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Calculation of characteristics of torsionally vibrating mechatronic system

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Analysis and modeling

ABSTRACT

Purpose: of this paper is the application of the approximate method to solve the task of assigning the frequency-modal analysis and characteristics of a mechatronic system.
Design/methodology/approach: was the formulated and solved as a problem in the form of a set of differential equations of motion and state equations of the considered mechatronic model of an object. To obtain the solution, Galerkin's method was used. The discussed torsionally vibrating mechanical system is a continuous bar of circular cross-section, clamped on its ends. A ring transducer, which is an integral part of the mechatronic system is assumed to be perfectly bonded to the bar surface.
Findings: this study is that the parameters of the transducer have an important influence on the values of natural frequencies and on the form of the characteristics of the said mechatronic system. The poles of the dynamical characteristic calculated with the use of mathematical exact method and Galerkin's method have approximately the same values. The results of the calculations were not only presented in a mathematical form but also as transients of the examined dynamical characteristic which are a function of frequency of the assumed excitation.
Research limitations/implications: is that the linear mechatronic system was considered, but for this type of systems, such approach is sufficient.
Practical implications: of this researches was that another approach is presented, that means in the domain of frequency spectrum analysis. The method used and the obtained results can be of some value for designers of mechatronic systems.
Originality/value: of this paper is that the mechatronic system, created from mechanical and electrical subsystems with electromechanical bondage was examined. This approach is other than those considered elsewhere.
Keywords: Applied mechanics; Torsionaly vibrating shaft; Approximate method; Flexibility

1. Introduction

The designed machines consist of various elements or components the properties of which are exactly specific.

The requirements concerning mechatronic systems, for example their working velocity, exact positioning, control and dimensions are challenging tasks for scientific research. These tasks, however, cannot always be approached from the point of view of traditional principles of mechanics. Therefore, it is necessary to investigate new possible methods of examination and analysis the mechatronic systems.

In the last few years a lot of attention has been given to research projects involving new construction solutions, especially as far as the technology of drives based on the phenomenon of piezoelectricity and electrostriction is concerned [11, 15-17, 19, 21,22]. The piezoelectric elements are also used to eliminate oscillation [18].

The first attempt to solve this problem, that is, to determine the dynamical characteristic of a longitudinally and torsiolnally vibrating continuous bar system and various classes of discrete mechanical systems in view of the frequency spectrum, by means of graphs and structural numbers methods, was made in the Gliwice Research Centre[1)] in [1-10, 12-14].

2. The torsionaly vibrating shaft with piezotransducer and shunting circuit

In the paper [8] the mechatronic system clamped at one of its end has been considered. The system was not excited by any mechanical moment. The harmonic electrical voltage which excites the system from the electric side was applied to the converter clips. This voltage evokes the deformation of the piezoelement, which interacts directly with the shaft (Fig. 1).

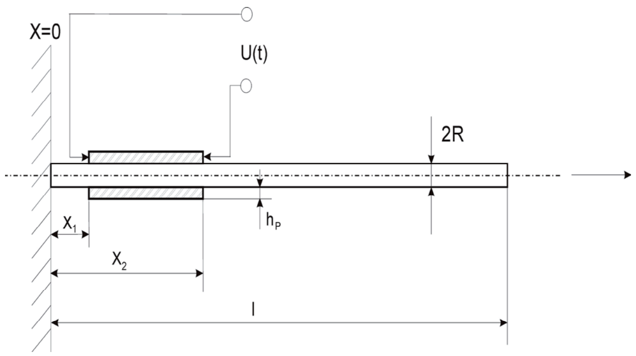

Fig. 1. The mechatronic system with electrical excitation

In this paper the torsionaly vibrating system is considered (see Fig. 2).

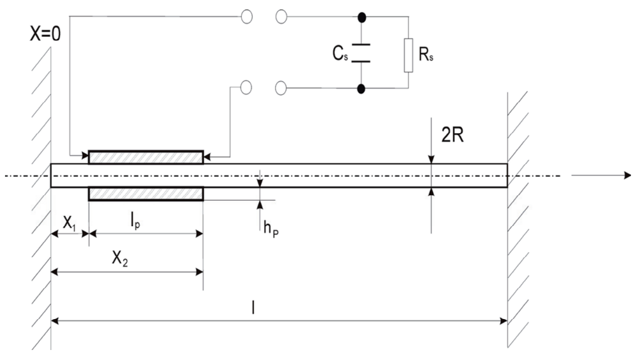

Fig. 2. Shaft with piezotransducer with mechanical exitatation and shunting circuit

The continuous elastic shaft with full section, constant along the whole length l, clamped at of its ends is made of a material with transverse modulus, i.e. Kirchoff's modulus G and mass density ρ . The system is excited by any mechanical moments.

To the surface of the shaft, an ideal ring piezotransducer is perfectly bonded at a certain position x_1.

The dynamical equation of the motion of the shaft, in view of the given system, takes the following form [18]

$$\ddot{\varphi} - \frac{G}{\rho}\varphi_{xx} = -\frac{\lambda^*}{I_o\rho l}U\left[\delta\left(x-x_1\right)-\delta\left(x-x_2\right)\right] + \frac{M}{I_0\rho l}\delta\left(x-l\right), \quad (1)$$

where: $\lambda^* = \frac{2}{3}\pi G_p\left[\left(R+h_p\right)^3 - R^3\right]\frac{d_{15}}{l_p}$,

G_p, l_p - transverse modulus and length of the piezoelement respectively,

d_{15} the electromechanical coupling coefficient [18,19,22].

Similarly, the dynamic equation of the piezotransducer is given in the form:

$$\dot{U} + \alpha_1 U + \alpha_2\dot{\varphi}\left(l_p, t\right) = 0, \quad (2\text{-}3)$$

where: $C_p = 2\pi R h_p \frac{e_{15}}{l_p}\left(1 - \frac{2d_{15}G_p}{e_1}\right)$

$\alpha_1 = \frac{1}{R_s C_p}$,

e_{15} - the dielectric constant [18, 19, 22].

Taking into consideration equations (1) and (2), a set of equations that will be a starting point of further considerations can be derived. The considered mechatronic system is described by the next set of equations in form

$$\begin{cases} \ddot{\varphi} - \frac{G}{\rho}\varphi_{xx} + \frac{\lambda^*}{I_0\rho l}U\left[\delta\left(x-x_1\right)-\delta\left(x-x_2\right)\right] = \frac{1}{I_0\rho l}M\delta\left(x-l\right) \\ \dot{U} + \alpha_1 U + \alpha_2\dot{\varphi}\left(l_p, t\right) = 0 \end{cases} \quad (3)$$

According to Galerkin's discretisation of the solutions of the differential equation system with partial derivative, the solution sought in this paper will involve the sum function, i.e. the function of the time and displacement variables, which are strictly determined and which fulfill the boundary conditions [8,19].

The boundary conditions on the shaft ends are given in the form of

$$\varphi(0,t) = 0;\ \ \Phi(0)T(t) = 0 \rightarrow \Phi(0) = 0 \quad (4)$$

$$\varphi(l,t) = 0;\ \ \Phi(l)T(t) = 0 \rightarrow \Phi(l) = 0\,. \quad (5)$$

It is accepted that dislocation, i.e. the angle of the torsion of the cross-section takes the following form:

$$\varphi(x,t) = \sum_{i=1}^{\infty}\varphi_i(x,t) = \sum_{i=1}^{\infty} A_i \sin\frac{n\pi x}{l}e^{i\omega t}\,. \quad (6)$$

1) Other diverse problems have been modelled by different kind of methods, next the problems were examined and analysed in the centre for the last several years (e.g. [17, 23-27].)

Moreover, it is assumed that the system is excited by harmonic moment as follows:

$$M = M_0 e^{i\omega t}\,. \tag{7}$$

When the mechanical excitation (7) has a harmonic character, then the voltage generated in the transducer as a piezoelectric effect will have the same character, that means

$$U = Be^{i\left(\omega t-\frac{\pi}{2}\right)}\,. \tag{8}$$

3. Frequency-modal analysis of mechatronic system

For the first vibration mode, i.e. when n=1, the angle of torsion (6) takes the form of

$$\varphi_1(x,t) = A_1 \sin\frac{\pi x}{l} e^{i\omega t}\,. \tag{9}$$

The solution of the examined set of differential equations (3), leads to appropriate derivatives. By substituting the derivatives of expressions (6-8) in equation (3) are derived

$$\begin{cases} AK_1 e^{i\omega t}\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] + Be^{i\left(\omega t-\frac{\pi}{2}\right)}\frac{\lambda^*}{I_o\rho l}D = \frac{1}{I_0\rho l}M_0 e^{i\omega t}E, \\ Ai\alpha_2\omega Ce^{i\omega t} + Bi\omega e^{i\left(\omega t-\frac{\pi}{2}\right)} + B\alpha_1 e^{i\left(\omega t-\frac{\pi}{2}\right)} = 0, \end{cases} \tag{10}$$

where: $K_1 = \sin\frac{\pi x}{l}$,

$C = \sin\frac{\pi}{l} l_p$, $D = \left[\delta(x-x_1) - \delta(x-x_2)\right]$,

$E = \delta(x-l)$ for predetermined x, x_1 and x_2

After arrangement (10) takes character

$$\begin{cases} A_1 K_1 e^{i\omega t}\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] I_o\rho l + B_1 e^{i\left(\omega t-\frac{\pi}{2}\right)}\lambda^* D = M_0 e^{i\omega t}E, \\ A_1 i\alpha\omega C_1 e^{i\omega t} + B_1 i\omega e^{i\left(\omega t-\frac{\pi}{2}\right)} = 0. \end{cases} \tag{11}$$

To designate the dynamical characteristic from these equations, the time function must be eliminated using Euler's theorem, accordingly, the following set of equations is obtained as the matrix shape

$$\mathbf{W}\,\mathbf{A} = \mathbf{F} = \begin{bmatrix} K_1\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] I_0\rho l & -i\lambda^* D \\ i\alpha\omega C_1 & \omega \end{bmatrix}\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} M_0 E \\ 0 \end{bmatrix} \tag{12}$$

The main determinant of square matrix $\mathbf{W}$ is equal to

$$|\mathbf{W}| = K_1\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] I_0\rho l\omega - \alpha\omega C_1\lambda^* D \tag{13}$$

By substituting in square matrix $\mathbf{W}$, the first column by matrix $\mathbf{F}$, is obtained

$$|\mathbf{W}_A| = \begin{vmatrix} M_0 E & -i\lambda^* D \\ 0 & \omega \end{vmatrix} = M_0 E\omega\,. \tag{14}$$

Thus, the amplitude of the dynamical characteristic is obtained as

$$A_1 = \frac{|\mathbf{W}_A|}{|\mathbf{W}|} = \frac{M_0 E}{K_1\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] I_0\rho l - \alpha C_1\lambda^* D}\,, \tag{15}$$

By substituting the received amplitude A_1 (15) to (9), the angle of the torsion of the cross-section for the first vibration mode, i.e. n=1, is determined

$$\varphi_1(x,t) = \frac{E\sin\frac{\pi x}{l}}{K_1\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] I_0\rho l - \alpha C_1\lambda^* D} M_0 e^{i\omega t}\,. \tag{16}$$

Out of (16) the dynamical flexibility for the first vibration mode takes the form of

$$Y_{x1} = \frac{E\sin\frac{\pi x}{l}}{\left[\frac{G}{\rho}\left(\frac{\pi}{l}\right)^2 - \omega^2\right] I_0\rho l - \alpha C_1\lambda^* D}\,. \tag{17}$$

In the same way the dynamical flexibility for the second, third and next vibration modes can be obtained (comp. [8]).

4. Conclusions

The presented approach makes it possible to consider the behavior of the mechatronic system in a global way. The influence of the change in values parameters, which directly depend on the type of the piezoelement and on its geometrical size in view of the characteristics, the sort of vibrations of the mechatronic system, mainly as far as the piezoelectric converter "activation" is concerned, shall be discussed in further research works.

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