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Crack arrest model for a piezoelectric strip subjected to Mode-I loadings

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Properties

ABSTRACT

Purpose: The present paper aims at proposing a crack arrest model for an infinitely long narrow, poled ceramic strip weakened by a finite hairline straight crack when the edges of the strip are subjected to combined mechanical and electrical loads.

Design/methodology/approach: (Model) As a consequence of loads the rims of crack open forming a yield zone ahead of each tip of the crack. To arrest the crack from further opening the rims of the yield zones are subjected to normal cohesive quadratically varying yield point stress. Two cases are presented when edges of the strip are subjected to: Case-I~in-plane stresses and electrical displacement or Case-II~in-plane stresses and in-plane electric field. Problems are solved using Fourier integral transform method.

Findings: The stress intensity factor, yield zone length, crack opening displacement, crack growth rate have been calculated. Their variation with respect to affecting parameters viz. yield zone length, width of the strip, material constant, electrical and mechanical loads has been depicted graphically.

Research limitations/implications: The material of the strip is assumed mechanically brittle and electrically ductile consequently mechanically singularity is encountered first. The investigations in this paper are carried at this level. Also the crack yielding under the loads is considered small scale hence the yield zone is assumed to be lying on the line segment ahead of the crack.

Practical implications: Piezoelectric ceramics are widely used as sensors and actuators, this necessity prompts the fracture study on such ceramics under different loading conditions.

Originality/value: The paper gives an assessment of the quadratically varying load required to be prescribed on yield zones so as to arrest the opening of the crack. The investigations are useful to smart material design technology where sensors and actuators are manufactured.

Keywords: Fracture mechanics; Smart materials; Yield zone; Crack opening displacement; Crack growth rate

1. Introduction

The work on piezoelectric strip weakened by a crack was started by Ozawa, Nowacki and Shindo by calculating the singular stresses and electric fields of a cracked piezoelectric strip [1]. A computer simulation of a high velocity impact experiment with aluminum projectile and ceramic cylinder rod has been carried in [2]. Fourier transform method together with linear theory of piezoelectricity is utilized [3] for calculating the singular stresses and electric field in an orthotropic piezoelectric ceramic strip containing a Giffith crack under longitudinal shear. Their study also investigates [4] the electro-elastic intensification near anti-plane shear crack in orthotropic piezoelectric ceramic strip. Crack tip field of an infinite piezoelectric strip under anti-plane impact is dealt in [5]. A shear zone model with parallel boundaries is used to evaluate the dynamic cutting forces in orthogonal cutting [6]. A fourth power stress intensity factor crack growth equation for an orthotropic piezoelectric ceramic strip is developed [7]. Under Mode-III loading for a straight crack symmetrically situated and oriented in a direction parallel to edges of strip, the dynamic electro-mechanical response of a piezoelectric strip with a constant crack vertical to the boundary is investigated [8] based on the superposition and integral transform technique. A crack growth rate equation is found [9] for a finite crack in a narrow transversely isotropic piezoelectric ceramic body under tensile loading based on yield strip method , the solution is found using integral transform method. Fracture behavior of cracked poled piezoelectric material strip under combined mechanical and electrical loads is investigated [10] when the crack is vertical to the top and bottom edges of the strip. The saturation strip model for piezoelectric crack is re-examined [11] in a permeable environment to analyze fracture toughness of a piezoelectric ceramic for a permeable crack. Generalized Dugdale model solution is presented in [12] for a piezoelectric plate weakened by two straight cracks. Fracture resistance behavior of Alumina-Zirconia composites is studied in [13]. A plane strain problem for an interface crack with an artificial contact zone near its tips, along the fixed edge of a piezoelectric semi-infinite strip under concentrated electro-mechanical loading is examined [14]. A crack arrest model is proposed [15] for a poled piezoelectric plate weakened by a straight crack. The crack opens due to the tension at infinity consequently the plastic zones are developed which are closed by with variable yield point stresses.

2. Mathematical formulation

Solution of governing equations using Fourier transforms may be written as

$$u(x,z)=-\frac{2}{\pi}\sum_{j=1}^{3}a_j\int_0^\infty[A_j(\alpha)\sinh(\gamma_j\alpha z)+B_j(\alpha)\cosh(\gamma_j\alpha z)]\sin(\alpha x)\,d\alpha, \tag{1}$$

$$w(x,z)=\frac{2}{\pi}\sum_{j=1}^{3}\frac{1}{\gamma_j}\int_0^\infty[A_j(\alpha)\cosh(\gamma_j\alpha z)+B_j(\alpha)\sinh(\gamma_j\alpha z)]\cos(\alpha x)\,d\alpha+a_h z. \tag{2}$$

$$\phi(x,z)=-\frac{2}{\pi}\sum_{j=1}^{3}\frac{b_j}{\gamma_j}\int_0^\infty[A_j(\alpha)\cosh(\gamma_j\alpha z)+B_j(\alpha)\sinh(\gamma_j\alpha z)]\cos(\alpha x)\,d\alpha-b_h z, \tag{3}$$

where u, w denote the displacement component in x and z-direction and ϕ is electrical potential; $A_j(\alpha)$ and $B_j(\alpha)$ are arbitrary functions to be determined ; a_h , b_h are constants to be determined from edge conditions of the strip and γ_j^2 $(j=1,2,3)$ are the roots of characteristic equation.

$$A\gamma^6-B\gamma^4+C\gamma^2-D=0, \tag{4}$$

and, $A=-c_{44}(c_{33}\varepsilon_{33}+e_{33}^2)$,

$$B=-2c_{44}e_{15}e_{33}-c_{11}e_{33}^2-c_{33}(c_{44}\varepsilon_{11}+c_{11}\varepsilon_{33})+\varepsilon_{33}(c_{13}+c_{14})^2+2e_{33}(c_{13}+c_{14})(e_{31}+e_{15})-c_{44}^2\varepsilon_{33}-c_{33}(e_{31}+e_{15})^2,$$

$$C=-2c_{11}e_{15}e_{33}-c_{44}e_{15}^2-c_{11}(c_{33}\varepsilon_{11}+c_{44}\varepsilon_{33})+\varepsilon_{11}(c_{13}+c_{44})^2-2e_{15}(c_{13}+c_{44})\times(e_{31}+e_{15})-c_{44}^2\varepsilon_{11}-c_{44}(e_{31}+e_{15})^2,$$

$$D=-c_{11}(c_{44}\varepsilon_{11}+e_{15}^2), \tag{5}$$

The constant a_j and b_j are given by

$$a_j=\{(e_{31}+e_{15})(c_{33}\gamma_j^2-c_{44})-(e_{13}+e_{44})\times(e_{33}\gamma_j^2-e_{15})\}/\{(c_{44}\gamma_j^2-c_{11})\times(e_{33}\gamma_j^2-e_{15})+(c_{13}+c_{44})(e_{33}+e_{15})\gamma_j^2\}, \tag{6}$$

$$b_j=\frac{(c_{44}\gamma_j^2-c_{11})a_j+(c_{13}+c_{44})}{e_{31}+e_{15}} \tag{7}$$

where c_{11}, c_{13}, c_{33} and c_{44} are elastic constants; e_{31}, e_{33} and e_{15} are electric constants ; ε_{11} and ε_{33} stand for dielectric constants.

As is well-known the governing equation for electric potential in vacuum reduces to Laplace equation whose solution may be written as

$$\phi^V(x,z)=\frac{2}{\pi}\int_0^\infty A_4(\alpha)\sinh(\alpha z)\cos(\alpha x)\,d\alpha, 0\le x\le c \tag{8}$$

where superscript $'V'$ denotes the quantities refer to vacuum. $A_4(\alpha)$ is the unknown.

Opening mode stress intensity factor (SIF), $K_I(a)$, is defined as

$$K_I(a)=\lim_{x\to a^+}\left\{[2\pi(x-a)]^{1/2}\,\sigma_{zz}(x,0)\right\}. \tag{9}$$

Crack opening displacement $\delta(x)$ is defined by

$$\delta(x)=2w(x)=\frac{4}{F_1}\int_x^a m(x,\alpha)K_I(\alpha)\,d\alpha, \tag{10}$$

with $m(x,\alpha)=\sqrt{\frac{\alpha}{\pi}}\frac{1}{\sqrt{\alpha^2-x^2}}$.

3. The problem & mathematical model

An infinitely long, finite width (2h), transversely isotropic piezoelectric strip occupies oxyz region. The strip is poled along oz-direction and is embedded with a finite hairline straight crack, L, lying in the interval $|x|<c$ on ox-axis. The edges of the strip are subjected to combined mechanical and electrical loading. Consequently the rims of the crack open in mode-I type deformation forming a yield zone ahead of each tip of crack, occupying the region $c\le|x|\le a$.

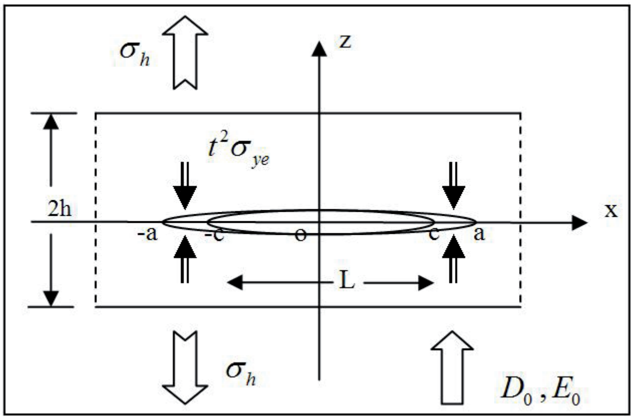

Fig. 1. Schematic presentation of the problem

The configuration is subjected to following conditions

(a) $\sigma_{zx}(x,0)=0, \sigma_{zx}(x,h)=0,$,for $0 \le x < \infty$,

(b) $\sigma_{zz}(x,0)=\{x^2\,\sigma_{ye}\,H(x-c)\}/c^2$, for $0 \le x \le a$,

(c) $u_z(x,0)=0$, for $a \le x < \infty$,

(d) $\phi(x,0)=0$, for $c \le x < \infty$,

(e) $D_z(x,0)=D^V{}_z(x,0), E_x(x,0)=E^V{}_x(x,0), 0 \le x < c$

where $H(x-c)$ is the Heaviside-step function .

4. Solution of the problem

For Case-I, $\sigma_{zz}=\sigma_h=\dfrac{\overline{c_{33}}}{c_{33}}\sigma_0-\dfrac{e_{33}}{\varepsilon_{33}}D_0$ and $D_z=D_0$ for $0 \le x \le \infty$, where σ_0 is the uniform normal stress at zero electrical load and $e^2{}_{33}=(\overline{c_{33}}-c_{33})\,\varepsilon_{33}$ for material under consideration. Using the boundary condition (a-e) and equations (1-8), the solution finally reduces to the solution of

$$C(\alpha)=\frac{\pi a^2}{2F_1}\int_0^1 \xi^{1/2}\,\Phi(\xi)\,J_0(a\alpha\xi)\,d\xi, \quad (11)$$

where

$$F_1=\frac{1}{d}[\gamma_1 g_1(b_2 f_3-b_3 f_2)+\gamma_2 g_2(b_3 f_1-b_1 f_3) \quad (12)$$
$$+\gamma_3 g_3(b_1 f_2-b_2 f_1)]$$

$$d=b_1(f_2-f_3)+b_2(f_3-f_1)+b_3(f_3-f_1) \quad (13)$$

and $J_0(a\alpha\xi)$ is the zero order Bessel function of first kind, $\gamma_j{}^2$ $(j=1,2,3)$ are the roots of equation (4).

The constants f_j and g_j are known quantities given by

$$f_j=c_{44}(a_j\gamma^2{}_j+1)-b_j e_{15}, g_j=c_{13}a_j-c_{33}+e_{33}b_j, \quad (14)$$

$$a_h=(\varepsilon_{33}\sigma_h+e_{33}D_0)/(c_{33}\varepsilon_{33}+e^2{}_{33}); \quad (15)$$

$$b_h=-(e_{33}\sigma_h-c_{33}D_0)/(c_{33}\varepsilon_{33}+e^2{}_{33}). \quad (16)$$

And, $\Phi(\xi)$ satisfies the integral equation

$$\Phi(\xi)+\int_0^1 K(\xi,\eta)\Phi(\eta)\,d\eta = \quad (17)$$
$$\begin{cases} -\sigma_h\xi^{1/2}, & \xi<\dfrac{c}{a} \\ -\sigma_h\xi^{1/2}+\dfrac{1}{2}\xi^{5/2}\dfrac{a^2}{c^2}\sigma_{ye}\times \\ \left[1-\dfrac{2}{\pi}\sin^{-1}\left(\dfrac{c}{a\xi}\right)+\dfrac{1}{\pi}\sin\left\{2\sin^{-1}\left(\dfrac{c}{a\xi}\right)\right\}\right] & ,\dfrac{c}{a}<\xi<1 \end{cases}$$

with Kernel $K(\xi,\eta)=\sqrt{\xi\eta}\int_0^\infty\left[\dfrac{G_1(\alpha/a)}{F_1}-1\right]\alpha\times$

$$J_0(\alpha\xi)J_0(\alpha\eta)\,d\alpha, \quad (18)$$

For Case-II, the edge conditions on the strip yield $a_h=(\sigma_h+e_{33}E_0)/c_{33}$, $b_h=E_0$.

All other unknowns $A_i(\alpha), B_i(\alpha)$ and $C(\alpha)$ remain the same as for the Case- I.

5. SIF, yeild zone, crack opening displacement and crack growth rate

Opening mode stress intensity factor at the tip x=b is given by

$$K_I(b)=-\sqrt{\pi b}\,\Phi(1), \quad (19)$$

Yield zone length is dertermined using equation

$$\frac{c}{a}=\cos\left[\pi\left(\frac{c}{a}\right)^2\frac{\sigma_h+U(h/a)}{\sigma_{ye}}\frac{c}{a}\sqrt{1-\left(\frac{c}{a}\right)^2}\right], \quad (20)$$

where $U(h/a)=\int_0^1 K(1,\eta)\Phi(\eta)\,d\eta$. (21)

Crack opening displacement at any point x on the rim of the crack is obtained from

$$COD(x)=\frac{8\sigma_{ye}}{\pi F_1}\left[\frac{\sqrt{a^2-x^2}}{2}\left\{\left(\frac{a}{c}\right)^2\cos^{-1}\left(\frac{c}{a}\right)+\sqrt{\left(\frac{a}{c}\right)^2-1}\right\}\right.$$
$$\left.-\frac{\pi}{4c^2}\int_x^a\frac{\alpha^3}{\sqrt{\alpha^2-x^2}}\left\{1-\frac{2}{\pi}\sin^{-1}\left(\frac{c}{\alpha}\right)+\frac{1}{\pi}\sin\left(2\sin^{-1}\left(\frac{c}{\alpha}\right)\right)\right\}d\alpha\right]$$

Crack growth rate per cycle , dc/dN , is calculated from

$$dc/dN=\{\pi\,(\Delta K_I)^4\}/\{192\,\gamma\,F_1\,\sigma^2{}_{yc}\},$$

where

$$\Delta K_I=\{\Delta\sigma+2U(h/a)\}\sqrt{\pi c}$$

$$\Delta\sigma=\begin{cases}\dfrac{\overline{c_{33}}}{c_{33}}\Delta\sigma_0-\dfrac{2e_{33}}{\varepsilon_{33}}D_0, & (Case-I) \\ \Delta\sigma_0-2e_{33}E_0, & (Case-II)\end{cases} \quad (22)$$

Under cyclic loading $\sigma_0 \to \Delta\sigma_0/2$, σ_{yc} is the cyclic yield strength.

6. Numerical results & discussion & conclusion

Case study has been carried for stress intensity factor with respect to strip width, yield zone to crack length ratio for ceramics PZT-4, PZT- 5H and $BaTiO_3$.

Figure 2, depicts the variation of normalized stress intensity factor as the strip width is increased for Case-I. As expected for larger strip width the SIF decreases for all the three ceramics for different values of $c_{33}e_{33}D_0/\sigma_0\overline{c_{33}}\,\varepsilon_{33}$. As the value this constant varies from -0.25 to 0.5 value the SIF decreases exponentially.

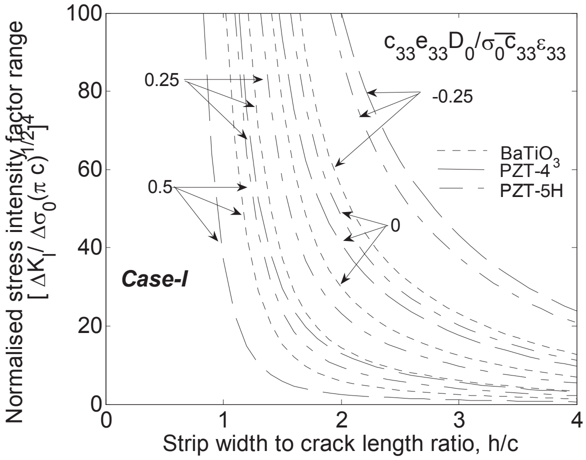

Fig. 2. plots Normalized SIF range vs. h/c variation

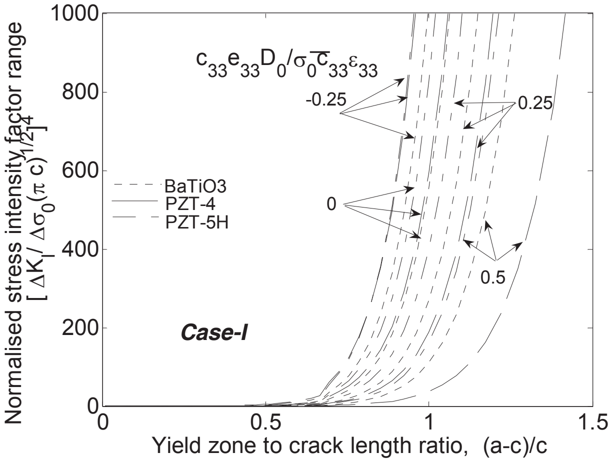

Fig. 3. plots Normalized SIF range vs. (a-c)/c variation

Variation of stress intensity factor with increasing yield zone to crack length ratio is ploted in figure 3 for Case-I. As yield zone size is increase this SIF also increases.

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