

## Curved thin shell buckling behaviour

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### Analysis and modelling

#### ABSTRACT

**Purpose:** The aim of the paper is to evaluate buckling instabilities behaviour of long curved thin shell. Both initially straight and curved tubes are investigated with numerical and experimental assessment methods, in the context of NPP applications with an illustrative example for IRIS LWR integrated Steam Generator (SG) tubes.

**Design/methodology/approach:** In this study structural buckling response tube with combination effects of geometric imperfections as well as initially bent shape under external pressure load are investigated using a non linear finite element (MSC.MARC FEM code) formulation analysis. Moreover results are presented, extending the findings of previous research activity works, carried out at Pisa University, on thin walled metal specimen.

**Findings:** The experiments were conducted on Inconel 690 test specimen tube. The comparison between numerical and experimental results, for the same geometry and loading conditions, shows a good agreement between the elastic-plastic finite-element predictions and the experimental data.

**Research limitations/implications:** The presented research results may be considered preliminary in the sense that it would be important to enlarge the statistical base of the results themselves, even if they are yet certainly meaningful to highlight the real problem, considering the relatively large variability of the geometrical imperfections and bending instabilities also in high quality production tubes.

**Originality/value:** From the point of view of the practical implication, besides the addressed problem general interest in industrial plant technology, it is worth to stress that straight and curved axis tubes are foreseen specifically in innovative nuclear reactors SG design.

**Keywords:** Numerical techniques

### 1. Introduction

The stability of circular cylindrical shells under uniform lateral pressure has been widely investigated. The behaviour of cylindrical shells under external pressure is very sensitive to geometric imperfections. There have been many theoretical studies investigating the strength of cylinders with specific imperfection forms, and it is well established that axisymmetric imperfections cause the greatest reductions in strength (Koiter 1963; Yamaki 1984).

When thin shells were subjected to external pressure, the collapse was initiated by yielding, which was often the dominant factor, but the interaction with the instability is meaningful. In fact, the presence of imperfections reduces the load bearing capacity by an amount of engineering significance; so the classical elastic

solution, like Timoshenko and Gere approach [1], appear to be not completely adequate from a practical application point of view.

The major factors that affected the collapse pressure of pipes were the geometrical and material parameters like the diameter-to-thickness ratio  $D/t$ , initial shell curvature value, Young's modulus as well as yield stress in the circumferential direction. Moreover the interaction with initial imperfections in the form of ovality as for eccentricity is considered.

The current paper deals with buckling issue of a thin circular cylindrical shell in the dimension range of possible interest for nuclear power plant steam generators (SG) of type foreseen in the innovative IRIS Reactor [2]. The IRIS SGs were of the once through type, with the primary fluid outside the tubes.

In general buckling analysis is used to predict failure of long pipelines, subjected to external over-pressure.

In the considered application a selected configuration was used to perform the different analysis significant for the specified field of interests (numerical as were as experimental).

The present study might also serve as a base for other loading models, which include different boundary and geometrical conditions. Buckling phenomenon occurs when most of the strain energy, which is stored as membrane energy, can be converted into bending energy required by large deflections. Nonlinear buckling pressures can be evaluated using a nonlinear stress analysis by observing the first change in the slope (i.e., stiffness of the structure) in the load–deflection curve [3-4].

In nonlinear buckling analysis, however, an initial imperfection, either in terms of the geometry or load, is necessary to trigger the buckling phenomenon. The load carrying capacity of shell subjected to an external pressure could be characterised by the yield load, which was not considered as a failure pressure, because it was not equivalent to an immediate loss of stability.

In the tests it was shown that buckling may occur before to or beyond the characteristic cross section tube ovalization, depending on the local instability processes.

The main characteristic of a curved tube under external pressure is the distortion (ovalization) of its cross-section (Fig.1) because of the inward stress components  $\sigma_y$ .

The tubes, initially straight may be bent inducing a deformation characterized by initial longitudinal radius of curvature R significantly larger than the cross-sectional radius r.

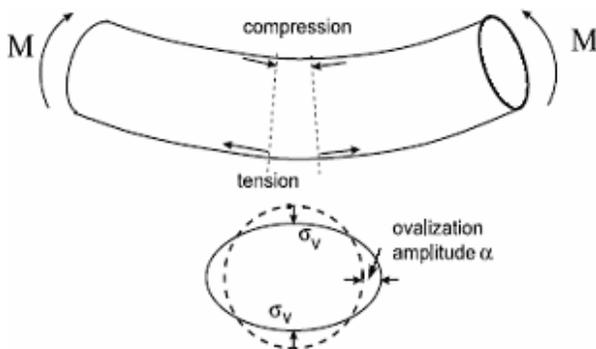


Fig. 1. Schematic representation of initially bent shell

A theoretical solution for straight circular cylindrical shells in a bending less state, before buckling, and subject to a homogeneous external pressure may be performed solving the elastic differential equation:

$$\frac{D}{h} \nabla^8 \omega + Ek_x^2 \frac{\partial^4 \omega}{\partial y^4} + 2Ek_x k_y \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + Ek_y^2 \frac{\partial^4 \omega}{\partial x^4} - \sigma_x^{(0)} \nabla^4 \left( \frac{\partial^2 \omega}{\partial x^2} \right) - 2\sigma_{xy}^{(0)} \nabla^4 \left( \frac{\partial^2 \omega}{\partial x \partial y} \right) - \sigma_y^{(0)} \nabla^4 \left( \frac{\partial^2 \omega}{\partial y^2} \right) = 0 \quad (1)$$

where  $\sigma_x^{(0)}$ ,  $\sigma_{xy}^{(0)}$ ,  $\sigma_y^{(0)}$  are the initial membrane stresses;  $\omega$  is the buckling deflection in the z direction; E is Young's modulus,  $\nu$  is Poisson's ratio; h and L are the thickness and the length of the shell;  $D = Eh^3/[12(1-\nu^2)]$  is the bending stiffness;  $k_x$ ,  $k_y$  are the curvatures in x, y direction [5].

For curved circular long shell under external pressure, on the contrary, an analytical formulation of buckling phenomenon is not well established or known.

## 2. Numerical analysis

The presence of imperfections, such as the ovality, the eccentricity makes elastic analysis inadequate to the purpose of determining the critical load, so a nonlinear analysis was required.

The present work is based on a nonlinear finite element (MSC.MARC FEM code) formulation. Using this numerical formulation, it is possible to investigate issues, which were not examined in previous works [6-7]. Tubes, in fact, may be affected by preloaded states due to initial curvature or other imperfections, such as ovality or eccentricity. Therefore, the effects of a possible pre-buckling state were investigated and taken into account through bending instabilities assuming a slight initial imperfection.

A nonlinear three-dimensional FE analysis is necessary to treat adequately bifurcation instability. This formulation has been ready adopted successfully for predicting the ultimate bearing capacity load for thin straight shell tubes under external pressure.

Some different elastic-plastic buckling analyses were set up to evaluate the collapse load and the influence of shell geometry.

The assumption of perfect plasticity permitted a better assessment of the effects of circumferential instability.

In such cases, the models were modeled by using solid 3-D finite elements; in particular 20-node brick elements for the cylindrical surface, because shell elements were not adequate to compute the contribution of radial compression.

The adopted mesh involved 60 elements over the circumferential section, and 5 elements through the thickness.

The considered tube was a long cylindrical shell of overall length L, uniform thickness t, cross section diameter D, radius of curvature R; the two ends were assumed as fixed hedges, under simultaneous action of uniform external pressure as sketched in Figure 2. The material was as previously said, Inconel 690, homogeneous and isotropic with Young's modulus  $E = 190$  GPa and Poisson's ratio  $\nu = 0.289$ .

It was assumed that all variables involved in the analysis were constant along the tube length.

This assumption is valid until a localized buckle occurs, for long tubes, in which the effect of end constraints can be neglected.

The overall length was large enough in respect to the tube diameter to allow to disregard the end conditions. Moreover, the residual stress due to the bending tube manufacturing was not taken into account in the present analysis.

The imperfections considered were of 20% of tube thickness for the eccentricity; 2.5% of nominal diameter for the ovality further the curved foresee geometry.

The boundary and load conditions were the same (real) ones, adopted for the experimental buckling tests on the Inconel 690 tube, previously mentioned.

The deepening of the initial buckle was either a result of a local buckling process, characterized by a local instability, and/or of progressive stress-deflection dependence. However, the reaching of a certain depth of the initial on going buckle can lead to the formation of another adjacent buckle, if the material local resistance is larger than the local deformation one.

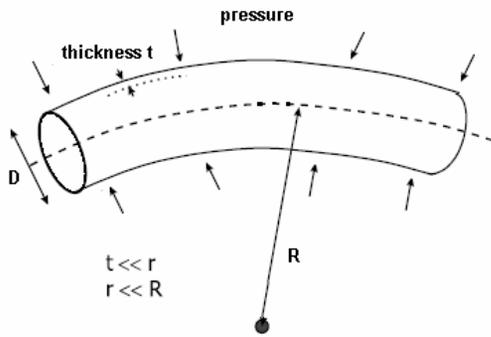


Fig. 2. Loading condition in a long bent shell

The influence of the mentioned imperfections appears clearly in comparing the buckling loads for perfect shell and imperfect one indicated in Table 1.

The number of waves around the circumference decreases as the length increases and takes the minimum value of two only for very long tubes ( $L/R \geq 50$ ).

The results, listed in the mentioned table, show the differences between the collapse loads for perfect shell and for imperfect ones. It is evident that the capacity of bearing load decreases in presence of important imperfections like eccentricity and the ovality.

Table 1. Buckling loads for perfect and imperfect curved shells

Buckling Load	Lateral Pressure (MPa)	Hydrostatic Pressure (MPa)
Perfect shell	35.53	35.70
Eccentricity	32.67	32.70
Ovality	33.52	33.57

The numerical results point out that collapse load level is strictly dependent from the initial imperfections, so the level of the first instability load initially falls with the increasing imperfection amplitudes.

In the following Fig. 3, there are shown some different views of the same deformed thin long curved shell shape in order to highlight the complexity to predict buckling phenomenon with a not simply geometry.

A further analysis has been performed with straight shells which have the same imperfections and material properties of previous curved shell ones.

The obtained numerical results seem to confirm the detrimental effects of imperfection, highlighting that in this case the ovality determines the largest reduction of structure bearing capacity as it seems clear observing the buckling pressure loads for perfect shell and imperfect one indicated in Table 2.

Table 2. Buckling loads for perfect and imperfect straight shells

Buckling Load	Lateral Pressure (MPa)	Hydrostatic Pressure (MPa)
Perfect shell	34.05	34.68
Eccentricity	33.62	34.01
Ovality	30.93	34.47

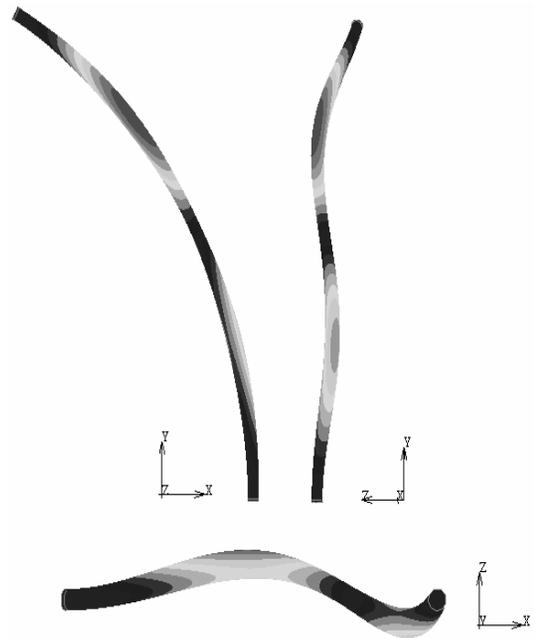


Fig. 3. FEM deformed shape model of long bent shell

### 3. Comparison between buckling pressure loads

The buckling response of a straight cylindrical shell has been experimentally determined and compared with the numerical analysis results.

Up today, the tests have been conducted only on straight Inconel 690 tube specimens with nominal diameters of 19 mm and nominal thickness of 1 mm and corresponding length of 828 mm.

Firstly, the diameter (D) has been measured at twelve equidistant points along the meridional length as well as along two equally spaced points on the same circumference.

Measured values of external diameter are in agreement with the nominal diameter for each test specimen

As second step, the specimen was confined between two rigid cylindrical sliding base supports, manufactured in the laboratory of Pisa University, which maintain the tube in the vertical position, so any rotations or preloaded states during assembling are prevented.

The required load and strain relationships recorded with strain gauge instrumentation and piezoelectric pressure transducer are used to determine the buckling pressure load.

These collapse loads are used to check the reliability of obtained numerical buckling ones.

Some recorded relations of pressure-strain circumferential in the upper part of diagrams and the axial in the bottom, for the mentioned Inconel 690 specimen are shown in Fig. 4.

In the above mentioned tests, the tubes were buckled to a final value of about 27 MPa pressure. In these graphs buckling phenomenon corresponds to the non-linear part observed.

A good agreement, instead, has been obtained between the predicted numerical and experimental buckling pressure values in

this test. A few discrepancies in the comparison values arise probably from the other imperfections, which could not be detected on the tube specimens.

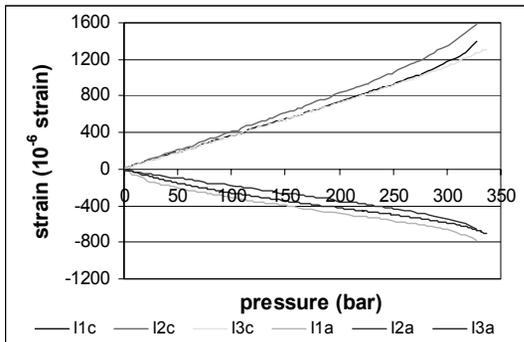


Fig. 4. Diagrams of strain versus pressure

The presence of the assumed geometrical imperfections on the shell produces a reduction of 10% on buckling pressure respect to that of perfect shell.

The tube specimens combine longitudinal with cross-sectional deformation (ovalization). The same behaviour and deformation mechanism has been thus recorded, in all the carried out test. Moreover the ovalization mechanism seems to result in loss of stiffness, referred to as ovalization instability [8].

a)



b)

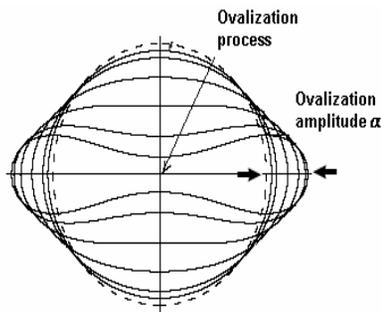


Fig. 5. Deformed collapsed straight tube (a) and ovalization process scheme (b)

Furthermore, the increased axial stress at the compression side due to ovalization may cause bifurcation instability buckling in a

form of longitudinal wavy-type “wrinkles” usually before a limit moment is reached.

Schematic picture of collapsed straight tube as well as ovalization process, are visible in Figs 5 (a) and (b). It is shown that buckling mode will always follow the lowest mode without additional constraints.

## 4. Conclusions

The present work has investigated the plastic collapse failure behaviour and related effect of curved shell as well as ovality and eccentricity.

On the basis of the overall obtained results it is worth to note that the tubes of an integrated PWR steam generator operate at a differential pressure of about 10 MPa, and that it is possible to show that the tube can bear fairly high external pressure levels.

The preliminary experimental results for the straight circular cylindrical specimen tubes are in good agreement with numerical ones.

Therefore they may be used to evaluate the reliability of the numerical approach and for the following evaluation of the long thin shells curved collapse pressure loads.

Moreover, it has been shown by means of the numerical set up analyses, that this method can accurately predict the buckling behaviour of long thin tubes, both in the linear field under external pressure and under the combined effects of material as well as geometrical non-linearity.

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