



## Multi-scale modelling of surface topography in single-point diamond turning

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co-operating with

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### Analysis and modelling

#### ABSTRACT

**Purpose:** A multi-scale model is proposed to explain the effect of material induced vibration and the quantitative relation between cutting force and the surface quality from dislocations, grain orientations, cutting tools, machine tools used in the simulation of the nano-3D surface topography in single-point diamond turning.

**Design/methodology/approach:** The model-based simulation system composes of several model elements which include a microplasticity model, a dynamic model and an enhanced surface topography model.

**Findings:** This research is the first attempt in which the microplasticity theory, theory of system dynamics and machining theory are integrated to address the materials problems encountered in ultra-precision machining. Indeed, this is a new attempt to link up the microplasticity theory to macro-mechanisms in metal cutting. The successful development of the model-based system allows the prediction of the magnitude and the effect of periodic fluctuation of micro-cutting force and its effect on the nano-surface generation in ultra-precision diamond turning of crystalline materials. It also helps to explain quantitatively the additional roughness caused by the variation of the crystallographic properties of the workpiece, and leads us to a better understanding of the further improvement of the performance of ultra-precision machines.

**Research limitations/implications:** The multi-scale model brings together knowledge from various disciplines to link up physical phenomenon occurring at different length scales to explain successfully the surface generation in single-point diamond turning of crystalline materials.

**Originality/value:** This model offers a new direction of research in ultra-precision machining.

**Keywords:** Numerical techniques; Mmulti-scale model; Mmesoplasticity; Ddynamic; Ssurface topography; Single point diamond turning

### 1. Introduction

Ultra-precision machining technologies include advanced design of machining facilities, precision control system, nano-metrology, mechanics of cutting, vibration as well as materials

sciences. The theory of ultra-precision machining has been studied from the perspectives of various disciplines from continuum mechanics, materials science, to computer modeling. However, there have been relatively few studies employing the multi-scale modeling. Ultra-precision machining differs from conventional machining as it deals with physical phenomenon

from the micro-scale of dislocations to the macro-scale of a cutting tool. Most researches of ultra-precision machining would focus on a particular aspects of the technology, and seldom link up the different length scales which all affect the quality of machined surface at the micro and nano levels.

The process of surface generation has attracted a lot of research interest. However, most of the work to date is based on empirical studies. Relatively little quantitative work has been reported. Although some attempts have been made in the development of machining models to simulate surface topography of a workpiece, most of them focused on the synthesis of surface topography from the data derived from interferometry or Scanning Electron Microscopy (SEM). Few deterministic models have been found to simulate the generation of surface topography based on machine kinematics, material science and cutting theories. The influences of crystallographic orientation of the work materials, and its effect on the materials induced vibration and hence on the surface generation have been overlooked in most of the current models.

To predict for the material induced vibration and its effect on surface generation, a multi-scale model-based simulation system from microscopic prediction of the shear angle to the simulation of tool loci was successfully established based on the theories of microplasticity, cutting dynamics, and system simulation. It allows the prediction of the magnitude and the effect of material induced vibration on the surface generation. This is essential for better understanding of the performance of ultra-precision machines.

Ultra-precision diamond turning is an expensive process. Nowadays, the achievement of a super mirror finish in many industrial applications still depends much on the experience and skills of a machine operator through an expensive trial and error approach when new materials or new machine tools are used. The

development of the surface topography model together with the multi-scale model-based simulation system contributes to the identification of the optimum cutting conditions for expensive diamond turning without the need for costly trial and error cutting tests. This pioneer work breaks down the traditional discipline barriers in successfully modeling the nano-surface topography from an understanding of the interplay between physical phenomenon at different length scales.

## 2. Developed of multi-scale model-based simulation system

As shown in Fig. 1, the multi-scale model-based simulation system is basically composed of several model elements which include a microplasticity model, a dynamic model and an enhanced surface topography model. The microplasticity model is used for the prediction of the periodic fluctuation of micro-cutting forces due to the changing crystallographic orientation of materials being cut. Apart from the fine vibration between the tool and the workpiece due to the machining environment, such a periodic variation of micro-cutting forces also induces additional tool-work vibration on the cutting system which is referred in the present study as materials induced vibration [1]. In the model-based simulation system [2], a dynamic model is deployed to determine the materials induced vibration caused by the periodic variation of the micro-cutting forces. The influence of this vibration on the surface roughness of the workpiece is predicted by an enhanced surface topography model which takes into account process parameters, tool geometry and resultant vibration between the tool and the workpiece.

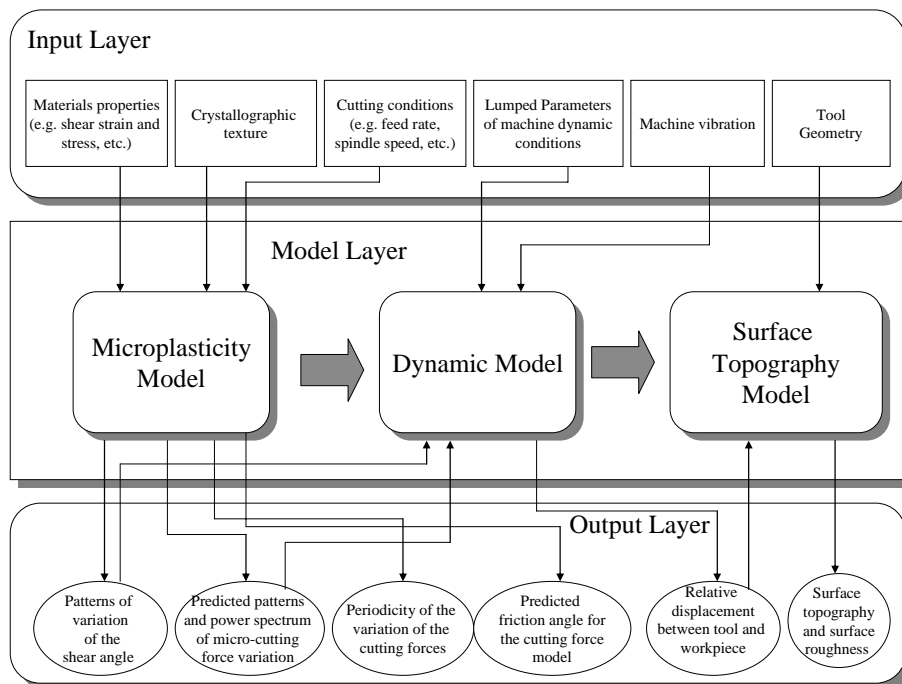


Fig. 1. The architecture of the multi-scale model-based simulation system

## 2.1. Prediction of periodic fluctuation of micro-cutting forces

In machining process, the shear angle affects the cutting forces and hence the surface roughness of the workpiece. There are various classical shear angle theories to correlate the relationship between the magnitude of the shear angle, cutting tool geometry, and the friction angles, the notable ones being the Merchant Theory and the slip line field. However, the magnitude of the shear angle could not be determined directly and has to be deduced from the measurement of the chip thickness obtained from cutting experiments. In the present study, a microplasticity model [3, 4] is used for the prediction of the periodic fluctuation of micro-cutting forces due to the changing crystallographic orientation of materials being cut.

Sato et al [5] attempted to use the continuum yield theory to analyse the shear stress and shear angle with material anisotropy. However, his attempt has been deemed to be unsuccessful since the value of the shear angle was in the reverse phase to shear stress. From the extensive experiments done by the research team (To, JMPT, Scripta, etc), there clearly exists a strong relationship between the crystallographic texture or orientation and the surface roughness obtained in diamond turning (Fig. 2). Based on the crystal plasticity, the shear angle can be inferred from the value of the Taylor factor for a given crystallographic orientation. The shear angle is predicted to occur at one which the minimum value of the effective Taylor factor determined from the load instability criterion is a minimum and the number of dislocation slip systems is also smallest [6]. A comparison between the predicted and the experimental shear angle is listed in Table 1. There is a good correlation between the predicted value and the observed one from the direct measurement.

Table 1.  
The measured and predicted shear angle for the different cutting direction on (230) plane

Cutting plane	Cutting direction	Measured shear angle	Predicted shear angle
(230)	[001]	38	40
(230)	[ $\bar{3}$ 24]	33	37

The shear angle thus calculated can be incorporated into various cutting mechanics model to predict the corresponding changes in the cutting force. During diamond turning operation, the crystallographic orientation of the workpiece material change with the cutting directions and this bring about the periodic fluctuations in the cutting forces which depends on the symmetry of the crystallography of the substrate materials and the spindle speed. The effect of the cutting force variation can be measured by a force transducer and analyzed by power spectrum.

Fig. 2 shows the predicted shear angle variation, the corresponding variation of the cutting force in diamond turning aluminium single crystals with (001) and (111) planes. It is found that the shear angle varies with the crystallographic orientation of the material being cut. There seems to exist a fundamental cyclic frequency of variation of the cutting force for each workpiece revolution which is determined to be four and three or six for (001) and (111) crystals respectively. These relate closely to the

crystallographic orientation of the crystals being investigated. The predicted and the measured spectral plots for the variation of cutting forces in diamond turning of (001) and (111) crystals are also shown. The predicted spectra are found to agree well with the measured spectra.

When comparing (001) and (111) single crystals, distinctive patterns of frequency distributions are observed for different crystals. Both the predicted and the measured spectra exhibit a dominant frequency component ( $f_2$ ) which can be correlated to the crystallographic orientation of the work material being cut. As shown in Fig. 2, the captioned component appears at almost the exact fundamental cyclic frequency for (001) crystal. For (111) single crystal, the measured dominant frequency component (300 Hz) appears almost double that of the predicted dominant frequency component (150 Hz) and is almost identical to its first harmonic (297 Hz). The cyclic cutting forces are shown to be made up of two patterns hereby referred to as the major pattern, with a higher amplitude, and a minor pattern, with a lower amplitude. The presence of six peaks is well predicted except that there is a discrepancy in the amplitude of the peak. This suggests that the (111) single crystal might possess two fundamental cyclic frequencies which are three for the major pattern of the cutting force variation as well as six for both of the major and the minor patterns of the cutting force variation.

The overall results support the argument that the variation of the cutting forces is related closely to the crystallographic orientation of the crystals being investigated. The microplasticity model is proven to be helpful in explaining the variation of micro-cutting forces in diamond turning crystalline materials. The main features of the cutting forces patterns are well predicted and confirmed by the cutting tests. There is a good agreement between the experimental findings and the predicted results.

## 2.2. Characterization of dynamic cutting system

The influence of vibration on the surface roughness machined with a single point tool has been previously studied by a number of researchers [7 - 9]. Mitsui [10] concluded that the machined surface roughness in the infeed cutting direction is more dominant than it is in the cutting direction, as the relative displacement of the tool in the cutting direction does not engage in the surface generation [11]. As a result, only the relative displacement between the workpiece and the tool in the infeed direction is considered and the cutting system is modelled with one degree of freedom in the consideration of the cutting tool and the workpiece.

As shown in Fig. 3, the complexity of the machine structure is represented in terms of a lumped-parameter for the purpose of the analytical study. The lumped parameters in such a representation are the equivalent mass, the rigidity and the damping for one of the modes of oscillation of the machine structure. The tool and work systems are represented by springs and dashpots connected in parallel. The tool and the workpiece are acted upon by the periodic fluctuation of thrust force  $F_z(t)$  as determined by the microplasticity model proposed by the project team [2]. The parameters of the dynamic cutting system can be estimated by the covariance equivalent ARMA (2,1) model [12-14].

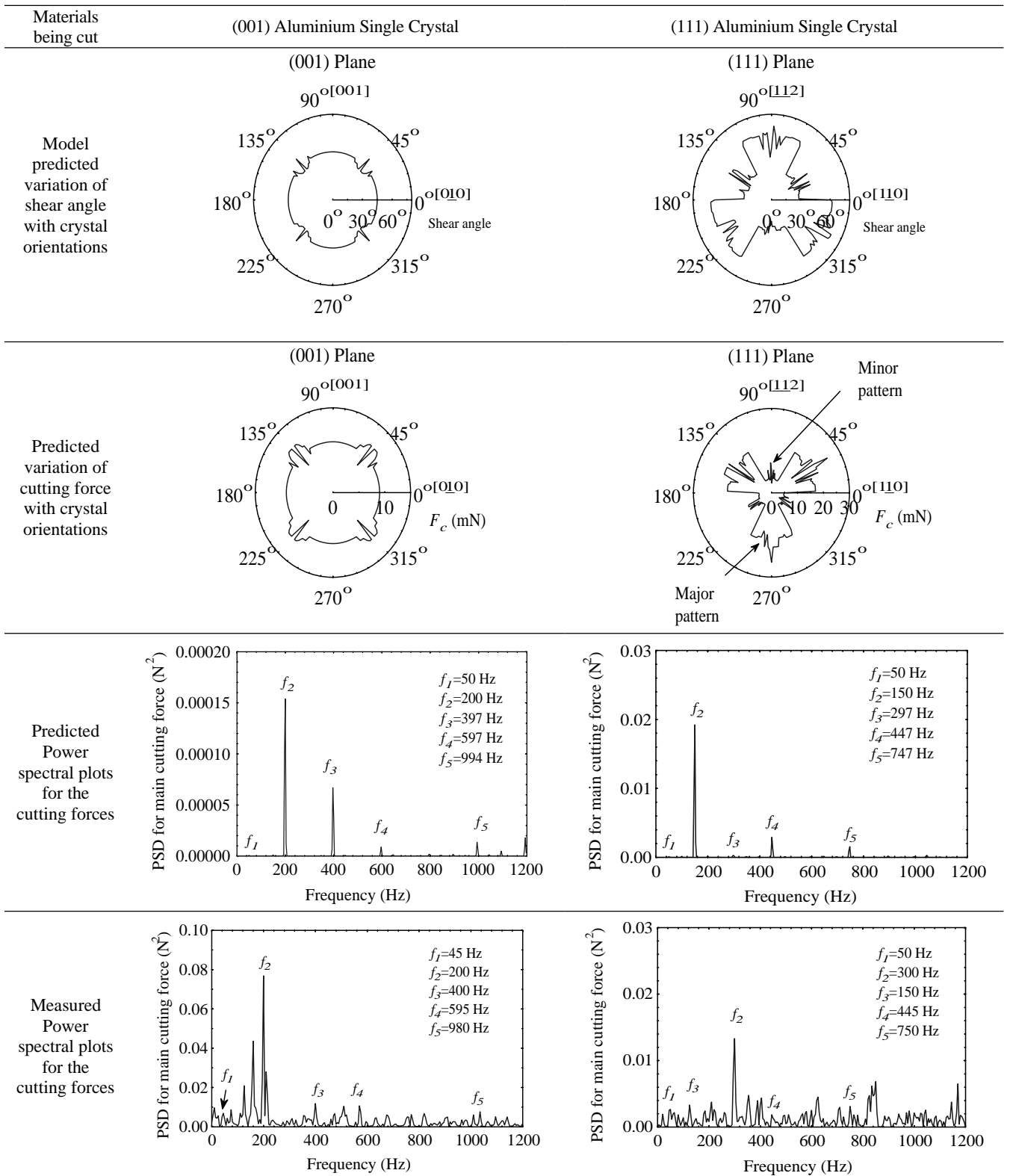


Fig. 2. A comparison of the variation of shear angle, cutting forces, predicted and measured of the cutting force in diamond turning aluminium single crystal with different crystallographic orientations

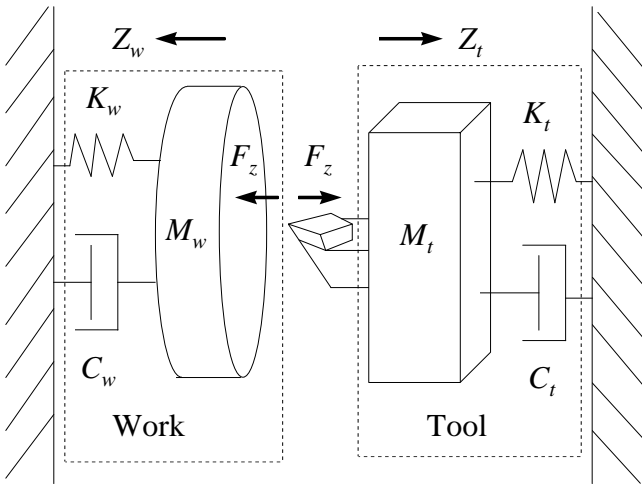


Fig. 3. A lumped-parameter representation of the system

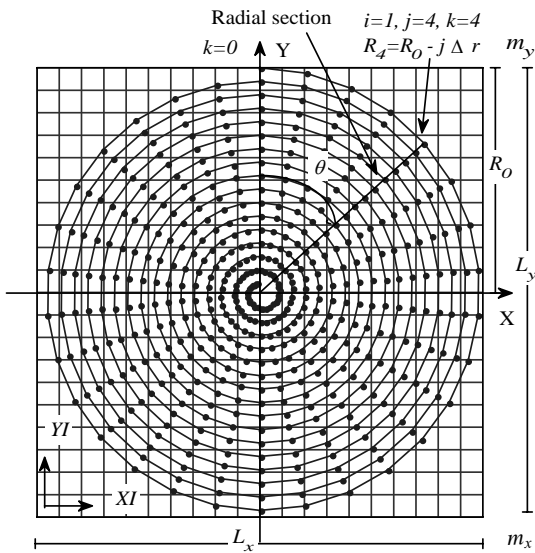


Fig. 4. Tool locus and linear mapping of surface dynamic cutting system data on a cross lattice

### 2.3. Generation of the three-dimensional nano-surface

Ultra-precision diamond turning is a complex metal removal process. The achievement of a super mirror finish still depends much on the experience and skills of the machine operators through expensive trial and error approach when new materials or new machine tools are used. In the modelling of surface topography, the real trick to the model is its new approach to the deterministic modelling and simulation of the 3D surface topography of a diamond-turned surface [15-17]. It uses the surface roughness profiles predicted at a finite number of equally spaced radial sections of the workpiece to construct the surface topography of a diamond turned surface. The roughness data are determined from the surface roughness profiles predicted at a finite number of equally spaced radial sections on the workpiece as shown in Fig. 4. The surface roughness profiles are

predicted based on a 2D surface topography model [15]. These surface roughness data from different radial sections of the surface are mapped on the surface elements of a cross lattice. The surface elements are used to build the mesh and the parametric surfaces [16] which are best fitted to the surface topography data. The contour levels of the parametric surfaces are proportional to the surface height.

A simulated 3D surface topography of the diamond turned surface is produced by adding the tool geometry component to the model as shown in Fig. 5. The model is found to give an estimate of the surface roughness value  $R_a$  of 92.6 nm which is closest to measured value of 94.4 nm. The experimental results agree well with the predicted results. Fig. 6 shows a virtual surface topography of a diamond turned workpiece. The use of virtual surface representation allows us to visualise and examine in detail the various features of the surface topography.

As the effect of material anisotropy caused by the changing crystallographic orientation of the work material is taken into account, an additional tool-work displacement is introduced into the cutting system. Since the force function  $F_z(t)$  varies with the crystallographic orientation of the workpiece, the patterns of the variation of cutting force is not a simple harmonic function but can be treated as an arbitrary function which varies with the crystallographic orientation of the work materials. The resultant tool-work displacement varies with the crystallographic orientation of the work material being cut. The surface topography model is modified to accomplish the additional displacement due to the changes induced by the crystallography of the material being cut.

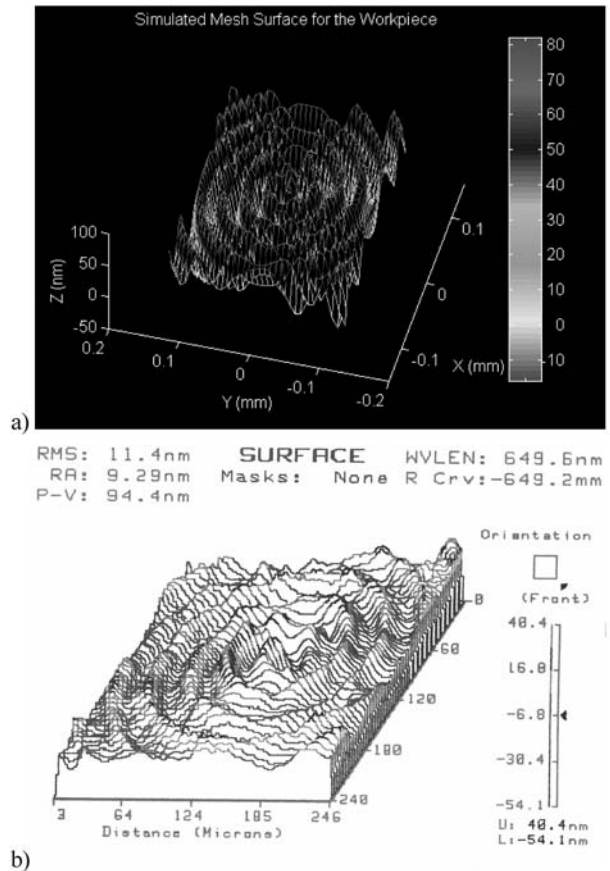


Fig. 5. a) Simulated and b) measured 3-D surface topographies

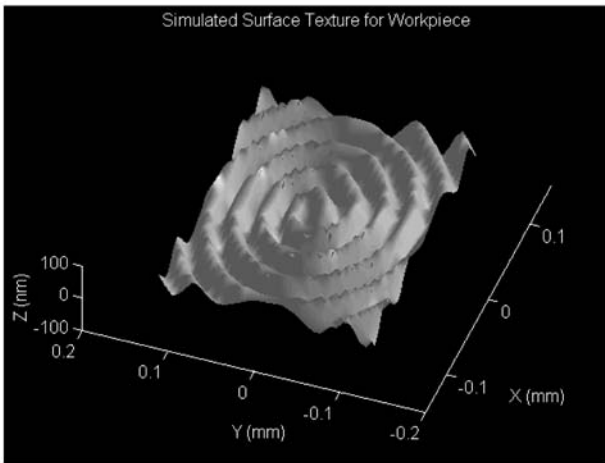


Fig. 6. Virtual surface topography of the turned workpiece

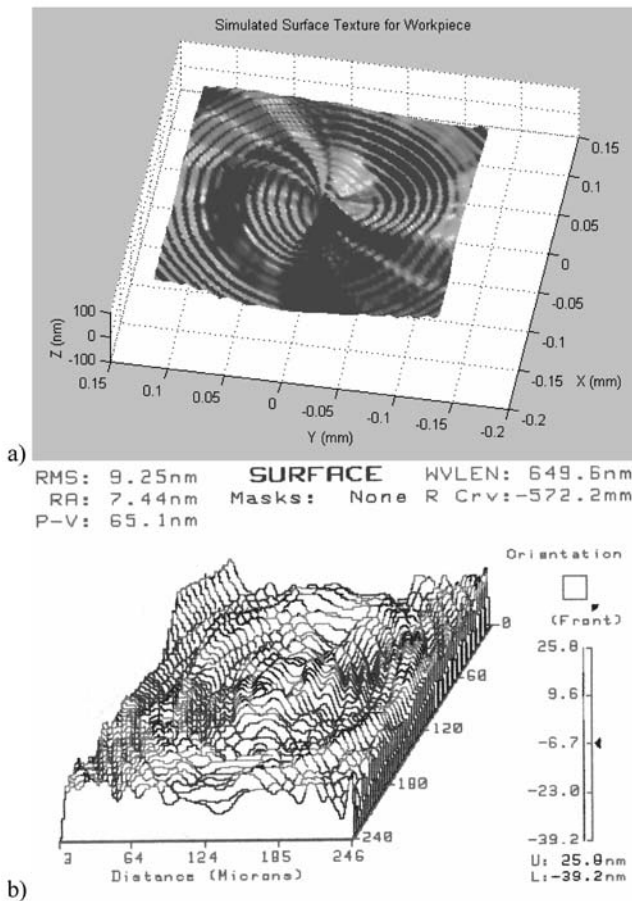


Fig. 7. a) Simulated and b) measured surface topographies for face cutting of aluminium single crystal with (110) plane (combined effects)

Figs 7 and 8 show the simulated and the measured surface topographies for the diamond turned (110) plane and (111) plane, respectively. The simulated surface topographies are found to be

similar to those obtained from the WYKO interferometric microscope for the work materials being investigated. The topographies exhibit combined features of surface modulation caused by the materials induced vibration and the relative tool-work vibration. Most of the surface features found in the measured patterns are reflected in the simulated patterns. Table 2 tabulates the predicted and the measured arithmetic roughness  $R_a$  values of the machined surfaces. The predicted results found to agree well with the measured results. As a whole, the model based simulation system is demonstrated to be helpful in explaining the additional roughness due to the variation of crystallographic orientation of the workpiece. There is a good agreement between the experimental findings and the simulation results.

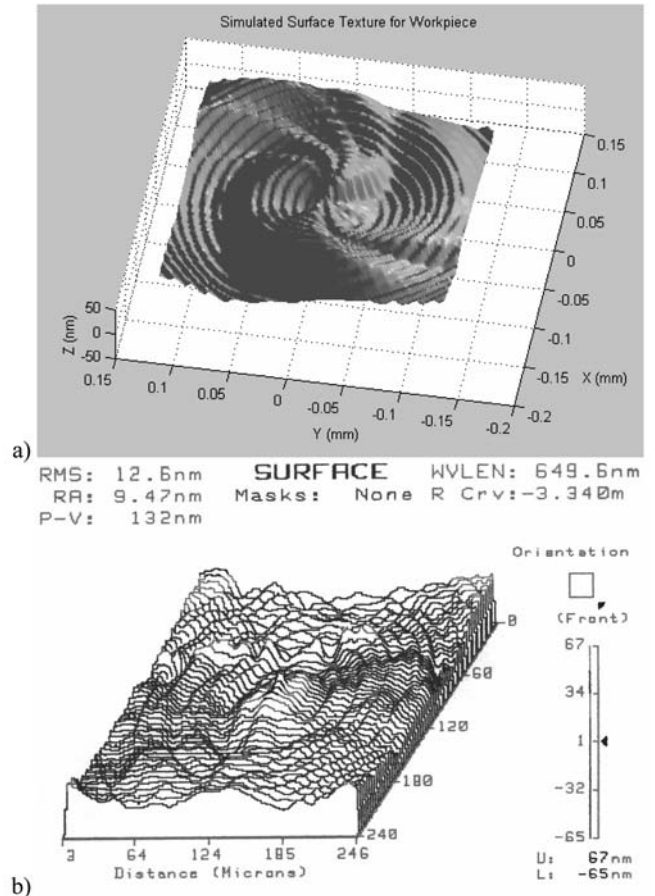


Fig. 8. (a) Simulated and (b) measured surface topographies for face cutting of aluminium single crystal with (111) plane (combined effects)

Table 2. A comparison between the predicted and the measured arithmetic roughness

Specimen no.	Mean Arithmetic roughness $\bar{R}_a$ (nm)	
	Predicted	Measured
(110) Aluminium Single Crystal	13.38	17.18
(111) Aluminium Single Crystal	15.28	19.99

### 3. Discussions and conclusions

This research presents a multi-scale model that is proposed to explain the effect of material induced vibration and the quantitative relation between cutting force and the surface quality from dislocations, grain orientations, cutting tools, machine tools used in the simulation of the nano-3D surface topology in single-point diamond turning. Hence, a model-based simulation system for determining quantitatively the variation of local surface roughness due to materials induced vibration in diamond turning is discussed. The system is based on several model elements which include a microplasticity model, a dynamic model and an enhanced surface topography model. The microplasticity model is used for predicting the variation of micro-cutting forces caused by the changing crystallographic orientation of the workpiece materials during cutting. A model-based simulation system has been built for assessing the vibration induced by the variation of micro-cutting forces. The surface roughness and the topography of the machined surface are predicted by an enhanced surface topography model. A software package was developed to implement the simulation system, and the performance of the system has been evaluated through a series of cutting experiments. Experimental results indicate that the variation of the cutting forces and the surface roughness are related closely to the crystallographic orientation of the crystals being cut. Overall, the simulation results are found to agree well with the experimental ones.

This research is the first attempt in which the microplasticity theory, theory of system dynamics and machining theory are integrated to address the materials problems encountered in ultra-precision machining. Indeed, this is a new attempt to link up the microplasticity theory to macro-mechanisms in metal cutting. The successful development of the model-based system allows the prediction of the magnitude and the effect of periodic fluctuation of micro-cutting force and its effect on the nano-surface generation in ultra-precision diamond turning of crystalline materials. It also helps to explain quantitatively the additional roughness caused by the variation of the crystallographic properties of the workpiece, and leads us to a better understanding of the further improvement of the performance of ultra-precision machines.

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