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New branched vibrating systems as result of synthesis of selected class of characteristics

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Analysis and modelling

ABSTRACT

Purpose: of this paper is the application of the synthesis method according to realization of mobility or immobility function into partial fraction when the level of the denominator of characteristic is higher than the level of its numerator.

Design/methodology/approach: was formulated and formalized by the problem of obtaining the "new" discrete vibrating mechanical systems with branched structures. The models of mechanical systems were represented by polar graphs. The reverse problem of dynamics of defined class of vibrating mechanical systems was also formalized and solved.

Findings: this study is that the same class of polar graph is a model of mechanical system with "old" and "new" branched structure. Obtaining structures of graphs as models of mechanical systems are a physical realization of dynamical characteristics, which may be considered as the immobility or mobility function.

Research limitations/implications: is that the linear vibrating discrete mechanical systems with branched structures were considered.

Practical implications: of this researches was that another approach was presented, which that means unclassical method of modeling of different structures of mechanical systems in form of polar graphs was used. The used method of synthesis and the obtained results can be of some value for designers of designated class of vibrating mechanical systems.

Originality/value: of this paper is that the only one polar graph is obtained as a model of not only one mechanical system. The results are obtained after distribution of two dynamical characteristics into partial fraction of finding structures of mechanical systems. This approach is other than those considered elsewhere.

Keywords: Applied mechanics; Distribution characteristic into partial fraction; Vibrating systems

1. Introduction

The synthesis consists in investigating the structure of a system with the discrete parameters and specific requirements set for the realization of the desired mechanical phenomena. In electrical and electronic systems the problems are known and solved (e.g. [2, 13, 18]). The first attempt at the solution to this problem, that means the synthesis of a continuous bar system and selected class of discrete mechanical systems concerning the frequency spectrum has been made in the Gliwice research centre[1)] in [3-8, 10-12, 23]. In these works the distribution of selected characteristics by using the continued fraction expansion method, recurrent cascade method and distribution of immobility function into partial fraction have been presented and used to

1) Other diverse problems have been modeled by different kind of graphs next they were examined and analyzed in the center (e.g. [14-17, 21, 22]).

continuous systems in a situation when the level of the numerator was higher than the level of denominator. However in this papers according to other cases the problem has been just signalized, similarly as in [9].

In the paper the realization of the characteristics by means of the distribution of immobility and mobility function into partial fraction has been presented and proposed by the synthesis method, when the level of the denominator was higher than the level of numerator.

2. Graph as a model of vibration systems

Using the system of significations proposed in [19,20], next used in [3-8, 10-12] and the model of discrete of mechanical vibration system represented by graph $X=\left({}_1X,\ {}_2X\right)$ (${}_1X$ finite set of vertices, ${}_2X$ - family of edges, being two-element subsets of vertices [1]), the idea of synthesis of the mobility function $V(r)$, that means mobility has been presented. This characteristic is in general the rational function of joint variable r as

$$V(r)=\frac{\sum_{i=0}^{k}c_i r^i}{\sum_{j=0}^{l}d_j r^j}, \tag{1}$$

or

$$V(r)=\frac{c_k\, r\left(r^2+r_2^2\right)\left(r^2+r_4^2\right)\cdots\left(r^2+r_{2n}^2\right)}{d_l\left(r^2+r_1^2\right)\left(r^2+r_3^2\right)\cdots\left(r^2+r_{2n+1}^2\right)}, \tag{2}$$

where: $r=j\omega$, $j=\sqrt{-1}$, $c_k,\ c_{k-1},\ \dots, c_0,\ d_l,\ d_{l-1},\ \dots,\ d_0$ are the real numbers, i,j,k,l are natural numbers, ω frequency, l-k=1

3. Essence of synthesis of vibrating discrete systems with branched structures

When $l-k=1$ and k is an even natural number, then the synthesis of transformed inverse function to the mobility $V(r)$, that means $U(r)=\frac{1}{V(r)}$, called immobility, and when the number of elements being synthesized is even, takes the form of

$$\frac{U(r)}{H}=\frac{\prod_{i=0}^{2n+1}\left(r^2+r_{2i+1}^2\right)}{\prod_{j=1}^{2n}\left(r^2+r_{2j}^2\right)}=$$
$$=\frac{d_l\left(r^2+r_1^2\right)\left(r^2+r_3^2\right)\cdots\left(r^2+r_{2n+1}^2\right)}{c_k\, r\left(r^2+r_2^2\right)\left(r^2+r_4^2\right)\cdots\left(r^2+r_{2n}^2\right)}, \tag{3}$$

where: $H=\frac{d_l}{c_k}$.

At first function (3) is distributed into partial fraction. Consequently

$$\frac{U(r)}{H}=b_\infty r+\frac{b_0}{r}+\sum_{k=1}^{n}\frac{B_{2k-1}}{r-j\,r_{2k}}+\sum_{k=1}^{n}\frac{B_{2k}}{r+j\,r_{2k}} \tag{4}$$

where: b_∞, b_0, B_1, B_2, ... , B_{2k-1}, B_{2k} - values of residuum pole suitably equal ∞, 0, $\boldsymbol{j}r_2$, $\boldsymbol{-j}r_2$, ... , $\boldsymbol{j}r_{2k}$, $\boldsymbol{-j}r_{2k}$.

The residua can be calculated as follow

$$\begin{cases} b_\infty=\lim\limits_{r\to\infty}\frac{U(r)}{r},\ b_0=\lim\limits_{r\to0} r\,U(r),\\ B_1=\lim\limits_{r\to \boldsymbol{j}r_2}(r-\boldsymbol{j}r_2)U(r),\ B_2=\lim\limits_{r\to -\boldsymbol{j}r_2}(r+\boldsymbol{j}r_2)U(r),\\ \vdots\\ B_{2k-1}=\lim\limits_{r\to \boldsymbol{j}r_{2k}}(r-\boldsymbol{j}r_{2k})U(r),\ B_{2k}=\lim\limits_{r\to -\boldsymbol{j}r_{2k}}(r+\boldsymbol{j}r_{2n})U(r). \end{cases} \tag{5}$$

Out of the equations (4) and (5) it is seen that B_1, B_2, ... , B_{2k-1}, B_{2k} are conjugate numbers but analysing the qualities of the real positive rational function it is obvious that all residua on the imaginary axis are real and positive i.e.

$$B_1=B_2=b_2,\ B_3=B_4=b_4,\ \dots,\ B_{2k-1}=B_{2k}=b_{2k}, \tag{6}$$

$$\begin{cases} \frac{B_1}{r-jr_2}+\frac{B_2}{r+jr_2}=\frac{2b_2 r}{r^2+r_2^2},\\ \frac{B_3}{r-jr_4}+\frac{B_4}{r+jr_4}=\frac{2b_4 r}{r^2+r_4^2},\\ \vdots\\ \frac{B_{2n-1}}{r-jr_{2k-1}}+\frac{B_{2k}}{r+jr_{2k}}=\frac{2b_{2k} r}{r^2+r_{2n}^2}. \end{cases} \tag{7}$$

Applying the results of (7), equation (4) can be written as

$$\frac{U(r)}{H}=b_\infty r+\frac{b_0}{r}+\sum_{k=1}^{n}\frac{2b_{2k} r}{r^2+r_{2k}^2}, \tag{8}$$

where: $b_\infty>0$, $b_0>0$, $b_{2k}>0$, $k=1,2,\dots,n$.

It is easily seen that separate components eq. (7) correspondents with following expressions

$$\begin{cases} b_\infty r = U_z^{(\infty)}(r) = m_z^{(\infty)} r, \text{ that means } b_\infty = J_z^{(\infty)}, \\ \dfrac{b_0}{r} = U_r^{(0)}(r) = \dfrac{c_r^{(0)}}{r}, \text{ that means } b_0 = c_r^{(0)}. \end{cases} \tag{9}$$

The immobilities which are the components of series $\frac{2b_{2k} r}{r^2 + r_{2k}^2}$ correspond the connections in series of the elastic and inertial elements which give

$$\frac{2b_{2k} r}{r^2 + r_{2k}^2} = U_{zr}^{(2k)}(r) = \frac{c_r^{(2k)} r}{r^2 + \dfrac{c_r^{(2k)}}{m_z^{(2k)}}}, \tag{10}$$

from this

$$c_r^{(i+1)} = c_r^{(2m)} = 2b_{2k}, \quad m_z^{(i)} = m_z^{(2k)} = \frac{2b_{2k}}{r_{2k}^2}; \; (k = 1, \ldots, n), \tag{11}$$

where: $m_z^{(\infty)}, m_z^{(i)}, m_z^{(2k)}$ - are polar moments of inertia of the cross-section of the synthesized bars, $c_r^{(0)}, c_r^{(i+1)}, c_r^{(2k)}$ - are constants of the springs.

Finally the formula (8) is taken in form

$$\frac{U(r)}{H} = m_z^{(\infty)} r + \frac{c_r^{(0)}}{r} + \sum_{k=1}^{n} \frac{1}{\dfrac{r}{c_r^{(2k)}} + \dfrac{1}{m_z^{(2k)} r}}. \tag{12}$$

Form (12) corresponds with immobility function of the polar graph $\underset{00}{X}$ [2,3,5,8] which is shown in Fig. 1.

The graph (Fig. 1) is a model of discrete system (Fig. 2).

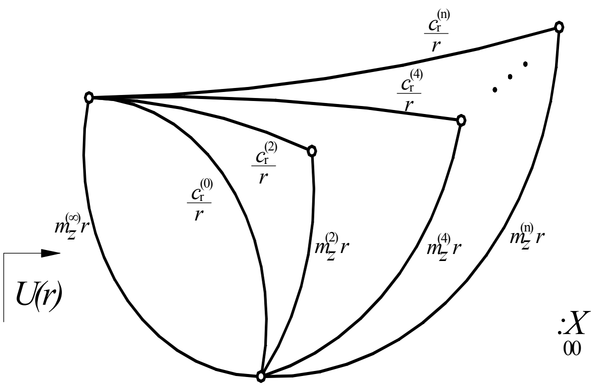


Fig. 1. Graphical illustration of equation (12)

The second case of synthesis of dynamical characteristic of searching structure of mechanical system is possible when $l - k = 1$ and k is an even natural number and immobility function $U(r)$, takes form

$$\frac{U(r)}{H} = \frac{\prod_{i=1}^{2n} \left(r^2 + r_{2i}^2\right)}{\prod_{j=0}^{2n+1} \left(r^2 + r_{2j+1}^2\right)} = \frac{d_l \, r \left(r^2 + r_2^2\right)\left(r^2 + r_4^2\right) \cdots \left(r^2 + r_{2n}^2\right)}{c_k \left(r^2 + r_1^2\right)\left(r^2 + r_3^2\right) \cdots \left(r^2 + r_{2n+1}^2\right)}. \tag{13}$$

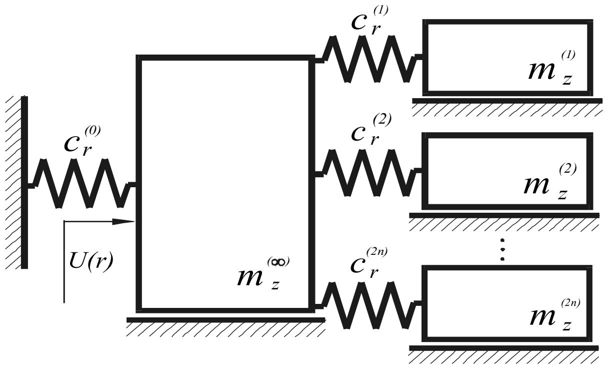


Fig. 2. The model of the mechanical system as an implementation of equation (12)

After distributing into a partial fraction immobility function (13), after operations (5) to (11), is given in form

$$\frac{U(r)}{H} = \sum_{k=1}^{n} \frac{1}{\dfrac{r}{c_r^{(2k-1)}} + \dfrac{1}{m_z^{(2k-1)} r}}. \tag{14}$$

The form (14) corresponds with immobility function of subgraph of polar graph $\underset{00}{X}$ (Fig. 1) without edges, with attributed immobilities: $U_r^{(0)}(r) = \frac{c_r^{(0)}}{r}$ and $U_z^{(\infty)}(r) = m_z^{(\infty)} r$. Mechanical model corresponds with that graph and sentence (14) has been shown in Fig. 3.

When $l - k = -1$, the method of synthesis of immobility function U(r), with changed requirements, is presented here, as well, assuming the odd natural elements. Then U(r), as third case of synthesis of the function of mechanical system, is given in the following form

$$\frac{U(r)}{H} = \frac{\prod_{i=1}^{2n} \left(r^2 + r_{2i}^2\right)}{\prod_{j=1}^{2n-1} \left(r^2 + r_{2j-1}^2\right)} = \frac{d_l \, r \left(r^2 + r_2^2\right)\left(r^2 + r_4^2\right) \cdots \left(r^2 + r_{2n}^2\right)}{c_k \left(r^2 + r_1^2\right)\left(r^2 + r_3^2\right) \cdots \left(r^2 + r_{2n-1}^2\right)}. \tag{15}$$

After transformations (5) to (11) immobility function (15) takes form

$$\frac{U(r)}{H} = m_z^{(\infty)} r + \sum_{k=1}^{n} \frac{1}{\dfrac{r}{c_r^{(2k)}} + \dfrac{1}{m_z^{(2k)} r}} . \qquad (16)$$

Equation (16) represents the immobility function of the dynamical structure in a form of the polar graph (Fig. 1), but without edges, with the attributed immobility $U_r^{(0)}(r) = \frac{c_r^{(0)}}{r}$, however the mechanical model has been shown in Fig. 4.

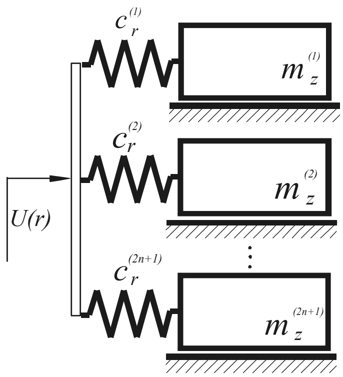


Fig. 3. The model of the mechanical system as an implementation of equation (14)

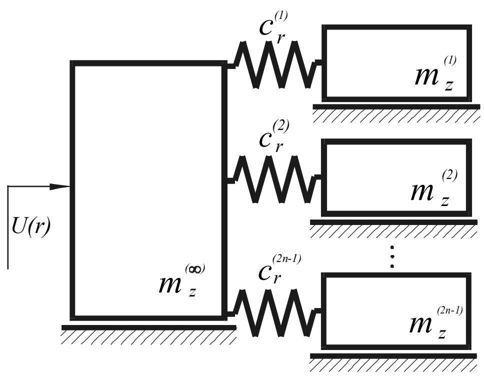


Fig. 4. The model of the mechanical system as an implementation of equation (16)

The synthesis of dynamical characteristic – immobility function $U(r)$, of a mechanical system is possible when k is a natural number, as well, and $l - k = -1$ and then immobility $U(r)$ is given in a form

$$\frac{U(r)}{H} = \frac{\prod_{i=1}^{2n-1}\left(r^2 + r_{2i-1}^2\right)}{\prod_{j=1}^{2n}\left(r^2 + r_{2j}^2\right)} = \frac{d_l \left(r^2 + r_2^2\right)\left(r^2 + r_4^2\right)\cdots\left(r^2 + r_{2n-1}^2\right)}{c_k\, r \left(r^2 + r_2^2\right)\left(r^2 + r_4^2\right)\cdots\left(r^2 + r_{2n}^2\right)} . \qquad (17)$$

Finally immobility function (17), after operations (5) to (11), takes form

$$\frac{U(r)}{H} = \frac{c_r^{(0)}}{r} + \sum_{k=1}^{n} \frac{1}{\dfrac{r}{c_r^{(2k)}} + \dfrac{1}{m_z^{(2k)} r}} . \qquad (18)$$

Form (18), which corresponds with mobility function (17) of the subgraph of the polar graph $\underset{00}{X}$ (Fig. 1) without edges, with attributed immobility $U_z^{(\infty)}(r) = m_z^{(\infty)} r$ and corresponds with the model of the mechanical system which has been shown in Fig. 5.

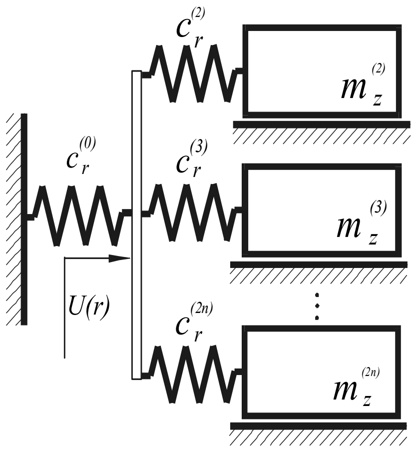


Fig. 5. The model of the mechanical system as an implementation of equation (18)

Further on the case in which the level of denominator is higher than the level of nominator of characteristic synthesis has been presented. This concerns also synthesis of mobility function $V(r)$.

Global case has been solved as a mobility, which is the next characteristic in the form

$$V(r) = \frac{c_k r^k + c_{k-2} r^{k-2} + \ldots + c_0}{d_{k-1} r^{k-1} + d_{k-3} r^{k-3} + \ldots + d_1 r} , \qquad (19)$$

where: k – is even natural number.

This kind of distribution is being made by the lowest cubes of variable r. In this case the numerator and the denominator have been divided into r in the highest potency

$$V(r)=\frac{c_k\frac{r^k}{r^k}+c_{k\text{-}2}\frac{r^{k-2}}{r^k}+\ldots+c_0\frac{1}{r^k}}{d_{k-1}\frac{r^{k-1}}{r^k}+d_{k\text{-}3}\frac{r^{k-3}}{r^k}+\ldots+d_1\frac{r}{r^k}} \tag{20}$$

Transforming equation (20) and introducing the new variable $r'=\frac{1}{r}$, we get

$$V(r')=\frac{c_k+c_{k\text{-}2}\frac{1}{r^2}+\ldots+c_0\frac{1}{r^k}}{d_{k-1}\frac{1}{r}+d_{k\text{-}3}\frac{1}{r^3}+\ldots+d_1\frac{1}{r^{k-1}}} \tag{21}$$

and finally

$$V(r')=\frac{c_k+c_{k\text{-}2}r'^2+\ldots+c_0r'^k}{d_{k-1}r'+d_{k\text{-}3}r'^3+\ldots+d_1r'^{k-1}}\,. \tag{22}$$

When in (22) the level of the denominator is lower than the level of the numerator, which means that the reverse argument and function have to be synthesized. Finally the mobility $V(r')$ as a function of argument r' is concerned, that means

$$V(r')=\frac{c_0r'^k+c_2r'^{k-2}+\ldots c_{k\text{-}2}r'^2+c_k}{d_1r'^{k-1}+d_3r'^{k-3}+\ldots+d_{k-1}r'} \tag{23}$$

or

$$\frac{V(r')}{H}=\frac{(r'^2+r_1^2)(r'^2+r_3^2)\ \ldots\ (r'^2+r_{2n+1}^2)}{r'(r'^2+r_2^2)(r'^2+r_4^2)\ \ldots\ (r'^2+r_{2n}^2)}, \tag{24}$$

where: $H=\frac{c_0}{d_1}$.

After transformations (5) to (11) the formula (24) is given in the form

$$\frac{V(r')}{H}=m_z'^{(\infty)}r'+\frac{c_r^{(0)}}{r'}+\sum_{m=1}^{n}\frac{c_r^{(2m)}r'}{r'^2+\frac{c_r^{(2m)}}{m_z^{(2m)}}}\,. \tag{25}$$

Form (25) corresponds with the mobility function of the polar graph $\underset{00}{X}$ (see Fig. 6). The mobility determined at the point indicated by the arrow is identical with (25). The graph (Fig. 6) is a model of a discrete system (Fig. 7).

For some edges it has to be qualify to dynamic characteristics according to order, which is a result of order of the characteristic implementation, the synthesized method of distribution the mobility $V(r)$ by partial the fraction method. The other cases of the synthesis function of an argument r and the same - r' have been presented in table 1.

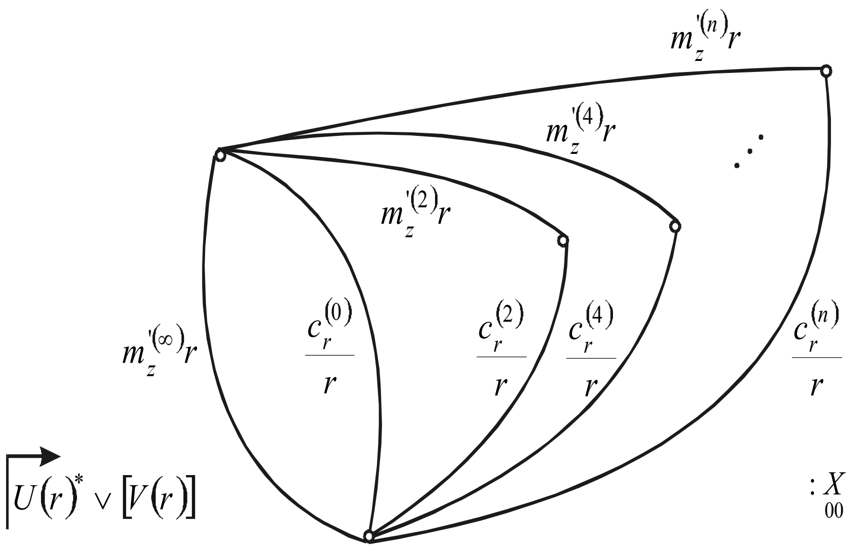
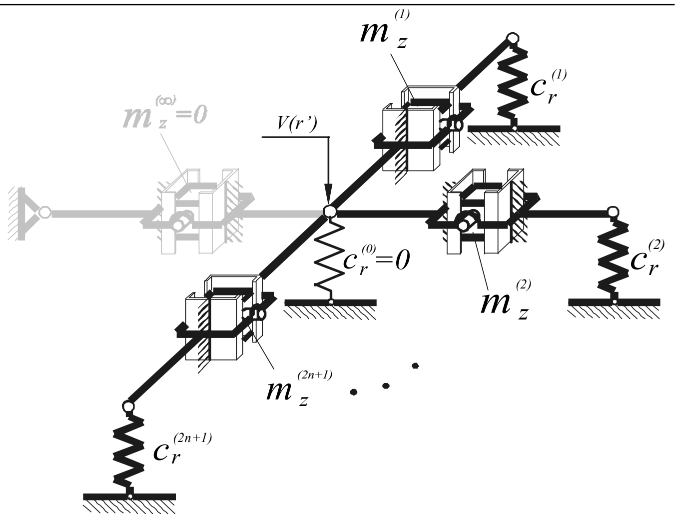
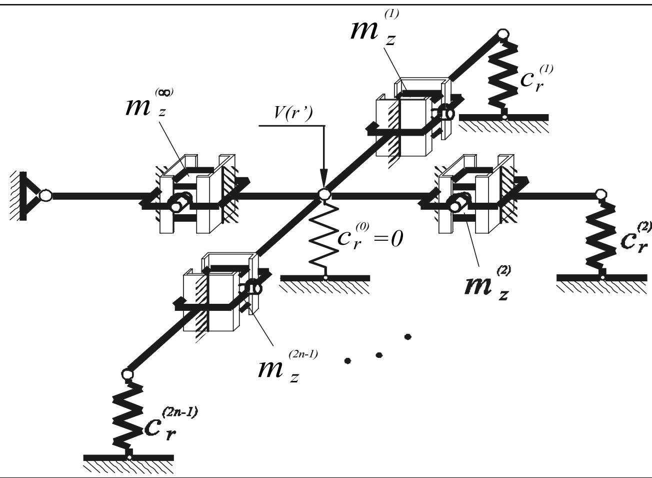
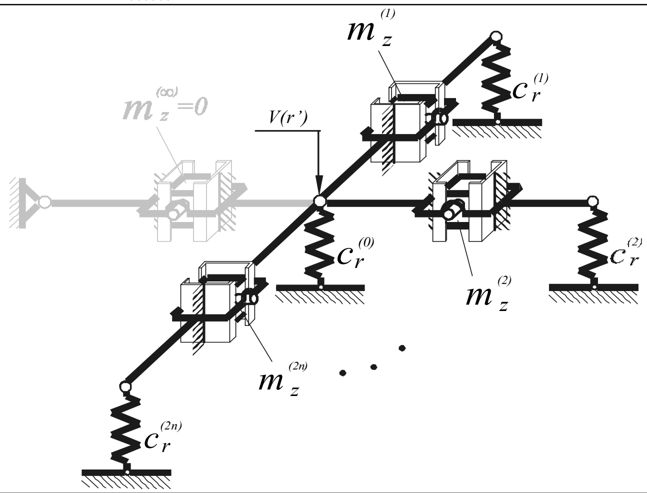


Fig. 6. Graphical illustration of equation (25)

Table 1.
The set of characteristics and their implementations

Characteristics	Discrete mechanical system as an implementation of characteristic
$\frac{V(r)}{H}=\frac{r(r^2+r_2^2)(r^2+r_4^2)\ \dots\ (r^2+r_{2n}^2)}{(r^2+r_1^2)(r^2+r_3^2)\ \dots\ (r^2+r_{2n+1}^2)}$	
$\frac{V(r)}{H}=\frac{r(r^2+r_2^2)(r^2+r_4^2)\ \dots\ (r^2+r_{2n-1}^2)}{(r^2+r_1^2)(r^2+r_3^2)\ \dots\ (r^2+r_{2n}^2)}$	
$\frac{V(r)}{H}=\frac{(r^2+r_1^2)(r^2+r_3^2)\ \dots\ (r^2+r_{2n-1}^2)}{r(r^2+r_2^2)(r^2+r_4^2)\ \dots\ (r^2+r_{2n}^2)}$	

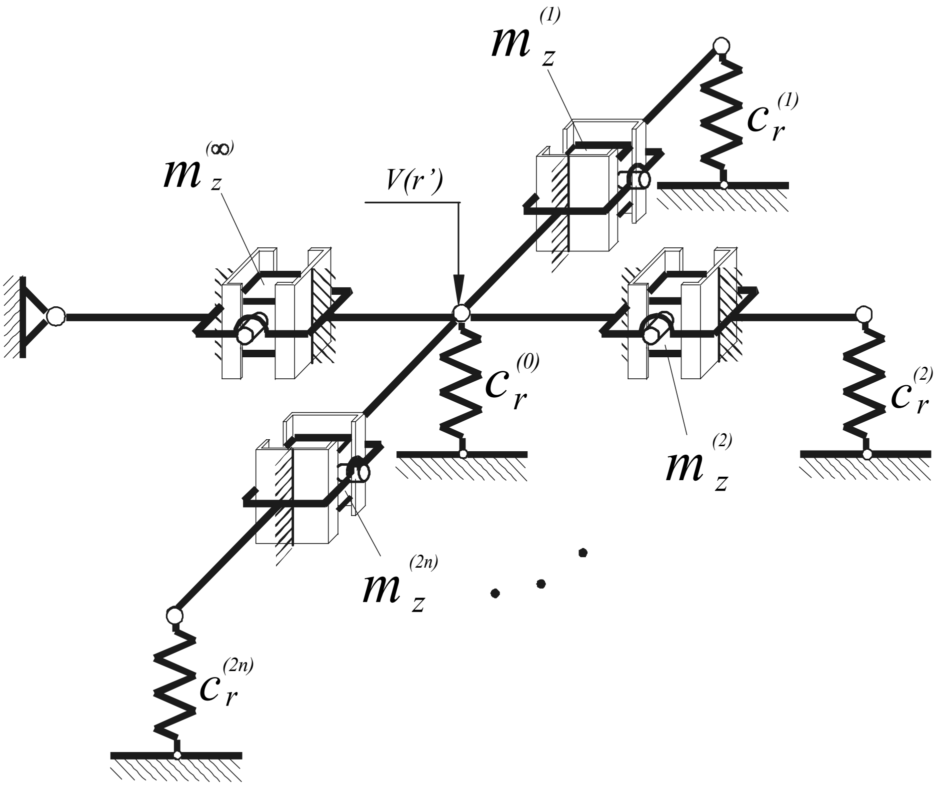


Fig. 7. The model of the mechanical system as an implementation of equation (25)

4. Last remark

Presented approach enables the expansion of synthesis of characteristics, that means mobility functions V(r) of discrete systems, in which a level of a denominator is lower than level of a numerator, towards new discrete vibration mechanical system with branched structures.

Acknowledgments

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