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Comparison of solutions obtained by exact and approximate methods for vibrating shafts

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Analysis and modelling

ABSTRACT

Purpose: The purpose of this paper is to compare the transients of characteristics obtained by the approximate and exact method and to answer to the question – if the method can be used to nominate the characteristics of mechatronic systems.

Design/methodology/approach: The main approach to the problem is to nominate the relevance or irrelevance between the characteristics obtained by different methods – especially the relevance of the pole values of characteristics. The main subject of the research is the continuous torsionally vibrating bar considered as a mechanical subsystem of the mechatronic system.

Findings: The values of zeroes and poles of continuous mechanical system depend on the boundary conditions. Foreseeing approximate solutions fulfill some conditions only, particularly for flexibly vibrating beams.

Research limitations/implications: The linear continuous torsionally vibrating bar is considered.

Practical implications: The main point is the analysis and the examination of torsionaly vibrating continuous mechatronic systems which characteristics can be nominated with approximate methods only.

Originality/value: The originality of the approach relies on the comparisation of the compatibility of the characteristics of the mechatronic and mechanical systems with demanded accuracy, nominated with the same approximate method.

Keywords: Applied mechanics; Galerkin's and exact methods; Mechatronic systems; Vibrating bar subsystems

1. Introduction

The mechatronic system[1], which has been clamped at one of its end, has been considered in the paper [3,11]. The mechanical part of considered mechatronic system is the continuous elastic shaft with full section, constant along the whole length *l*. The shaft is made of a material with mass density ρ and Kirchoff's modulus *G*. The electric part of the mechatronic one is an ideal piezotransducer ring is perfectly bonded at the shaft. The system was excited by the harmonic electrical voltage from the electric side which was applied to the converter clips. The next torsionaly vibrating mechatronic system clamped at of its ends was considered in the paper [4]. The main purpose of those papers was application of approximate method of solving the task of assignment the frequency-modal analysis and characteristics of mechatronic system.

[1] The problems concerned of piezoelectricity and electrostriction were presented for example in [5, 7-10, 12-14].

The synthesis of a continuous bar system and selected class of discrete mechanical systems concerning the frequency spectrum has been made in the Gliwice research centre in [1,2, 6]. Other diverse problems have been modeled by different kind of graphs next they were examined and analyzed in (e.g.[15]).

In the paper the transients of characteristics obtained by the approximate and exact method were compared.

2. Excitated torsionally vibration of the shaft

2.1. The dynamical flexibility of continuous system - solution with the exact method

The shaft with the constant cross section is considered, clamped on left end and free on the right one. The shaft is made of material with a transverse modulus G and mass density ρ. The shaft inertia moment - J, and its angle dislocation $\varphi(x,t)$ at the time moment t of the lining shaft section within the distance x from the beginning of the system. Moreover the shaft is excited by harmonical moment $M(t)$ acted on the free end.

The equation of motion of the shaft takes form

$$c_\varphi \frac{\partial^2 \varphi}{\partial x^2} = J \frac{\partial^2 \varphi}{\partial t^2}, \qquad (1)$$

where: $c_\varphi = \frac{GI_0}{l}$ - torsional rigidity of the shaft, I_0 -polar inertia moment of the shaft cross section, l- length of the shaft.

The boundary conditions on the shaft ends are following

$$\begin{cases} \varphi = 0 \text{ when } x = 0, \\ c_\varphi \frac{\partial \varphi}{\partial x} = M_0 \cos\omega t \quad \text{when } x = l. \end{cases} \qquad (2)$$

Dislocation is the harmonic function because the excitation is harmonic one, that is why it is

$$\varphi(x,t) = X(x)\cos\omega t . \qquad (3)$$

Substituting (3) to (1) is obtained

$$c_\varphi \frac{\partial^2 X}{\partial x^2} + J\omega^2 X = 0 . \qquad (4)$$

Afterwards it searches the general solution of expression (4) in form

$$X(x) = A\cos\lambda x + B\sin\lambda x . \qquad (5)$$

where: A and B are any real constants and

$$\lambda l = \omega \sqrt{\frac{I}{c_\varphi}} . \qquad (6)$$

The solution (5) fulfills the boundary conditions for:

$$\begin{cases} A = 0, \\ B = \frac{M_0}{c_\varphi \lambda \cos\lambda l}. \end{cases} \qquad (7)$$

It means that dislocation of section x of the shaft takes shape:

$$\varphi(x,t) = \frac{\sin\lambda x}{c_\varphi \lambda \cos\lambda l} M_0 \cos\omega t . \qquad (8)$$

On the base of expression (8) the dynamical flexibility is equal

$$Y_{xl} = \frac{\sin\lambda x}{c_\varphi \lambda \cos\lambda l} . \qquad (9)$$

To enable the further analysis of the gained process, the absolute value of dynamical flexibility was considered on the right end of the shaft, that means when $x=l$. Dependence (9) will be given as:

$$\left|Y_{ll}\right| = \left|\frac{1}{c_\varphi \lambda} \mathrm{tg}\lambda l\right| . \qquad (10)$$

2.2. The dynamical flexibility of continuous system - solution with the approximate method

According to the approximate - Galerkin's method, the final solution will be searched within the sum of own functions which will respond to the variables of the time and dislocation, which are strictly accepted and fulfill the boundary conditions in form

$$\begin{cases} \varphi(0,t) = 0, \; X(0)T(t) = 0, \\ c_\varphi \left.\frac{\partial \varphi}{\partial x}\right|_{x=l} = M, \; c_\varphi X'(l)T(t) = M. \end{cases} \qquad (11)$$

where: $M = M_0 \cos\omega t$.

Using the Galerkin's method the solution of the equation (1) is given n following form

$$\varphi(x,t) = \sum_{k=1}^{\infty} \varphi_k(x,t) = A_k \sum_{k=1}^{\infty} \sin\left[(2k-1)\frac{\pi}{2l}x\right] \cdot \cos\omega t . \qquad (12)$$

The solution of the differential equation (1) comes to fulfilling the appropriate derivatives of the above expression (11)

$$\begin{cases} \dfrac{\partial^2\varphi_k(x,t)}{\partial t^2} = -A_k\omega^2\sin\left[(2k-1)\dfrac{\pi}{2l}x\right]\cos\omega t, \\ \dfrac{\partial^2\varphi_k(x,t)}{\partial x^2} = -A\left[(2k-1)\dfrac{\pi}{2l}\right]^2\sin\left[(2k-1)\dfrac{\pi}{2l}x\right]\cos\omega t. \end{cases} \quad (13)$$

Putting the derivatives (13) to the equation of motion is obtained

$$-c_\varphi A_k\left[(2k-1)\frac{\pi}{2l}\right]^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t + \\ + JA_k\omega^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t = M_0\cos\omega t. \quad (14)$$

Making appropriate transformations, the amplitude value A_k of the vibrations takes form of

$$A_k = \frac{M_o}{J\omega^2 - c_\varphi\left[(2n-1)\dfrac{\pi}{2l}\right]^2} \quad (15)$$

Using the equation (14) and substituting it to (11) the dynamical flexibility equals

$$Y_{xl} = \sum_{k=1}^{\infty} Y_{xl}^{(k)} = \sum_{k=1}^{\infty}\frac{\sin\left[(2k-1)\dfrac{\pi}{2l}x\right]}{J\omega^2 - c_\varphi\left[(2k-1)\dfrac{\pi}{2l}\right]^2} \quad (16)$$

When k=1,2,3 the transient of expression (15) is shown in fig. 1.

3. Last remark

On the base of the obtained formulas and them transients, which were determined by the exact and approximate method, it is possible to make the analysis of the considered vibrating system by only Galerkin's method. In case of others of boundary conditions of mechanical or mechatronic systems and others kinds of their vibrations it is necessary to achieve of offered researches in this paper. The problems will be presented in future works.

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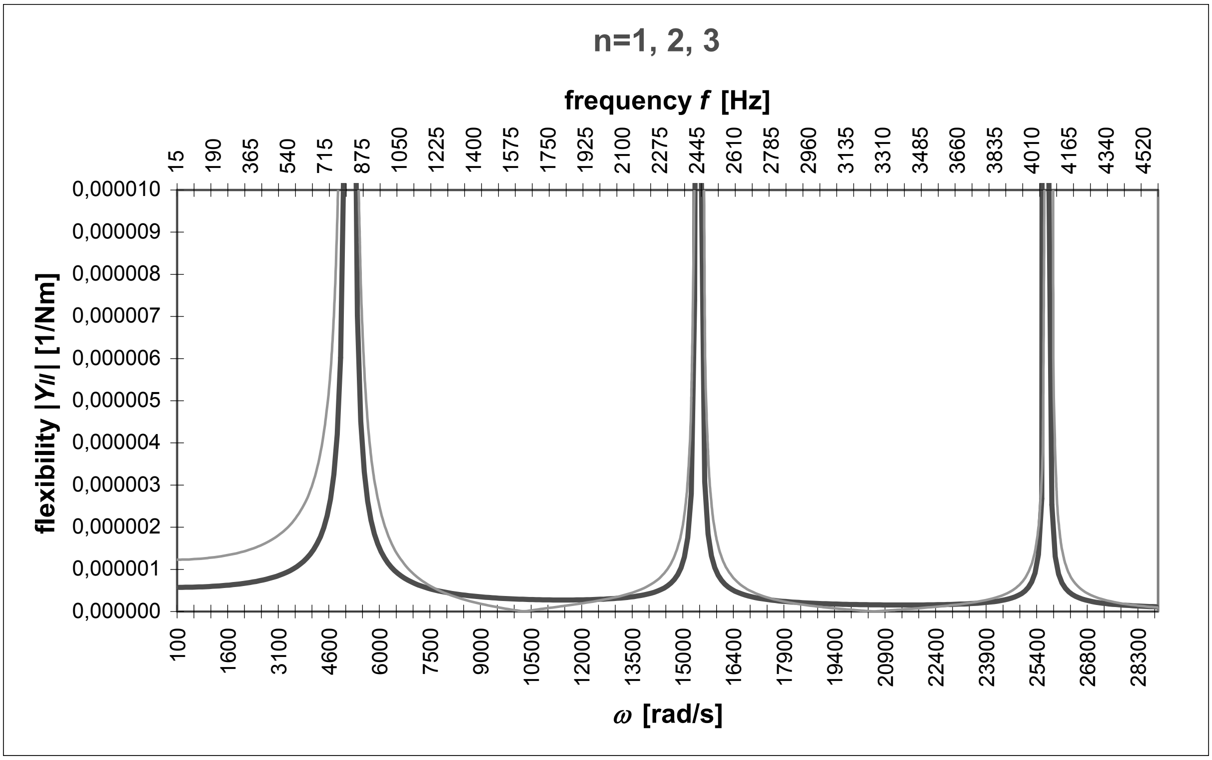

Fig. 1. Transient of the sum for k=1, 2, 3 mode vibration

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