameter it is necessary to use all pairs of data $\{\Gamma, N\}$ from tests carried out at the same temperature and at all strain rates [4]. Therefore, it is necessary to build the $\Gamma(\dot{N})$ functions at every N value. However, the construction of these functions is based, in general, on few data points and, therefore, the calculation of the m parameter may have a large error.

The algorithm used to calculate the strain rate sensitivity has the following steps:

i.   Use of all the tests carried out at the same temperature.

ii.   Determination of the final N value of each test and the minimum among these values, $N_{\min}$. An equal-spaced vector from 0 to $N_{\min}$ is build.

iii.   Evaluation of the phenomenological function, Eq. (4), at each element of the vector and determination of $\{\dot{N}, \Gamma\}$ data at N and T constants

iv.   Construction of the $\Gamma(\dot{N})$ function for each N value, whose derivate provides the m coefficient. We usually used 3 to 6 pairs of data, (as much as tests carried out at different strain rates and at the same temperature), to built this function. Therefore, its construction is associated to high levels of uncertainty. Then, among the different possibilities of interpolating these pairs (spline, piecewise cubic hermite polynomials (pchip), etc.), we chose a piecewise linear interpolation for the logarithmic data. In this way, the obtained value for m, is the average of the slopes of the lines that reach a given point $\{\dot{N}, \Gamma\}$. In the case of extreme points, the slope of the associated line is chosen.

v.   Calculation of the m coefficient at every measured $\{\Gamma, N\}$. based on both, the values of m calculated at every N value, and the pchip interpolation.

3.5. Application of the conversion methods to torsion data of a UHCS

Figs. 3 and 4 are two examples of stress-strain curves. The tested material is the UHCS-1.3%C and the figures show the results of torsion tests carried out at $\dot{\varepsilon} = 10\,s^{-1}$ and different temperatures. The figure 3 shows the stress-strain curves when the Tresca criteria is used for calculating true stress values and the Von Mises method is applied to calculate true strain values. Fig. 4 shows the same tests, but the Von Mises method is used for true stress values and the Eichinger method for true strain values.

It can be seen that the maximum strain that reached the material depends on the reduction method for the strain. The Von Mises method provides a value of ∼ 30 while the Eichinger method gives a value of ∼ 4. Since the transition from stress values at peak to stress values at rupture in the Eichinger method occurs during a shorter interval of strain, the change of the stress as a function of the strain is larger than the one obtained by the Von Mises method. Therefore, if the Eichinger method is used, the steady state is difficult to be determined.

According to Von Mises, the true strain of the cylindrical sample is proportional to the rotation of the machine, i.e., it is assumed that the sample behaves as a rigid solid. However, in the Eichinger method, this proportionality does not appear and the sample behaves as a Newtonian fluid under deformation.
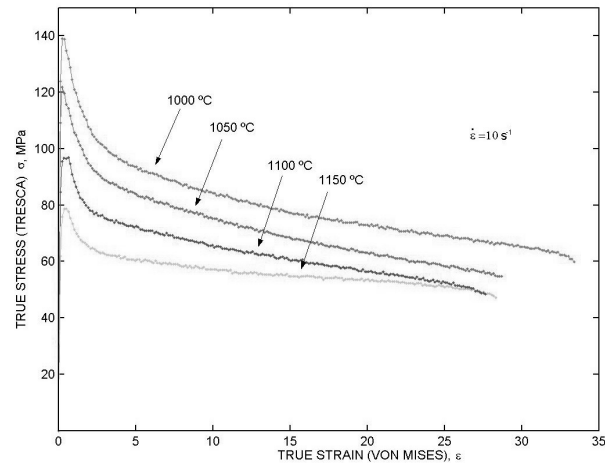

Fig. 3. Tresca stress versus Von Mises strain for a UHCS-1.3%C, at $\dot{\varepsilon} = 10\,s^{-1}$ and various temperatures.

4. Garofalo equation and its fitting

The Garofalo equation has been fitted by two methods, RCR and NLD. This equation is highly nonlinear, and is also non-convex, and its convexity varies along the entire working range.

These two methods are very different. The NLD algorithm works on the linearized form of the Garofalo equation and a set of initial values are needed in the design domain to obtain the optimal solution. In contrast, the RCR method is a multi-staged algorithm that in its first stage works on the linearized form and

then, at the later stages, works on the non-linear form. This method is auto-consistent and does not need initial values.

Therefore, the results obtained by the NLD method can be only compared with the ones obtained in the first stage of the RCR method. We have checked that both results, for the UHCS, are very close, which allows the validation of both methods.
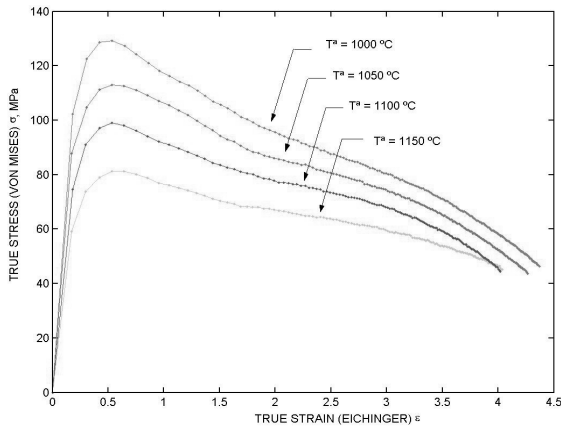

Fig. 4. Von Mises Stress versus Eichinger Strain for a UHC steel, at $\dot{\varepsilon} = 10 \ s^{-1}$ and different temperatures.

4.1. RCR method

The RCR method has been explained, in detail, in previous works [10-12]. A brief description of this method is given in order to compare it with the NLD method.

The parameters $\theta_1, \theta_2, \theta_3, \theta_4$ are defined as the coefficients of the Garofalo equation $\{\theta_1 = Q, \theta_2 = \alpha, \theta_3 = n, \theta_4 = A\}$. At the first stage, the method minimizes the following equation that defines the total error.

$$\min_{\theta_1, \theta_2, \theta_3, \theta_4} \frac{1}{N} \sum_{i=1}^{N} \left[\theta_4' - \frac{\theta_1}{x_i} + \theta_3 \log\left(\sinh(\theta_2 y_i)\right) - \log\left(z_i^{exp}\right) \right]^2 \quad (13)$$

If θ_2 parameter is a constant, Eq. (13) is linear in $\theta_1, \theta_3, \theta_4$. We define a minimum and maximum value for θ_2, and a $\Delta \theta_2$ that is the increment of θ_2. These three values are the initial data for the algorithm.

The minimization problem is transformed into a problem of multilinear regression. The optimal solution are the $\theta_1, \theta_2, \theta_3, \theta_4$ values that reach a maximum of R^2, and a maximum of the estimator F experimental of Fisher – Snedecor. They are the initial values for the second stage that are conducted by a direct non-linear regression method based on the modified Gauss-Newton method [10-12] in order to obtain the optimal Garofalo equation. This second stage works on the non-linear expression of the Garofalo equation. This method has been implemented with suitable parameters for the convergence control.

Using the definitions: $\underline{\theta} = (Q, \alpha, n, A)$ and $F(\underline{\theta}) = \frac{\partial}{\partial \underline{\theta}^t} f(\underline{\theta})$, where

$$f(\underline{\theta}, T, \sigma) = \theta_4 \cdot e^{-\frac{\theta_1}{RT}} \cdot [\sinh(\theta_2 \sigma)]^{\theta_3} \quad (11)$$

is the Garofalo equation [13-15], the equation for the algorithm of the Gauss-Newton method is:

$$\underline{\theta_i} = \underline{\theta_{i-1}} + (F^t(\underline{\theta_{i-1}}) \cdot F(\underline{\theta_{i-1}}))^{-1} \cdot F^t(\underline{\theta_{i-1}}) \cdot \left[\underline{y} - f(\underline{\theta_{i-1}}) \right] \quad (12)$$

where [y] is the strain rate matrix [13-15].

The optimal solution found is the beginning for the last stage where a fine-tuning of the global optimum is obtained. This third stage is an iterative method by paths in R^2 that corresponds to two types of lines associated to alternated values of constant Q and α [13-15].

This method has been applied to several materials, for example with the C-Mn-V-N steel [16].

4.2. NLD method

This algorithm works on the linearized form of the Garofalo equation. The method solves the minimization problem defined in Eq. 13 with the following restrictions for the design variables:

$$1 \cdot 10^{-3} \le \theta_1 \le 1000; \quad 1 \cdot 10^{-3} \le \theta_2 \le 1$$
$$1 + 1 \cdot 10^{-3} \le \theta_3 \le 30; \quad 10 \le \theta'_4 \le 60 \quad (13)$$

being $\theta'_4 = \log(\theta_4)$.

An equal-spaced grid over the design domain is defined as follows:

$$\left\{ \theta^{min}_i \le \theta^{(k)}_i \le \theta^{max}_i \right\} \quad \theta^{(k)}_i = \theta^{min}_i + (k-1) \cdot \Delta \theta_i$$
$$\Delta \theta_i = \frac{\theta^{max}_i - \theta^{min}_i}{9} \quad k = 1,2,...,10 \quad i = 1,2,3,4 \quad (14)$$

This is a grid over a $\Re^4$ domain that consists on 10.000 initial values. The minimization problem is solved over every point of the grid and the optimal solution corresponds to that with the minimum error. The convergence criteria that has been applied to the optimization process are very restrictive, in the sense that the stop of the algorithm occurs for a very small variation of the cost function.

The method to solve this minimization problem is based on Matlab algorithms. Concretely, the *lsqcurvefit* function has been used as central part of this method. This function uses a subspace trust region method based on the interior-reflective Newton method [17-18].

In the case of the UHCS, we have fitted the Garofalo equation at different strains and by different conversion methods. It has been observed that, for the solved fittings, around 9800 initial points converge to the same optimal solution.

5. Evolution of the fitting parameters of UHCS

The fitting of the Garofalo equation for the UHCS is based on 30 torsion tests at different strain rates and temperatures.

Sets of $\{T, \dot{\varepsilon}, \sigma\}$ data at different strains have been used to fit the Garofalo equation. The following strain values have been used: strain at maximum (~0.6), 0.8, 1, 2 and 3.

For each strain, the Garofalo equation has been fitted by the RCR and NLD methods. All the conversion processes have been used to check their influence on the parameters obtained.

In Table 1 shows the results obtained for the UHCS at all the strains under consideration. All the conversion processes and the two fitting methods are included.

We find that the parameters of the Garofalo equation do not depend significantly on the fitting method used, which means that both numerical methods work correctly.

Comparing the results obtained by the NLD method and the first stage of the RCR method, it is observed that the n exponent changes less than 2%, the Q parameter changes less than 0.1%, α changes less than 5 % and ln(A) changes less than 1%.

Table 2 show the change of the parameters of the Garofalo equation between the stage 1 and stage 3 of the RCR method. The Von Mises method for stress and the Eichinger and Von Mises

methods for strain have been used. The strain value is at maximum of the stress-strain curves. It can be seen that the total error decreases once the second and the third stages are applied. These decrease is very high, at least of one order of magnitude. Table II also shows the variation of the parameters of Garofalo's equations from stage 1 to 3 when the RCR method is applied. The variation of the n parameter is of the order of 2% in the Von Mises – Von Mises reduction case, and of the order 4% in the Von Mises – Eichinger reduction case. For the Q parameter, the relative variation is of the order of 3.5% and 4.5% respectively. This variation can be considered as an improvement of the RCR method while it is not possible to conduct with the NLD method.

Figures 5 to 8 show the evolution of the four parameters of the Garofalo equation with increasing strain. The four conversion processes have been plotted. The results showed in these figures, have been obtained by means of the NLD method. It can be observed that these parameters are not strongly influenced by, either, Tresca or Von Mises conversion method used.

The conversion methods for the strain have a significant influence on the obtained parameters. The n and A parameters decrease with increasing strain. At high strains the n parameter approaches the value of 2.

The determination of an accurate value for n is an important subject in order to control the forming stability [19].

Table 1.
Parameters of the Garofalo equation for UHCS-1.3%C for various strain values of flow curves. The parameters have been obtained by the stage 1 of the RCR method, and the NLD method. The usual reduction methods are used.

Work	Stress	Strain	Strain Rate	Strain value	Cases	Nonlinear direct NLD					Stage one of RRC method				
						n	Q[kJ/mol]	α[MPa^{-1}]	ln(A)	error	n	Q[kJ/mol]	α[MPa^{-1}]	ln(A)	error
1	VM	VM	VM	Max.	26	4,66	274,31	0,0052	29,376	0,029	4,68	275,95	0,0050	29,535	0,033
2	VM	VM	VM	ε=0,8	26	3,83	200,02	0,0036	24,008	0,024	3,84	201,34	0,0035	24,095	0,028
3	VM	VM	VM	ε=1	26	3,83	195,63	0,0033	23,963	0,024	3,83	196,87	0,0033	23,940	0,027
4	VM	VM	VM	ε=2	26	3,02	201,01	0,0150	18,490	0,015	3,04	202,26	0,0150	18,524	0,016
5	VM	VM	VM	ε=3	26	2,00	218,12	0,0350	17,358	0,021	2,07	220,42	0,0340	17,480	0,024
6	VM	Ei	Ei	Max.	26	4,32	251,72	0,0050	26,167	0,030	4,32	253,27	0,0050	27,167	0,034
7	VM	Ei	Ei	ε=0,8	26	3,96	202,61	0,0030	24,920	0,020	3,96	203,90	0,0030	24,859	0,022
8	VM	Ei	Ei	ε=1	26	3,75	192,21	0,0042	22,430	0,021	3,85	191,76	0,0030	23,711	0,035
9	VM	Ei	Ei	ε=2	26	1,66	214,04	0,0044	15,610	0,021	1,65	217,34	0,0430	15,765	0,030
10	Tr	VM	VM	Max.	26	4,66	274,31	0,0045	29,380	0,029	4,68	275,95	0,0043	29,532	0,033
11	Tr	VM	VM	ε=0,8	26	3,83	200,01	0,0031	24,005	0,024	3,84	200,35	0,0030	24,132	0,028
12	Tr	VM	VM	ε=1	26	3,83	195,62	0,0029	23,950	0,024	3,82	196,02	0,0030	23,775	0,027
13	Tr	VM	VM	ε=2	26	3,02	200,99	0,0131	18,500	0,015	3,04	201,36	0,0030	18,521	0,016
14	Tr	VM	VM	ε=3	26	1,99	218,12	0,0305	17,360	0,021	1,91	219,62	0,0320	17,280	0,024
15	Tr	Ei	Ei	Max.	26	4,32	251,72	0,0043	27,170	0,030	4,32	253,29	0,0043	27,165	0,034
16	Tr	Ei	Ei	ε=0,8	26	3,96	202,58	0,0026	24,940	0,020	3,95	205,29	0,0027	24,964	0,030
17	Tr	Ei	Ei	ε=1	26	3,75	192,23	0,0036	22,430	0,021	3,32	189,22	0,0031	20,335	0,034
18	Tr	Ei	Ei	ε=2	26	1,66	214,04	0,0380	15,620	0,021	1,67	213,39	0,0370	15,574	0,023

This is a typical value of a superplastic material. It is worth noting that this steel have shown this behavior at lower strain rates

The evolution of Q and α is more anomalous because it shows an increase with increasing strain. This increase could be due to a change in the mechanism controlling plastic deformation. For instance, a change from slip creep to a grain boundary sliding mechanism is usually connected to a change in the activation energy.

Table 2.
Parameters obtained at the first and third stages of the RCR method. Eichinger and Von Mises conversion methods have been used.

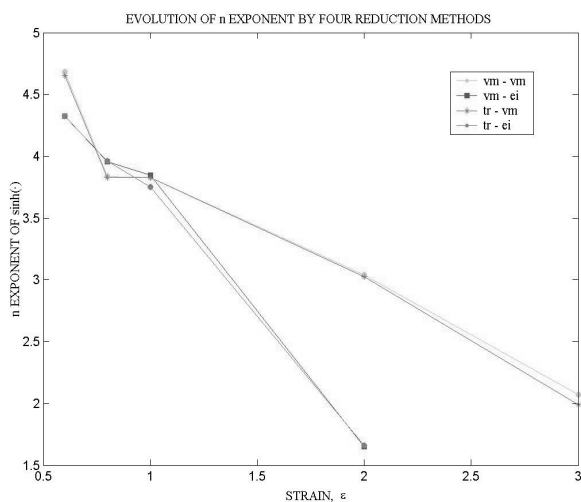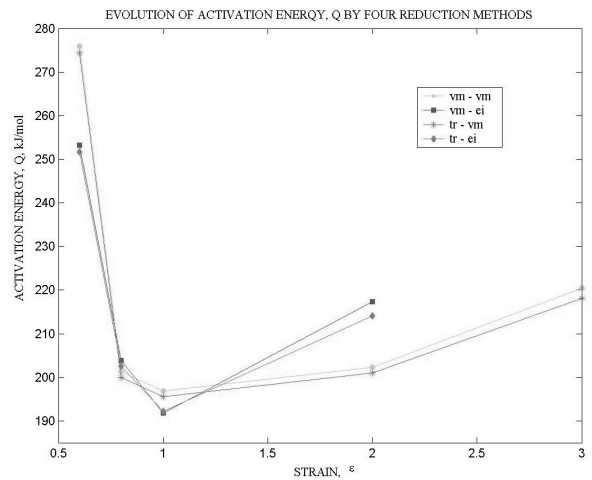| Stress conversion	Von Mises	Von Mises	Von Mises	Von Mises
Strain conversion	Von Mises	Eichinger	Von Mises	Eichinger
Strain location	Maximum	Maximum	Maximum	Maximum
Obtained values	First stage		Third stage	
N	4,68	4,32	4,78	4,50
Q [kJ/mol]	275,95	253,27	285,76	265,28
α [Mpa^{-1}]	0,0050	0,0050	0,0053	0,0050
ln(A)	29,535	27,167	30,3322	28,47
Total error	0,0327	0,0342	0,0012	0,0015
Relative errors%				
N	2,1	4,0		
Q [kJ/mol]	3,5	4,5		
α [Mpa^{-1}]	5,7	0,0		
ln(A)	2,6	4,6		

Fig. 6. Evolution of Q with strain for UHCS-1.3%C.

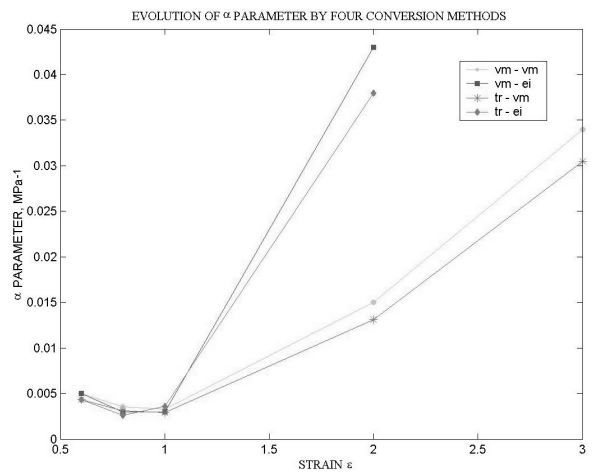Fig. 7. Evolution of α with strain for UHC 1.3% C
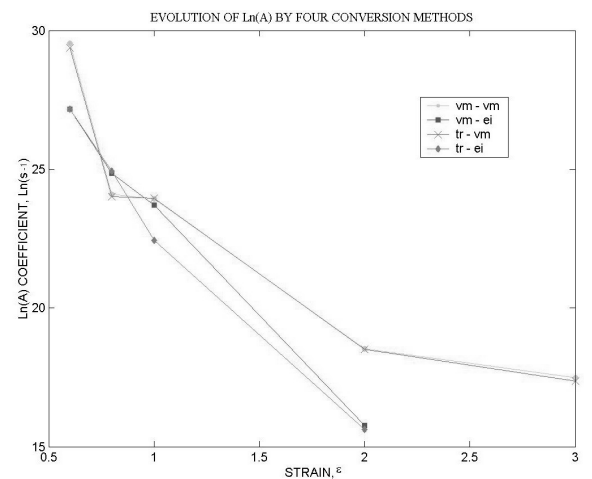

Fig. 5. Evolution of n with strain for UHCS-1.3%C

Fig. 8. Evolution of ln(A) with strain for UHC 1.3%C

6. Conclusions

i. The RCR fitting method is accurate, fast and convergent. The NLD method is an acceptable one and its error levels are similar to those obtained by the first stage of the RCR. For the UHCS-1.3%C, the optimal solution {Q, n, ln(A), α}obtained by the NLD method differs less than 5 % that the one obtained by the first stage of RCR.

ii. The parameters of the Garofalo equation do not depend significantly on the conversion methods for the stress, Tresca or Von Mises. However, these parameters are influenced by the conversion method for the strain. This is because, for a given strain for each method, the deformation of the material is larger according to Eichinger than according to von Mises.

iii. From a physical viewpoint, the Eichinger method considers the material as a Newtonian fluid since the strain is not proportional to the rotation of the torsion machine.

iv. When the Eichinger method is used, the stress does not reach a steady state.

v. The n and Q parameters do not depend on the conversion method used. The α and A parameters are somewhat more dependent.

vi. The parameters of the Garofalo equation show a evolution with increasing strain. Therefore, it is necessary to study the differential constitutive equations to simulate the process of the plastic deformation outside of the steady state.

vii. The Garofalo equation is a useful tool to provide necessary conditions to the future integration of the constitutive equations.

Acknowledgements

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