Modelling, synthesis, modification, sensitivity and analysis of mechanic and mechatronic systems

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ABSTRACT

Purpose: The main purpose of this thesis is the research of the authors scientific problematic which were presented during the AMME and CAMS’s conferences.

Design/methodology/approach: The main approach to the problem is to present the variety of the conferences and on this background – the abilities of the propositions for the potential participants. According to the author of this article these were the topics concerning - modelling, synthesis, design, examining the sensitivity, modifications of discrete, continuous, discrete continuous mechatronic systems and mechatronic active or passive and also taking into consideration the transportation.

Findings: The flexibility impact of the manipulators chains in fix position with the transportation was examined. The problem of the synthesis and examining the sensitivity of continuous and discrete continuous systems with the usage of method of examining this problem with discrete models was formulated and solved. The problem: of structural modification of different type of systems was formulated and solved; of active discrete structures. The methods of synthesis according to selected class of dynamical characteristic in order to generate the “new” structures – physical realizations in a shape of models of mechatronic systems were given.

Research limitations/implications: The various linear mechanical and mechatronic systems were modelled, synthesised, design and analysed.

Practical implications: Obtained results can be used as a field of improving and developing the modelling methods as far as design and examining the various class of vibrating mechanical and mechatronic systems are concerned. Mentioned results can be also useful for the engineers which are working with this or similar problems.

Originality/value: New modelling methods, synthesis, examination of sensitivity and modifications of selected class of mechatronic and mechatronic systems represented by graphs and structural numbers were presented.

Keywords: Applied mechanics; Robotics; Sensitivity; Synthesis; Modification;

1. Introduction

Developing the modelling method, synthesis, projecting, examining the sensitivity and analysis of discrete, discrete – continuous and discrete continuous mechanical and mechatronic systems analysis I had the opportunity to take part in the scientific conferences organized with participation of Prof., L. Dobrzanski. Presented paper concerns the review of the problems which was presented during these conferences and in the publications which were the result of mentioned conferences.
2. Robotics [1, 5]

The problem of modelling and investigation of manipulator of robot were considered in [1,5]. A model of the manipulator of robot as mechanical system consisting of two links, with continuously dispersed parameters. Next, a hypergraph of two-blocks has been built to represent the model. The loaded and oriented skeleton of hypergraph was used to determine a matrix of rigidities for the system.

In Fig. 1 the robot diagram is presented.

![Fig. 1. A scheme of the robot](image)

As the manipulator model a system of two bars positioned in any spatial configuration is considered. We assume the possible existence of some constraints, which influence the system nodes. These constraints may appear in the form of harmonic forces (Fig. 2a) and moments (Fig. 2b) during the realization of a technological process.

![Fig. 2. Harmonical forces (a), moments of harmonical forces (b) applied to the ends of manipulator links](image)

A completed graph of this hypergraph consists of edges, loaded with rigidities that describe connections between generalized dislocations and generalized forces working on ends of the manipulator (Fig. 3).

![Fig. 3. Geometrical representation of the lengthwisely-flexibly-torsionally vibrating bar model projection into a hypergraph of 13 vertices](image)

Examples of some amplitude characteristics of a manipulator links are shown in Fig. 4. Moreover examples of diagrams of trajectory and linear deflections in 3-D are shown in Fig. 5.

![Fig. 4. The rotation displacement in relation to axis: Xg](image)

![Fig. 5. Examples of linear deflections from the trajectory in 3-D](image)

3. Sensitivity [4, 10, 13]

The design mechanical systems with certain requirements was presented in [4, 10, 13]. Then across sensitivity of parameters,
understood as quantitative measure of system, making easy estimation of influence of changes of size received in result of synthesis of computational parameters on changes of exit – size, one can estimate area resonance and to qualify which for parameters has decisive part on given resonance area. Number these of parameters limits oneself to basic unit’s model of system that is to say to received coefficients of inertia and stiffness or pulses and stiffness. Then executing of correction values of parameters of system one can obtain reduction of level of amplitudes of twinkles to founded values. One can so ascertain, that analysis of sensitivity of modification of parameters or of structural system fundamentally has in view as greatest adaptation of dynamic property received model to accepted criterions. System has to trellis not only definite requirements, relating of realising required of dynamic occurrences, but also required by changes statically and dynamic.

The analytical form of logarithm function, sensitivity from dynamical flexibility $Y_\alpha$, according to the weight of sensitivity edge can be appointed in a following manner

$$S_{Y_\alpha}^{(r)} = a_r \left[ \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} - \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} \right] =$$

$$= \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} - \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}$$

In case of direct flexibility $Y_\alpha$ the formula number (1) can be wrote as

$$S_{Y_\alpha}^{(r)} = a_r \left[ \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} - \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} \right] = \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} - \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}.$$  

The polar graph of the as realisation of transforming characteristic [3,10] is shown in Fig. 6.

The sensitivity from flexibility $Y_\alpha$ is nominated as

$$S_{Y_\alpha}^{(r)} = a_r \left[ \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} - \frac{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}}{\operatorname{det}^2 A_{z^2} A_{h_{1,1}}} \right].$$  

The function of sensitivity according to (3) for the edge of graph, nominated as [4] it means the stiffness $c_1$ (Fig. 6) gets the shape of

$$S_{Y_\alpha}^{(r)} = \frac{r^4 m_1 m_2 c_1 + r^2 (m_1 c_1 + m_1 c_2 + m_2 c_2) + c_1^2 c_2 - c_1 c_2^2}{r^4 (m_1 m_2) + r^2 (m_1 c_1 + m_1 c_2 + m_2 c_2) + c_2 c_1}.$$  

Graphical representation of sensitivity function (4) is shown in Fig. 7.

![Graphical representation of sensitivity function (4)](image)

**4. Synthesis [14]**

The problem of synthesis of discrete physical, mechanical and, first of all, electrical and electronic systems is widely in scientific research. However, it is considerably to find examples of methods of synthesis of discrete-continuous mechanical systems. Authors of the paper approaching these problems emphasise that the synthesis of systems with the discrete-continuous distributions of parameters is only beginning to be developed, so its exact formulation and solution is still to deal with. The partial fraction method of the synthesis of the dynamic characteristics distribution makes it possible to obtain branched structures of the discrete-continuous system [14]. The investigation into the structures of branched systems was conducted by means of the software method for distributing the dynamic characteristics to the partial fraction based on graphs and structural numbers.
5. Modification [2, 3, 6]

Basing on the design as the result of synthesis of continuous free systems with definite frequency spectrum, with cascade structure, the dynamical flexibility of such systems has the alternating: pole, zero, pole, zero, etc. Nevertheless, the frequency with the zero value is always a pole. On the other hand, in the case of systems with cascade structure but fixed at one side, the dynamical flexibility has the alternating: zero, pole, zero, pole, etc. respectively. The frequency with the zero value is always a zero. In those paper [2, 3, 6] the exactly method was applied in order to structural modification of the vibrating mechanical system with cascade structure as the task of synthesis these systems. This is the method called cascade one represented by hypergraphs and structural numbers.

Exemplary courses of characteristics of free systems as well as of systems fixed at one side, composed of seven elements with constant cross-section are presented in Fig. 8 and 9.

\[
V(r) = \frac{r}{c_r^{(i)}} + \frac{1}{J_z^{(2)} r} + \frac{1}{r c_r^{(3)}} + \frac{1}{J_z^{(4)} r} + \frac{1}{r c_r^{(k-1)}} + \frac{1}{J_z^{(k)}}. \quad (5)
\]

The form (5) corresponds with mobility function of a polar graph \( X \) (see Fig. 10). The mobility determined at the point \( 0_0 \) indicated by the arrow is identical with (5). This graph is a model of discrete system but after transformation it is a continuous system.

![Fig. 10. Graphical illustration of equation (5)](image)

In order to attain these function, the third category graph \( \overline{X} = \{ X^{(i)}, X^{(i+1)} \} \) [4], as a model of longitudinally or torsionally transforming vibration system, is considered in Fig. 11 (hypergraph \( \overline{X}^{(i)} = \overline{X}^{(i)} \) is a model of the "n" basic element, whereas hypergraph \( \overline{X}^{(i+1)} \) is a model of the other part the system contains elements from \((i+1)\) to \(n\).

![Fig. 11. Illustration of transformation of the dynamical characteristic and of the graph of a free bar and a clamped bar [2,3,6]](image)

The construction scheme of the synthesized of transforming longitudinally or torsionally vibration system is obtained (see Fig. 12).
6. Active systems [19, 20]

The modelling, analysis and synthesis of active mechanical systems incorporating the method of polar graphs and their relationships with structural numbers were considered in [19,12]. The work was an attempt at the comprehensive development of a reverse task of the dynamics of active mechanical systems. The introduction of active elements into the elimination of vibration offers the possibility to overcome the limitations of the methods of passive elimination of vibration, such as, in particular, low efficiency in case of low-frequency vibration and the impossibility to reduce the vibration of specific parts of machinery. In those works, the method of polar graphs and their relationships with structural numbers were used in order to derive equations determining the values of amplitudes of forces generated by active elements. The use of such a method enables the automation of calculation during the determination of dynamic characteristics of a system and the algorithmisation of calculations.

7. New structures [7, 8, 9]

The purpose of the papers was the application of the synthesis method according to realization of mobility or immobility function into partial fraction when the level of the denominator of characteristic is higher than the level of its numerator [7, 8, 9].

The problem of obtaining the “new” discrete vibrating mechanical systems with cascade and branched structures was formulated and formalized. The models of mechanical systems were represented by polar graphs. The reverse problem of dynamics of defined class of vibrating mechanical systems was also formalized and solved.

Findings those study were that the same class of polar graph is a model of mechanical system with “old” and “new” branched and cascade structure. Obtaining structures of graphs as models of mechanical systems are a physical realization of dynamical characteristics, which may be considered as the immobility or mobility function. Practical implications of these researches were that another approach was presented, which that means unclassical method of modeling of different structures of mechanical systems in form of polar graphs was used. The used method of synthesis and the obtained results can be of some value for designers of designated class of vibrating mechanical systems. The results were obtained after distribution of two dynamical characteristics into partial fraction of finding structures of mechanical systems. This approach is other than those considered elsewhere.

Global case has been solved as a mobility, which is the next characteristic in the form

\[ V(r) = \frac{c_k r^k + c_{k-2} r^{k-2} + \ldots + c_0}{d_{k-1} r^{k-1} + d_{k-3} r^{k-3} + \ldots + d_1 r}, \]

where: \( k \) – is even natural number.

This kind of distribution is being made by the lowest cubes of variable \( r \). In this case the numerator and the denominator of sentence (6) have been divided into \( r \) in the highest potency

\[ V(r) = \frac{c_k + c_{k-2} r^{2} + \ldots + c_0 r^6}{d_{k-1} r^{4} + d_{k-3} r^{3} + \ldots + d_1 r^{2}}, \]

Transforming equation (7) and introducing the new variable \( r^* = \frac{1}{r} \), we get

\[ V(r^*) = \frac{c_k + c_{k-2} r^{2} + \ldots + c_0 r^6}{d_{k-1} r^{4} + d_{k-3} r^{3} + \ldots + d_1 r^{2}}. \]

When in (8) the level of the denominator is lower than the level of the numerator, which means that the reverse argument and function have to be synthesized. Finally the mobility \( V(r^*) \) as a function of argument \( r^* \) is concerned, that means

\[ V(r^*) = \frac{(r^{2} + r_{01}^2)(r^{2} + r_{02}^2) \ldots (r^{2} + r_{0n}^2)}{r^* (r^{2} + r_{01}^2)(r^{2} + r_{02}^2) \ldots (r^{2} + r_{0n}^2)}, \]

where: \( H = \frac{c_s}{d_i} \).

Finally the formula (9) is given in the form

\[ \frac{V(r^*)}{H} = m_i^2 \left( c_i^2 r^2 + \frac{c_i(2m_i)^2 r^2}{r^*} + \sum_{m=1}^{n} \frac{c_{2m}^2 (2m_i)^2 r^2}{m_i^2} \right). \]

Form (10) corresponds with the mobility function of the polar graph \( X \) (see Fig. 13). The mobility determined at the point indicated by the arrow is identical with (10). The graph (Fig. 13) is a model of a discrete system (Fig. 14).

![Fig. 12. Model of four-bar free system — torsionally vibrating, - - - longitudinally vibrating](image-url)
For some edges it has to be qualify to dynamic characteristics according to order, which is a result of order of the characteristic implementation, the synthesized method of distribution the mobility $V(r)$ by partial the fraction method. The other cases of the synthesis function of an argument $r$ and the same - $r'$ have been presented in Table 1.

Fig. 13. Graphical illustration of equation (10)

8. Vibration in transportation [16,17,18]

Main purpose of those papers [16,17,18] was analysis and modelling of mechanical systems in transportation. The contemporary technical problems are lashed with high work demands such as high speeds of mechanisms, using lower density materials, high precision of work, etc. The main objective of this thesis was the dynamical analysis with taking into consideration the interaction between main motion and local vibrations during the model is loaded by transverse forces. Equations of motion were derived by classical methods, the Lagrange equations with generalized coordinates and generalized velocities assumed as orthogonal projections of individual coordinates and velocities of the beam and manipulators to axes of the global inertial frame. Presented mathematical model of the transversally vibrating systems in planar transportation can be put to use to derivation of the dynamical flexibility of these systems, moreover those equations are the starting point to the analysis of complex systems. In particular we can use those equations to derivation of the substitute dynamical flexibility of multibody systems. There were considered mechanical systems vibrating transversally in terms of plane motion. Next problem of dynamical analysis is the analysis of systems in non-planar transportation and systems loaded by longitudinal forces. Results of that thesis can be put to use into all machines and mechanisms running in transportation such as wind power plants, high speed turbines, rotors, manipulators and in aerodynamics issues, etc. Some results ought to be modified and adopted to appropriate models. High requirements applying to parameters of work of machines and mechanisms are caused the new research and new ways of modelling and analyzing those systems. One of these ways are presented in this thesis. There was defined the transportation effect for models vibrating transversally.

9. Mechatronics [11,12,15]

It is generally well know engineering applications of standard materials for example aluminum, gold, steel etc. In recent years, piezoelectric materials being used to construct new mechatronic systems. Equivalent circuit of piezoelectric was delivered and it was modeled by graph methods [15]. This kind of modeling by graph methods it is author’s method. Piezoelectric layer was modeled by Mason’s equivalent circuit. Graph method for complex system was delivered. This method is base on the matrix method and the application of the graph aggregation. After examination of piezoelectric layer, the matrix of impedance was delivered and complex system was analysis by graph methods. This matrix was author’s idea for the impedance calculation of systems compound of many elements. Recurrent formula and the analysis of other kinds of smart materials are proposed in the future research. Piezoelectrics are widely applied as sensors and actuators. The advantages such as: little dimensions, simple structure, low noise factor at operating provide wide applications at vibration generating, damping, converging of mechanical energy into electrical energy, elements of precise positioning and many others. Graph method for piezoelectric layer for modeling complex system.

Application of approximate method was main purpose of work to solution task of assignment of frequency-modal analysis and characteristics of mechatronic system [11, 12]. The problem in the form of set of differential equation of motion and state equation of considered mechatronic model of object has been formulated and solved. Galerkin’s method to solving has been used. The considered torsionally vibrating mechanical system is a continuous bar of circular cross-section, clamped on one end. A ring transducer, which is the integral part of mechatronic system, exterted by harmonic voltage excitation is assumed to be perfectly bonded to the bar surface. Parameters of the transducer have important influence of values of natural frequencies and on form of characteristics of considered mechatronic system.
Table 1.
The set of characteristics and their implementations

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Discrete mechanical system as an implementation of characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(r) = \frac{r(r^2 + r_1^2)(r^2 + r_2^2) \ldots (r^2 + r_n^2)}{(r^2 + r_1^2)(r^2 + r_2^2) \ldots (r^2 + r_{2n}^2)}$</td>
<td>![Discrete mechanical system diagram]</td>
</tr>
</tbody>
</table>
The mechanical part of considered mechatronic systems is the continuous elastic shaft with full section, constant along the whole length \( l \). The shaft is made of a material with mass density \( \rho \) and Kirchhoff’s modulus \( G \).

Transient of sum of \( n=1, 2, 3 \) mode vibration is shown in Fig. 17. For example the influence of thickness of piezoelectric on characteristics of mechatronic system are shown in Fig. 18.

### 10. Last remark

Presented problematic is not more than the small part of the eighth year authors works and their cooperation with the AMME and CAM’S conferences.

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Fig. 18. Influence of thickness of piezoelectric $h_p$ in [m] on characteristics of mechatronic system

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