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Analysis of a degenerated standard model in the piercing process

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Analysis and modelling

ABSTRACT

Purpose: The purpose of this paper is the mathematical description of the impact phenomenon of a bullet of the speed ca. 400 m/s, with the use of a degenerated model.

Design/methodology/approach: In the study, an attempt has been made to apply an untypical model for the piercing phenomenon analysis. Basing on the model, the theoretical analysis of the piercing phenomenon in quasistatic and dynamic load conditions, in the impact load form, has been carried out.

Findings: This analysis enabled derivation of significant conclusions useful in the design process of effective ballistic shields.

Research limitations/implications: In the study, the concept has been assumed that a dynamic model, simple as possible, that may be analyzed not only by numerical methods but also (at least approximately) with the mathematical analysis methods, may provide significant directions concerning material piercing.

Practical implications: The use of so called degenerated model allows to describe the phenomenon in more detail at various piercing speeds what extends the possibilities in the sphere of designing the optimum ballistic shields and of identification of the properties of materials applied for construction of shields.

Originality/value: The proposed method of identification of material properties in the piercing process, within the relation: force – deformation, is a novel one since the essence of the identification is the standard rheological model in an adequate plastic component describing the viscous attenuation with dry friction.

Keywords: Computational mechanics; Impact load; Impact; Composites

1. Introduction

In the military applications for light weight ballistic shields (e.g. body armors, vehicle armoring), light weight composite materials are more and more often applied; the materials include, for instance, structures made of laminates and sandwich panels reinforced with plastic fibers. The fibers are characterized with high resistance characteristics for impact loads. The piercing process for the lightweight material structures is extremely complex. It is connected with the fact that the damage process consists of several stages. For instance, Greaves [1-2] divides destruction occurring during the dynamical penetration into two phases, stating that the first phase during which indentation and shear happens is predominating (the biggest part of the energy is absorbed in this phase). Many other researchers [3-5] tried to determine the piercing process phases. Non classical methods of modeling concerning synthesis, analysis and researches of sensitivity various models of objects can be found in [6-12]. However, modeling of this process is done exclusively on the basis of determination of substitute rigidity values originating from the general elasticity theory for isotropic bodies. In the present study, the author proposes another approach wherein not only pure elastic interactions are assumed in the theoretical model, but also dissipative components in an appropriate (non parallel) configuration with the elastic components. Such model enables for a more accurate description of the piercing phenomenon in which the dependence of the characteristic curves "force – deformation" on the speed of the deformations occurring is observed often, what does not happen for the pure elastic liner model case.

2. Description of the approach

In the present study, the following premises have been used for constructing the piercing model:

1) the dominating role is played by the material sphere directly adhering to the piercing material (bullet),
2) it has been assumed that the bullet is non deformable,
3) shield vibration after the impact are of no influence upon the bullet movement in the shield (the bullet movement happens fast with relation to the wave propagation speed in the shield),
4) the material damage process within the impact scope has been divided into two stages:
   - stage I, wherein the reversible deformation (that does not destroy the material in a permanent way) happens,
   - stage II, wherein the irreversible deformation (permanent destruction of the material) happens.
5) it has been accepted that the shield has been secured in an ideally elastic way to an inertia-type impact system.

The diagram of the model accepted a priori has been presented in Fig. 1.

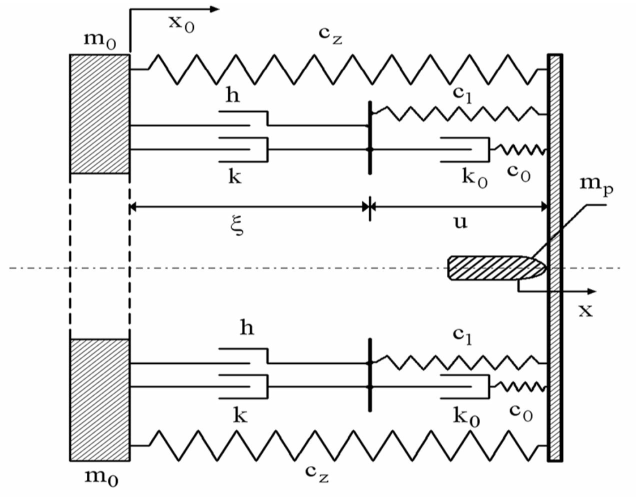

Fig. 1. The diagram of the model assumed in the piercing process, where: x – a constant describing the bullet movement in the shield, x_0 – a constant describing the shield shift, u – a variable describing the II stage of deformation, ξ – a variable describing the deformation stage II, c_1 – a constant describing the static rigidity of the material of stage I, c_0 – a constant describing the dynamic rigidity of the material of the stage I, c_z – a constant describing the shield fixing, k_0 – attenuation of the deformation stage I, h – dry friction of the deformation stage II, k – attenuation of the deformation stage II.

In the stage II, it has been assumed that the character of the acting force changes significantly after the force exceeds the value of h as dry friction during movement of the bullet in the material being pierced. The bullet movement with respect to the inertia-type (immovable) reference system is defined with a variable x(t) being the sum of the shift of the shield x_0, the reversible deformation u of the shield and the irreversible deformation ξ describing the values of the shield damage, i.e. in accordance with the relation (1).

$$x = x_0 + u + \xi \tag{1}$$

As it can be seen, within the scope of reversible deformations, the standard rheologic model has been wored out. The model includes the Maxwell component described with the constants k_0, c_0, in a parallel connection with the Hooke's component of the constant c_1. Let us note that at a rigid shield fixing, it may be assumed:

$c_z = \infty$.

In addition, the model allows also a description of simpler material models, namely:

- ideally elastic material (it should be assumed that $c_0 = 0$),
- ideally plastic material (assume: $c_1 = 0$, $c_0 = \propto$),
- elastic / plastic material ($c_0 = \propto$),
- material of a constant (independent of the deformation speed) plasticity limit (assume $k = 0$).

All constants included in the model may be determined experimentally, e.g.: at static loads, at quasistatic piercing, at dynamic load conditions, applying adequate identification methods for degenerated models [13-15].

3. Description of achieved results

3.1. Static load

In the case of a constant piercing force, $S(t) = S_0 = \text{const}$, the movement differential equations take the form:

$$m_0 \ddot{x}_0 + c_z x_0 = S_0 \tag{2}$$

$$h \,\mathrm{Sgn}\,\dot{\xi} + k\dot{\xi} = S_0 \quad \text{for } S_0 > h \tag{3}$$

$$\dot{\xi} = 0 \text{ for } S_0 \le h \tag{4}$$

$$c_1 u + c_0 (u - z) = S_0 \tag{5}$$

$$k_0 \dot{z} = c_0 (u - z) \tag{6}$$

From the above equations, it follows:

$$x_0(t) = \frac{S_0}{c_z} - \frac{S_0}{c_z}\cos\omega_0 t \,, \quad \omega_0 = \sqrt{\frac{c_z}{m}} \tag{7}$$

$$\xi(t) = \xi_0 = \text{const} \quad \text{for } S_0 \le h \tag{8}$$

$$\xi(t) = \xi_0 + \left(\frac{S_0 - h}{k}\right) t \quad \text{for } S_0 > h \tag{9}$$

where ξ_0 is the plastic strain at the initial instant, i.e. at the instant that the force S_0 is applied. For the virgin system (i.e. the system unloaded earlier), $\xi_0 = 0$. The plastic deformation increases in time at a constant speed $\dot{\xi} = \frac{S_0 - h}{k}$, if the force $S_0 > h$. On the other hand, if the force $S_0 \le h$, then the plastic deformation is constant and equal to the initial; value ξ_0. On the basis of the study [15], for the zero initial conditions, the relation u(t) takes the form:

$$u(t)=\frac{S_0}{c_1}\left[1-e^{-\frac{c_1}{k_z}t}\right] \tag{10}$$

The situation is, however, quite different in the case of short-term dynamic loads, wherein the momentary behavior of the material may be of the deciding influence upon the system motion. This happens in particular in the case of penetration of the shield by the bullet. In a case of $c_z = \propto$ and the piercing force lower that some constant h, $\xi = 0$ is obtained. Then x = u while the relation x(t) is described by the plot depicted in Fig. 2. However, for $S_0 > h$, we have:

$$\xi=\frac{S_0-h}{k}\cdot t=v_p\cdot t \tag{11}$$

where the constant $v_p=\frac{(S_0-h)}{k}$ defines the permanent deformation speed for the material in the static load conditions. Thus, in the case, the total deformation x will be equal to:

$$x=u+\xi=\frac{S_0}{c_1}\left(1-e^{-\frac{c_1}{k_z}\cdot t}\right)+v_p t \tag{12}$$

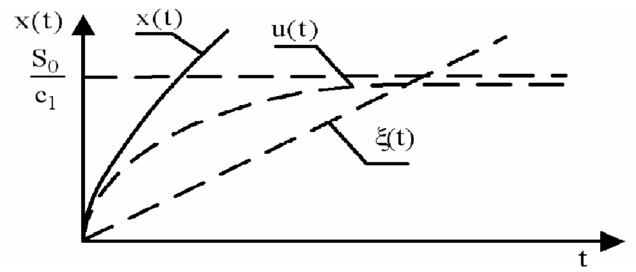

Fig. 2. Material behavior at rigid fixing under influence of the piercing force $S_0 > h$

3.2. Quasistatic load

In quasistatic loads, the piercing force is usually variable in time. For the analysis, the constant piercing force v_0 is accepted. A result of that test are, usually, the relations S(x). For $c_z = \infty$ (rigid fixing), the movement equation will be as follows:

$$\dot{\xi}=0 \quad \text{for } S\le h \tag{13}$$

$$c_1u+c_0(u-z)=S \tag{14}$$

$$k_0\dot{z}=c_0(u-z) \tag{15}$$

Upon transformations and elimination of the variable ”z”, as well as upon maintaining the case, if $S \le h$ then $\dot{\xi}_0 = 0$ and then $\dot{x}=\dot{u}$. Assuming piercing at a constant speed v_0, the final form S(u) in the quasistatic test will take the form:

$$S(u)=k_0v_0+c_1u-k_0v_0e^{-\frac{c_0}{k_0v_0}u} \tag{16}$$

The relation S(u), in its graphic form, for individual models, has been presented in Fig. 3. It can be easily noticed that, when the value v_0 approaches zero (slow piercing), all relations of S(u) become more and more similar to the relation for the Hooke’s model. On the other hand, for $v_0 = 0$, all of them are exactly equal to $S(u) = c_1u$. Significant differences occur, however, at high piercing speeds ($v_0 >> 0$) what happens in cases that the shield is shut through by the bullet (Fig. 4). Therefore, it should be noted that, by applying the Hooke’s model, both for the case (a) and (b), an apparent increase of the material rigidity (the Young’s module increase) is observed together with an increase in the piercing speed, what, as it is known, occurs in the material investigations. However, it is not the Young’s module that changes, but the reason is in that the constant k_0 is not taken into consideration in the constitutive interconnections.

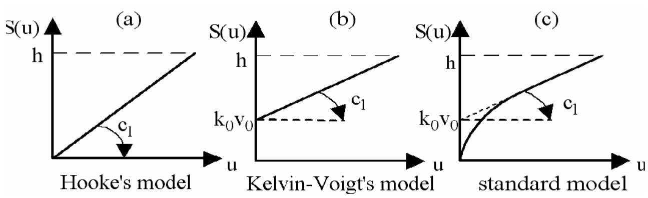

Fig. 3. The relation S(u) of quasistatic piercing for the models under consideration

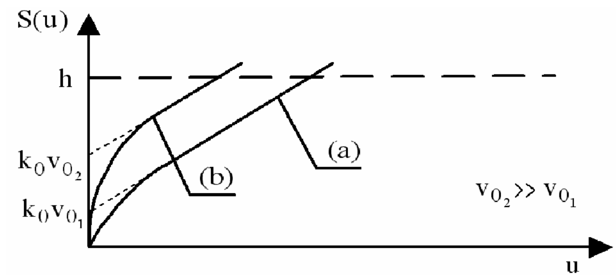

Fig. 4. A comparison of the relation S(u) for the standard model in a case of the piercing speed values v_0: small (a) and big (b)

3.3. Dynamic loads

Substituting $S_0 = -m\ddot{x}$ to the equations (2-6), the differential equations of the final mathematical form of the model accepted are obtained:

$$m_0\ddot{x}_0+c_zx_0=-m\ddot{x} \tag{17}$$

$$(hSgn\dot{\xi}+k\dot{\xi})+m\ddot{x}\cdot H(-m\ddot{x}-h)=0 \tag{18}$$

$$m\ddot{x}+c_1u+\frac{k_0}{c_0}[(c_1+c_0)\cdot\dot{u}+m\dddot{x}]=0 \tag{19}$$

where H is the Heaviside function.

For the high impact speeds, the equation system has been solved by the computer simulation technique, making use of Simulink. Exemplary piercing process results for the given constant values of the model have been depicted in Fig. 5.

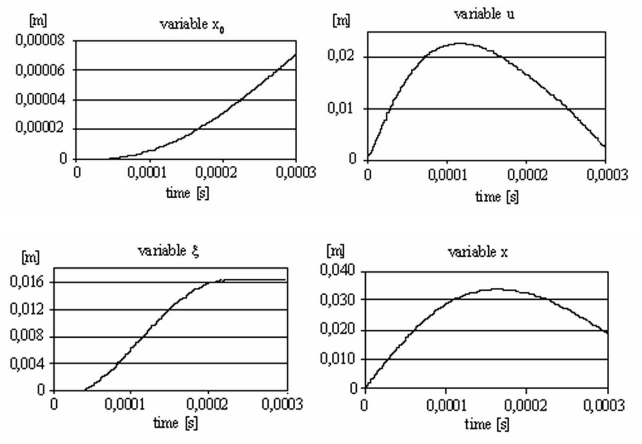

Fig. 5. Exemplary computer simulation results for given model values

4. Conclusions

The piercing analysis done on the basis of the degenerated model enabled:

- determination of the influence of the piercing speed on the force in the quasistatic tests (Fig. 4),
- determination of the impact of the characteristics defining the system behavior within the scope of reversible deformations (parameters c_0, k_0, c_1) on the permanent deformation (Fig. 5).

In addition, it has been found that, when the degenerated model is applied, there happen some differences in the piercing process for various ratios m/v_0, though the kinetic energy of the impact is the same. However, the investigation within this scope are in progress and the problem has not been discussed in the present study.

References

[1] L.J. Greaves, Failure Mechanisms in GFRP Armour, Unpublisched UK DRA Report, 1992.

[2] L.J. Greaves, Progress in modelling the perforation of GFRP by ballistic projectiles, Unpublisched UK DRA Report, 1994.

[3] J.K. Kim, T.X. Yu, Impact response and dynamic failure of composites and laminate materials, Part 2: Strain-Rate Effect, Energy Absorption and Modelling, Trans Tech Publications Ltd., Switzerland, 1998.

[4] P.D. Smith, J.G. Hetherington, Blast and Ballistic Loading of Structures, Butterworth-Heinemann, Oxford, 2003.

[5] E. Rusiński, J. Karliński, K. Jamroziak, The chosen aspects from researech of ballistic shields, Proceedings of the 22nd DANUBIA-ADRIA Symposium on Experimental Methods in Solid Mechanics DAS'2005, Parma, 2005, 22–23.

[6] A. Buchacz, Calculation of characterisics of torsionally vibrating mechatronic system, Journal of Achievements in Materials and Manufacturing Engineering 20 (2007) 327-330.

[7] A. Buchacz, Sensitivity of Mechatronical Systems Represented by Polar Graphs and Structural Numbers as Models of Discrete Systems, Journal of Materials Processing Technology 175 (2006) 55–62.

[8] A. Buchacz, Influence of a piezolectric on characteristics of vibrating mechatronical system, Journal of Achevements in Materials and Manufacturing Engineering (2006) 229-232.

[9] A. Buchacz, The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs, Journal of Materials Processing Technology 164–165 (2005) 1277-1280.

[10] A. Buchacz, Hypergrphs and their subgraphs in modelling and investigation of robots, Journal of Materials Processing Technology 157–158 (2004) 37–44.

[11] A. Buchacz, Modifications of Cascade Structures in Computer Aided Design of Mechanical Continuous Vibration Bar Systems Represented by Graphs and Structural Numbers, Journal of Materials Processing Technology 157–158 (2004) 45–54.

[12] L.A. Dobrzański, R. Honysz, S.D. Fassois, On the identification of composite beam dynamics based upon experimental data, Journal of Achievements in Materials and Manufacturing Engineering 16 (2006) 114-123.

[13] M. Kulisiewicz, S. Piesiak, The methodology of modelling and identification mechanical dynamic systems, Publishing House of the Wroclaw University of Technology, Wroclaw, 1994 (in Polish).

[14] M. Kulisiewicz, Identification of nonlinear models of vibration damping of dynamic mechanical systems subjected to random series of impulses, Systems Analysis Modelling Simulation 36/1 (1999) 49-62.

[15] K. Jamroziak, M. Kulisiewicz, The calculating models used to identify light weight ballistic shields subject to impact loads, Wroclaw WSOWL Report, 2003 (in Polish).