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A simple algorithm for formability analysis

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Analysis and modelling

ABSTRACT

Purpose: of this paper is to develop a simple algorithm for local and diffuse necking analysis, which covers different yield criteria and strain hardening laws.
Design/methodology/approach: Theoretical study, application of plasticity theory. Numerical analysis, FLD determination. Experimental verification (material parameters and FLD). Comparison of obtained results with the results available in literature. Both, stress and strain based FLD-s are considered.
Findings: The dimensionless instability tensors are introduced. The plastic instability criterion in tensor notation is derived. The capabilities of the derived instability criteria are improved using different anisotropic yield criteria from Hill48 up to BBC2003. A test procedure for determining material properties and forming limits in plane strain condition for sheet metal is performed.
Research limitations/implications: The study is based on classical instability conditions. The stress-strain behavior of the material is described with empirical equation (the strain rate and also temperature dependence of the flow stress are not considered).
Practical implications: The forming limit curve determined defines boundary between elastic or stable plastic deformation (below curve) and unsafe flow (above curve). The risk of failure is determined by the distance between the actual strain condition in the forming process and the forming limit curve.
Originality/value: A simple algorithm for local and diffuse necking analysis is proposed. The dimensionless instability tensors introduced can be used for theoretical improvements.
Keywords: Numerical techniques; Formability; Plastic anisotropy

1. Introduction

The traditional forming limit diagram is described by a curve in a plot of major strain vs. minor strain. This curve defines boundary between elastic or stable plastic deformation (below curve) and unsafe flow (above curve). The risk of failure is determined by the distance between the actual strain condition in the forming process and the forming limit curve.

The concept of FLD was proposed by Keeler and Backofen [1] for the biaxial stretching process and was extended by Goodwin [2] to include the tension-compression states. According to this concept the limit strains can be calculated on the basis of certain plastic instability criteria. The most widely used instability criteria have been proposed by Swift ([3], diffuse necking), Hill ([4], localized necking), Marciniack & Kuczynski ([5], initial imperfections) and Storen and Rice ([6], the corner theory) and Tvergaard ([7], damage model). The yield function calibration problems for orthotropic sheet metals are investigated in [8].

Most commonly, the FLD analysis is performed under the assumption of proportional loading. However, in actual sheet metal manufacturing processes the forming limit is generally strain path dependent. During the last decade a theoretical basis for a strain path-independent stress based approach is provided (Sing and Rao [9-10], Stoughton and Zhu [11]). In [12] the stress-based forming limit diagram (FSLD) is applied to forming limit prediction for the multi-step forming of auto panels.

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Recently, it is pointed out in [13], that the FLSD offers advantages over the traditional strain-based FLD in cases when unloading occurs between two successive loadings only.

An alternate, crystal plasticity based approach for forming limit prediction is given in [14]. The experimental FLD construction techniques are reviewed by Avila and Vieira [15].

In the current paper, an attempt is made to develop an algorithm for determing local and diffuse necking, which meets the following requirements:

- different anisotropic yield criteria must be included (the critical strain is sensitive to the yield surface shape),
- different strain hardening laws must be included in order to describe the stress-strain behavior of various materials.

The capabilities of the algorithm proposed are improved using different yield criteria: Hill ([4], quadratic), Hill ([16], non-quadratic), Hill ([17], user friendly), Logan and Hoshford [18], Barlat and Lian [19], BBC2003 (Banabic et al. [20]). It appears that the Hill's 1993 criterion ([16]) needs detailed attention due to its non-homogeneity with respect to the stress components. Both, the strain and stress based FLD are considered. A test procedure for determining material properties and forming limits in plane strain for sheet metal is performed.

2. Condition of continuous yielding

Assuming orthotropic symmetry, numerous yield criteria have been proposed to account in plane and normal anisotropy of the sheet metal. In the following, the yield function is considered as

$$f = \bar{\sigma}(\sigma_{ij}, R_\alpha, \sigma_\alpha, \sigma_b) - \sigma_y = 0, \quad (1)$$

where $\bar{\sigma}$ is the equivalent stress and σ_y the yield parameter, R_α and σ_α stand for the Lankford coefficients for the uniaxial yield stresses measured along the rolling, diagonal and transverse directions of the sheet, respectively ($\alpha \in \{0^0, 45^0, 90^0\}$). The equivalent stress $\bar{\sigma}$ is assumed to be a homogeneous function of degree 1 with respect to the stress components. The Lankford coefficients are defined as width to thickness strain increment ratios $R_\alpha = \varepsilon_{width} / \varepsilon_{thickness}$. The yield stress in equi-biaxial tension is denoted by σ_b.

However, the condition in form (1) describes the initiation of yielding, only. Equalizing the yield stress σ_y in (1) with the flow stress of the material one obtains the condition for continuous yielding as

$$\bar{\sigma}(\sigma_{ij}, R_\alpha, \sigma_\alpha, \sigma_b) = \sigma_{SH}(\bar{\varepsilon}_P, m_i) \quad (2)$$

In (2) $\bar{\varepsilon}_P$ and m_i stand for the the equivalent plastic strain rate and the material parameters, respectively.

3. Instability conditions

For analysis of the negative minor strain regime of FLD Hill's localized necking condition is employed. According to Hill's instability condition the localized necking (through thickness neck) occurs when the rate of strain hardening is equal to the rate of geometric softening. The localized neck is expected along the line of zero extension and the constraints are following

$$\dot{\sigma}_1 = \sigma_1(\dot{\varepsilon}_1 + \dot{\varepsilon}_2), \quad \dot{\sigma}_2 = \rho\dot{\sigma}_1. \quad (3)$$

In (3) ρ stands for strain ratio ($\rho = \sigma_2 / \sigma_1$).

The second condition in (3) states that the stress ratio remains fixed through the formation of the neck. In order to express the condition (3) in compact form let us introduce the Hill's instability tensor as

$$A_{ij}^{Hill}(\rho) = \begin{pmatrix} 1 & 1 \\ \rho & \rho \end{pmatrix} \quad (4)$$

In notation (4) the Hill's localized necking condition reads

$$\dot{\sigma}_i = \sigma_1 A_{ij}^{Hill} \dot{\varepsilon}_j. \quad (5)$$

Similarly, the Swift diffuse necking condition can be presented in tensor notation as

$$\dot{\sigma}_i = \sigma_1 A_{ij}^{Swift} \dot{\varepsilon}_j, \qquad A_{ij}^{Swift}(\rho) = \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \quad (6)$$

Obviously, the instability conditions (5) and (6) are covered by the following more abstract condition

$$\dot{\sigma}_i = \sigma_1 A_{ij}^{Instability} \dot{\varepsilon}_j, \quad (7)$$

where $A_{ij}^{Instability}$ is taken equal with A_{ij}^{Hill} and A_{ij}^{Swift} in the case of $d\varepsilon_2 / d\varepsilon_1 < 0$ and $d\varepsilon_2 / d\varepsilon_1 > 0$, respectively.

The instability conditions in tensor notation (5-7) simplify numerical implementation of the FLD.

4. Plastic instability criterion

Let us proceed from the condition of continuous yielding (2), instability condition (7) and assume that:

- the plasticity model is rate-independent and satisfies the so-called consistency condition $\dot{f} \equiv 0$,
- the equivalent stress in (2) is a homogeneous function of degree 1 with respect to the stress components.

By appying classical plasticity theory the instability criterion is derived in terms of the stress ratio $\rho = \sigma_2 / \sigma_1$, the equivalent plastic strain, the plastic anisotropy and hardening parameters as

$$\frac{1}{\varphi}\frac{\partial\bar{\sigma}}{\partial\sigma_i} A_{ij}^{Instability} \frac{\partial\bar{\sigma}}{\partial\sigma_j} = \frac{1}{\sigma_{SH}}\frac{\partial\sigma_{SH}}{\partial\bar{\varepsilon}_P}, \quad (8)$$

where the function φ is given by ratio $\varphi = \bar{\sigma} / \sigma_1$.

5. Strain based forming limit

The strain based FLD is defined by a plot of pairs ($\varepsilon_1^{limit}, \varepsilon_2^{limit}$). In general the equivalent limit strain $\bar{\varepsilon}^{limit}$ can be obtained by solving the instability criterion (8) with respect to the equivalent plastic strain. The expression of the equivalent limit strain corresponding Voce hardening law (sample) reads

$$\bar{\varepsilon}^{\,limit} = \frac{1}{n}\ln\left[\left(1-\frac{A}{B}\right)\left(1+n\left(\frac{1}{\varphi}\frac{\partial\bar{\sigma}}{\partial\sigma_i}A_{ij}^{Instability}\frac{\partial\bar{\sigma}}{\partial\sigma_j}\right)^{-1}\right)\right] \qquad (9)$$

Assuming proportional loading, the limit strains corresponding to the instability condition (7) can be determined from the associated flow rule as

$$\varepsilon_i^{\,limit} = \frac{\partial\bar{\sigma}}{\partial\sigma_i}\bar{\varepsilon}^{\,limit}, \qquad i=1,2. \qquad (10)$$

6. Stress based forming limit

One of the disadvantages of the conventional strain based FLD-s is the strain path dependence. Using the instability criterion (8), a simple approach for determining the stress based FLD can be given. Inserting the equivalent limit strain given with (9) in the strain hardening law yields (Voce law is employed as an example)

$$\bar{\sigma}^{\,limit} = \frac{Bn\left(\frac{1}{\varphi}\frac{\partial\bar{\sigma}}{\partial\sigma_i}A_{ij}^{Instability}\frac{\partial\bar{\sigma}}{\partial\sigma_j}\right)^{-1}}{1+n\left(\frac{1}{\varphi}\frac{\partial\bar{\sigma}}{\partial\sigma_i}A_{ij}^{Instability}\frac{\partial\bar{\sigma}}{\partial\sigma_j}\right)^{-1}} \qquad (11)$$

The limit stresses σ_1^{limit} and σ_2^{limit} can be determined in terms of equivalent limit stress $\bar{\sigma}^{limit}$ as

$$\sigma_1^{limit} = \frac{\bar{\sigma}^{limit}}{\varphi}, \qquad \sigma_2^{limit} = \rho\sigma_1^{limit} \qquad (12)$$

7. Experimental procedure

In the current study, the stainless steel AISI 304 sheet with 1 mm thickness is considered. The constitutive relations are defined on the basis of experimental results. The total length of the specimen was 150mm, the reduced section with gage length 100mm and width 12.5mm. The yield stresses σ_0, σ_{45}, σ_{90} and the Lankford coefficients R_0, R_{45}, R_{90} are determined from uniaxial tensile test of a specimen cut out at 0^0, 45^0 and 90^0 with the rolling direction. The strain hardening parameters are obtained from the tensile test of a specimen cut out at angle α with the rolling direction. The measured data are used for modeling empirical flow curves. The FLD is constructed by measuring major and minor strains just outside necks and fractures. The limit strains were determined from a circular grid path with a circle diameter of 3 mm. Due to limitations of the equipment, an in plane formability analysis was performed. The specimens with total length of 100 mm cut out at angle α with the rolling direction were evaluated. The width of the free section was varied from 12.5 to 50 mm. The free section length was varied from 30 to 50 mm. In order to achieve the exact plane strain conditions the specimen with maximal width and minimal length of the free section should be considered.

8. Numerical results

The strain and stress based FLD for stainless steel AISI 304 is depicted in Figures 1 and 2, respectively.

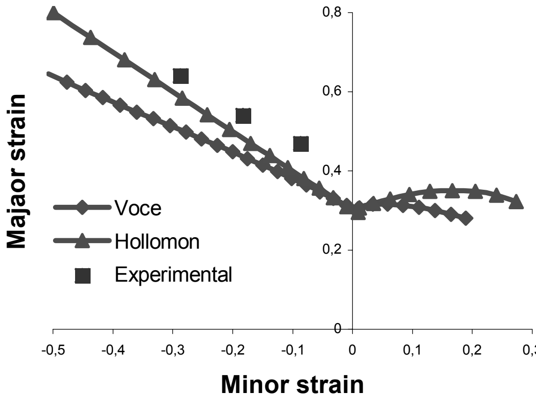

Fig. 1. Strain-based FLD of stainless steel AISI 304

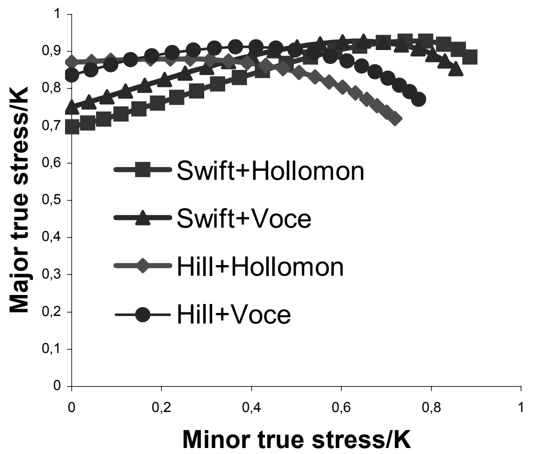

Fig. 2. Stress-based FLD of stainless steel AISI 304

In Figures 1-2 the Swift and Hill instability conditions are combined with the Hollomon and Voce strain-hardening rules.The forming limit curve corresponding to a recent plane stress yield criterion BBC2003 [20] is given in Figure 3. It can be seen from Figures 1-3 that the shape and position of the forming limit curves are sensitive to the yield criteria used. The forming limit curves corresponding BBC2003 and Barlat-Lian 1989 yield criteria are close to each other (Figure 3).

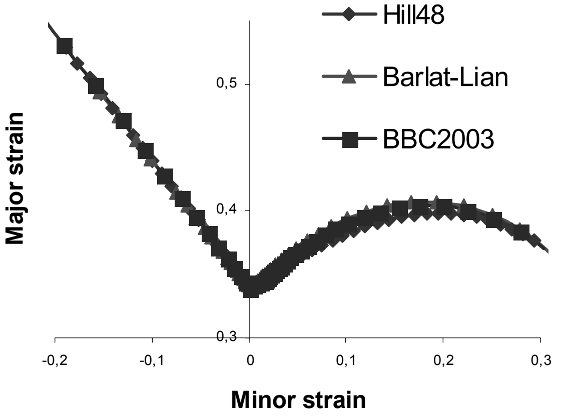

Fig. 3. Strain-based FLD of aluminum alloy sheet A6181-T4. Planar anisotropy

The material parameters are determined experimentally for the stainless steel AISI 304, but for the 6000 series aluminum alloy sheet are found in literature [20].

9. Conclusions

The instability criterion (8) has been derived by use of classical plasticity theory. Some new features can be outlined as:

- the dimensionless instability tensors are introduced,
- the non-homogeneous yield functions are covered, the non-homogeneity can be incorporated in function φ as

$$\varphi_{NonHom} = \varphi_{Hom} N_F, \quad N_F = \frac{\sigma_1 \dfrac{\partial \bar{\sigma}}{\partial \sigma_1} + \sigma_2 \dfrac{\partial \bar{\sigma}}{\partial \sigma_2}}{\bar{\sigma}} \quad (13)$$

As result no separate treatment is needed for each yield condition and strain hardening law. A number of yield criteria are examined: Banabic et al. [20], Barlat and Lian [19], Logan and Hoshford [18], (Hill [4, 16, 17]). The instability criterion was derived for homogeneous yield functions and was completed latter in order to cover Hill's 1993 criterion. A simple algorithm for determining FLD is based on use of classical instability conditions and abstraction. The cost of the simplicity is that lower accuracy can be achieved in comparison with the FLD obtained by use of more sophisticated instability conditions.

Acknowledgements

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