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Dynamic properties identification for laminated plates

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Properties

ABSTRACT

Purpose: The study aims to identify elastic properties of laminated plates from the measured dynamical properties.
Design/methodology/approach: Elastic constants of laminates have been determined by using an identification procedure based on experiment design, and multi-level theoretical approach.
Findings: The present paper is the first attempt at proposing a novel procedure to derive stiffness parameters from forced sandwich plates. The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical ore approximation method.
Research limitations/implications: In the future the extension of the present approach to sandwich plates will be performed in order to test various experimental conditions.
Practical implications: Structures composed of laminated materials are among the most important structures used in modern engineering and especially in the aerospace industry. Such lightweight and highly reinforced structures are also being increasingly used in civil, mechanical and transportation engineering applications.
Originality/value: The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical ore approximation method.
Keywords: Mechanical properties; Identification; Laminated plates; Elastic constants; Vibration

1. Introduction

Structures composed of laminated materials are among the most important structures used in modern engineering and especially in the aerospace industry. Such lightweight and highly reinforced structures are also being increasingly used in civil, mechanical and transportation engineering applications. The rapid increase in the industrial use of these structures has necessitated the development of new analytical and numerical tools that are suitable for the analysis and study of the mechanical behavior of such structures. The determination of stiffness parameters for complex materials such as fibre-reinforced composites is much more complicated than for isotropic materials since composites are anisotropic and non-homogeneous. In the meantime many different approaches were produced for identification of the physical parameters directly characterising structural behaviour. During recent year's investigations for developing a new technique for material identification, the so-called mixed numerical-experimental technique, have been conducted [1-3]. A novel procedure to derive stiffness parameters from forced vibrating sandwich plates is proposed**.**

2. Some aspects of the identification and optimization of construction parameters

We will consider some constructions for which it is necessary to build an adequate numerical scheme (N.S.) and on this basis to conduct the optimization of certain parameters. Let us consider set M that has N elements. On M are set the criteria of adequacy. These criteria are usually deviations of some experimental values from the corresponding values obtained on the basis of N.S. Designate the criterion of adequacy by K. It is set on M. Then

$$\min\left[K(m), m \in \mathrm{M}\right] = \alpha \quad .$$

It is natural to select, so that K(*m*) takes a minimum value on *m*. But often *m* is most complicated for application and least flexible and universal. We will consider, for example, the process of distribution of vibrations in a elaborate design. If we consider N.S., which examines every part of construction as a three-dimensional body, without even mentioning the difficulties in solving fairly accurately the dynamic tasks for every component, then it is necessary to take into account their interaction. An error in the calculation of even one of the component elements introduces large distortions in all the calculations. That is why in practice the method of discretisation, traditional methods of structural mechanics and the method of finite elements are used. Moreover, we need to mention methods, where the calculation of some heterogeneous construction is replaced by the appropriate homogeneous construction. One of the often-applied methods that is considered in this study is N.S. of the discrete-continuum type. Often the construction can be represented as a beam, plate or shell with concentrated masses. In this work the area of contact between the continue and discrete elements has a dimension that is one order higher than usual. For example, let us consider the thin-walled element not with concentrated masses with added massive hard bodies. It is necessary to note, that in general selection of some components as absolutely hard, and others as deformable is very problematic. After the selection of N.S. a question arises about its optimum and its adequacy.

Let ***m*** be some N.S., which is described by the following set of parameters:

1. Parameters unchanging in the process of optimization: geometrical, mechanical. Some of these parameters can be with a set of random distributions. These are stochastic external influences or stochastic distributions of physical-mechanical properties of materials. Possibly, the discrete analogue of both the external influences and internal properties must be obtained. We will designate the set of these parameters by E_1. We will describe the discrete analogue for the set of the inner parameters by E_2. We will describe the set of all parameters by E ($E = E_1 + E_2$). The task of optimization and the task of identification take a similar form: on the subset of E_s to find such a set of real numbers of $\lambda_1, \lambda_2, ..., \lambda_N$, so that:

Y - solution of the system of equations

$$L(Y, E_s) = 0 \, , \qquad (1)$$

that describes the stress-strain state. The task is to find such elements λ_i of E_i, so that:

$$\Phi(Y) \Rightarrow \min \qquad (2)$$

Here Φ is either functional of optimum or functional of adequacies.

3. Some aspects of identifying the parameters of laminated elements

Various displacement models have been developed by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer to investigate the phenomenon of wave propagation as well as vibrations in laminated composite plates [4-6]. Numerical evaluations obtained for wave propagation and vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the efficient displacement field for economic analysis of wave propagation and vibrations in laminated composite plate. The numerical method developed follows a semi-analytical approach with analytical field applied in longitudinal direction and layer-wise displacement field employed in transverse direction. The present work aims at developing a simple numerical technique, which can produce very accurate results in comparison with the available analytical solution and also to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis. As can seen the order for this approximation in normal direction to the laminated plate is 3 – for normal displacement *w* and 2 – for in plan displacement *u*. This coincide with, for example, theory [4] ore [5,6]. If the frequency level is higher, ore plate is not simply supported, but clamped (we usually in industry) ore has rigid inclusions ore holes, in such the case the more convenient theories must be discussed. For the each case of investigation must be solved the problem, what the theory is needed. Every theory has there limitation, thru them, such as finite elements method. It is well known, that the exact solutions of elasticity theories are singular near the corners. Thus, the oscillation solutions found, for example in [7] by ABAQUES are not obviously physically convenient to the case of study.

We will consider the discrete-continuum N.S., that contains the thin-walled laminated elements of K_s, $s=1,2,...,N$. We will represent excitation as a set of the discrete type 6- measured vectors, where every vector shows itself as an alternative set of power or kinematics factors:

$$G_i = \begin{pmatrix} U_x \vee P_x, U_y \vee P_y, U_z \vee P_z, \varphi_x \vee \\ M_x, \varphi_y \vee M_y, \varphi_z \vee M_z \end{pmatrix}$$

These vectors G_i are excitation from the side of the absolutely rigid bodies fastened on the thin-walled stratified elements. Vectors of G_i form the matrix of the $[G]$ dimension of $p \times 6$. Now we will consider some discretisation *D* of the initial element ***A***. To obtain the discrete analogue of ***A*** it is necessary to discretise each of the thin-walled elements. Its N.S. would contain the complete number of parameters of $\lambda^C{}_1, \lambda^C{}_2, ..., \lambda^C{}_N$. Let us now consider the part of these parameters $\lambda^C{}_{i1}, \lambda^C{}_{i2}, ..., \lambda^C{}_{iN}$ that change during optimization. Now the task of parametric optimization of A can be formulated thus:

$$\Phi(\lambda_k)_{\lambda_k \in D} \Rightarrow \min\,;\ \Psi_i(\lambda_k)_{\lambda_k \in D} = \phi_i \quad i = 1,2,\dots,N_\phi\,;$$
$$\Xi_i(\lambda_k)_{\lambda_k \in D} < \xi_i \quad i = 1,2,\dots,N_\xi\,, \qquad (3)$$

here D is the set of parameters of $\Gamma_1, \Gamma_2, \dots, \Gamma_N$ of the elements $\boldsymbol{A}_i$, ϕ_i, ξ_i are some numbers.

We will consider in more detail the discretisation of the laminated thin-walled element of ***A***. The parameters of $\lambda_1, \lambda_2, \dots, \lambda_{N_R}$, are parameters that describe the geometrical form of A: amount and thickness of the layers, mechanical properties of the materials of the layers. Their amount is limited. For composite materials it is often necessary to determine the mechanical properties from the structure of the materials. In such case some of the parameters of $\lambda_1, \lambda_2, \dots, \lambda_{NR}$ will be derived from a great number of primary parameters of $\eta_1, \eta_2, \dots, \eta_{NP}$, which characterize the mechanical properties of fibers, the structure of composition material and some features of the process of polymerization. It should be noted that D is not directly dependent on all the aggregate of parameters of $\lambda_1, \lambda_2, \dots, \lambda_{NR}$, only on their combinations. It is actually possible to consider only optimization on the parameters $\nu_1, \nu_2, \dots, \nu_{NC}$ which on $\lambda_1, \lambda_2, \dots, \lambda_{NR}$. If we consider the set accordingly:

$$\eta_i \in \mathrm{I}\ , \quad \lambda_i \in \mathrm{M}\ , \quad \nu_i \in \mathrm{O}$$

then the surjectiv reflection of A, B will take place

$$\mathrm{I} \xrightarrow[\mathrm{A}]{} \mathrm{M} \xrightarrow[B]{} \mathrm{O} \qquad (4)$$

The actual calculation and consequently the optimization will be conducted in the set of O, far narrower than M, and even more than I. N.S. which uses hypotheses for the entire package of the laminated elements will especially demonstrate this narrowing. The scheme of the condensed modeling of the sandwich element is presented in the fig.1. The variety of NS and its interaction with the testing methods and experimental modeling conditions are presented in fig.3. Some aspects of this scheme application may be found in [8-10].

For the elastic module identification the procedure was applied by comparing the elastic energy of the two beams: one of them – non-uniform, and other – uniform. Two methods were applied: analytical and approximation [8,9]. Figure 2 presents the theoretically found elasticity constants (E,G) for the Tymoshenko-beam analogue of sample ***B*** [8]. A small variance in values is caused by the different elastic energy components taken into account in both cases. In the approximation method the transverse energy component $\int_V (\sigma_{zz}\delta\varepsilon_{zz})dV$ (see [8,9]) is taken into account.

4. Conclusions

The present paper is the first attempt at proposing a novel procedure to derive stiffness parameters from forced vibrating sandwich plates. The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical ore approximation method. In the future the extension of the present approach to sandwich plates will be performed in order to test various experimental conditions.

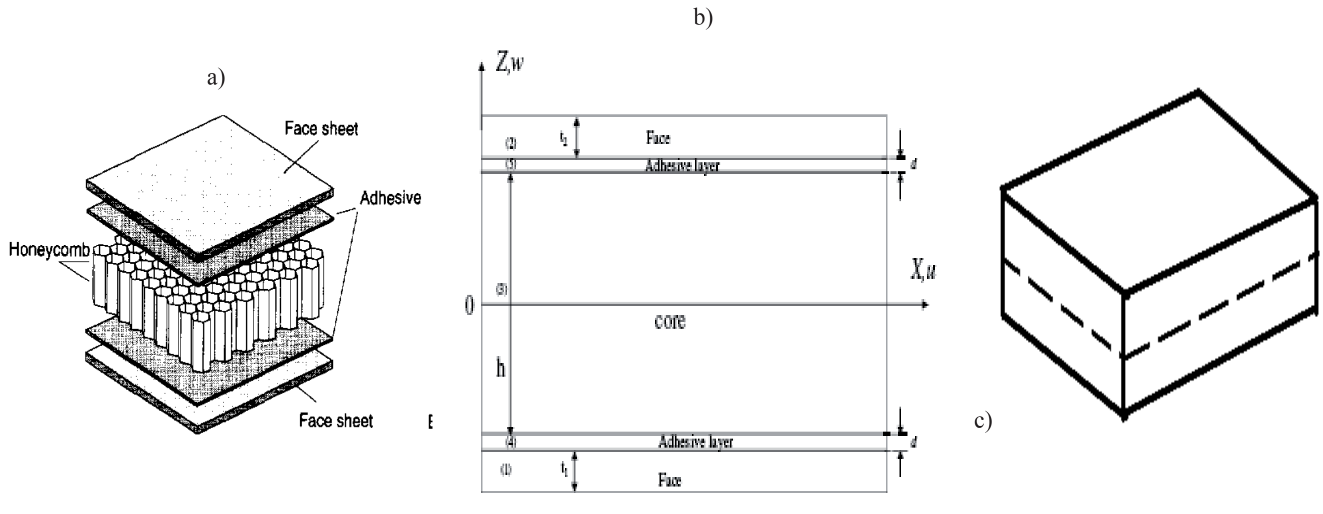

Fig. 1. Heterogeneous; a) 3-dimension model, b) 2-dimension model, c) 2-dimension model

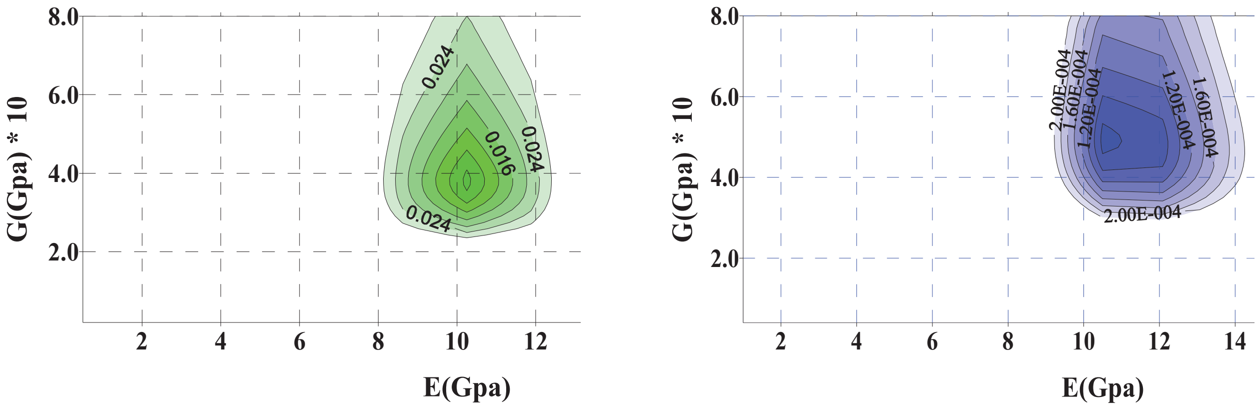

Fig. 2. a) Analytical approach, b) Approximation approach

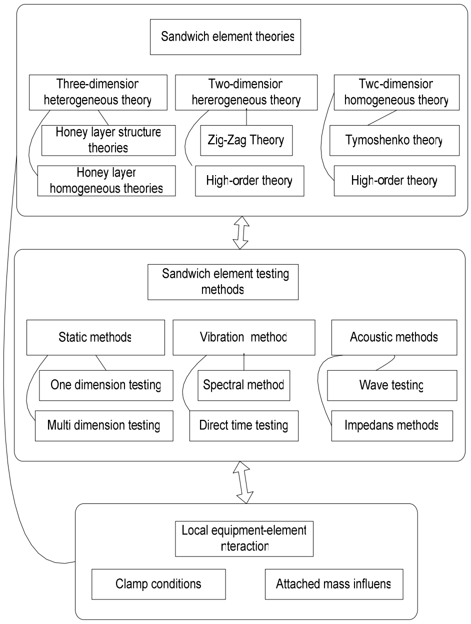

Fig. 3. The scheme of parameter identification

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