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Complex mathematical model and optimisation of vibration volumetric treatment for surfaces of machine parts

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Analysis and modelling

ABSTRACT

Purpose: The objective of this work is to develop the principles of the theoretical investigation of vibration volumetric treatment for the surfaces of articles manufactured by friable dry medium and to create the method for choice and optimization of machine parameters.

Design/methodology/approach: Theoretical investigations are based on using the analytic methods of theory of non-linear oscillations and asymptotic methods of solving non-linear differential equations for describing dynamic processes of volumetric vibration treatment. Experimental investigations of dynamic processes that take place in the vibromachines were carried out with using the computer and vibration measuring system.

Findings: Complex non-linear mathematic model of the system "vibromachine – treating medium" is created. Due to their non-linearity, models of such kind are adequate to real situation of vibrotreatment, they can also be applied in investigation of dynamics of the vibromachine and its medium.

Research limitations/implications: Processes of vibration volumetric treatment need additional investigations.

Practical implications: The obtained complex mathematical model of the system "vibromachine – trading medium" enables us to analytically determine the influence of its parameters upon the intensity of articles treatment in it according to the chosen criterion; thus it saves us from carrying out time taking cumbersome experiments both for designing a vibromachine and for selecting modes of treating articles in it.

Originality/value: The new approach to the investigations of volumetric vibration treatment by means non-linear mathematic model of the system "vibromachine – treating medium" have been shown.

Keywords: Mathematical modeling; Vibration treatment; Friable medium; Technological modes; Vibromachine

1. Introduction

It is known that physico-mechanical parameters of machine parts and constructions surface quality are the prevalent factor both in forming stress and structural state and in increasing durability and ensuring reliability of the articles manufactured. They also determine wear resistance, strength, corrosive resistance and other qualities of machine parts and constructions. The problem of the technology of surface treatment, in particular finishing and forming necessary surface engineering of machine parts, occupies an especial place in the general set of technological tasks concerning the improvement of performance characteristics of machine parts and units.

Application of strengthening finishing operations by means of surface plastic deformation (SPD) treatment is an efficient and perspective method of finishing in the improvement of performance characteristics of the articles manufactured. As to the efficiency of application, vibratory methods of strengthening finishing treatment deserve attention among the group of SPD methods. It is expediently divided indicated methods into the methods of vibration treatment by loose solid balls (treating medium) and the methods of vibratory-centrifugal treatment (hardening) by special tools on the basis of machine parts design and mass, types and designs of vibromachines, treated materials and tools using for treatment. Works written by researchers of various scientific schools are dedicated investigations of these methods. Works of the group of scientists from National University "Lvivska Polytechnika" are among them.

2. Statement of the problem

Technological process of forming necessary physico-mechanical parameters of surface quality by methods of vibration treatment has many parameters. It is realized by vibromachines, in which dynamics of moving its working tool and the motion of treating medium (loose abrasive or treating bodies) is useful. Modeling of the process is extremely needed for its optimization and selecting modes of treating articles.

3. Techniques of modelling

Description (studying) of the motion of friable medium in the container of the vibromachine, of the motion of device or mechanism during volumetric vibration treatment, vibrotransportation of friable medium, their vibroseparation, their milling and compacting is a complicated problem of dynamics. It is not solved fully to this time. However, solving the problems by creating the models of the motion of treating medium with the nearest approach to actual vibration processes is an important task which takes place in the case of the elaboration and the application in industry of the new technologies of treating machine parts and constructions of vibromachines, to take into account vibration technologies being widespread. Creation and investigation of the friable medium motion model for the vibration volumetric treatment, which should be adequate to real situation of vibrotreatment, enables us to determine the influence of the parameters (for different their combinations with each other) of the vibromachines, the devices, the mechanisms, the treating medium upon the character of their motion and the intensity of articles treatment. This enables us to cut down expenses of materials, energy and time during designing vibromachines and technological processes, their optimization and automation.

Widespread application of volumetric treatment calls for a new approach to calculating and designing vibromachines (unbalanced drives, elastic elements, container cases, and other dynamically loaded parts), investigation of the processes which take place in the loaded container of the vibromachine while it moves as well as the influence of its parameters upon the process of treating, creation of the methods of computer designing of machines in general, and therefore thorough mathematical description of dynamic processes which take place in vibromachines during their work. Dynamic processes which take place in such complicated systems were described mainly in linear formulation that usually unadequately describes real physical phenomena of the process. The necessities of practice call for prognostication of treatment results depending on the form of the parts treated, their structures, physico-mechanical properties kind of friable treating medium as well as vibromachine characteristics taking into account the non-linear forces which take place in the case of their interaction. Solution of such complex problems which take into account the influence of the drive upon dynamics of different kinds of the medium and the body treated enables us to prognosticate the intensivity of interaction between them (process of removing metal layer or superficial deformation of the parts treated), to properly choose geometrical and kinematic parameters of the vibromachine, and finally to lower material – and energy imputs as well as to increase the intensity of the treatment of articles.

It is suggested to solve the problems of this type through the elaboration and investigation of complex non-linear mathematic models of the "vibromachine - treating medium" - system by means of creating interconnected models of the motion of working tool of machine and friable medium; and on the basis of this dynamic phenomena of the system should be investigated in order to determine the influence of machine parameters and the medium upon factors of intensivity of article treatment. The models are derived with taking into account unification and adequacy. The former (unification) enables us to apply them for designing and operating many types of machines provided they are parameterized; and this leads to time and material imputs reduction. The latter, because of the complication of the dynamics of "vibromachine - treating medium" - system, is possible only when the models obtained are non-linear.

By an example of vibromachine with two independently driven unbalance vibrational exciters, the complex model of the vibromachine and its medium is shown below; graphic correlations between the amplitude (as the main factor of article treating intensity in vibromachine) of the oscillation of the container and parameters of the machine are obtained; the parameters which according to the chosen criterion provide the maximum intensivity of treating are calculated.

Vibromachine with two independently driven unbalance vibrational exciters are of several peculiarities which advantage them over other vibromachines. These advantages are the following: a) generation of oscillations of various forms (because of the two unbalance vibrational exciters), b) the possibility of the separation of the articles from the treating medium by means of redistribution of the oscillation amplitude of the working container on its plane during the treatment of articles or at the completion of treatment process due to the two independently driven vibrational exciters, c) relative simplicity of their designs and relatively high reliability of their parts in operation, d) their universality for modelling: the model includes, for example, the values of the masses of both unbalanced masses arbitrarily placed in the vibromachine; taking zero - value for one of the unbalanced masses enables us to employ this model for single - balance vibromachines with the unbalanced mass arbitrary placed with respect to its working container. In its turn, by means of the model derived, this enables us to investigate dynamic processes in vibromachines of other types; e) the possibility of automation of the process of volumetric vibrotreatment using the software elaborated on the basis of adequate mathematical models obtained as well as using results of investigating dynamic processes in vibromachines.

For investigation, the diagram of volumetric treatment vibromachine with two independently driven vibrational exciters [1] is shown in Fig 1.

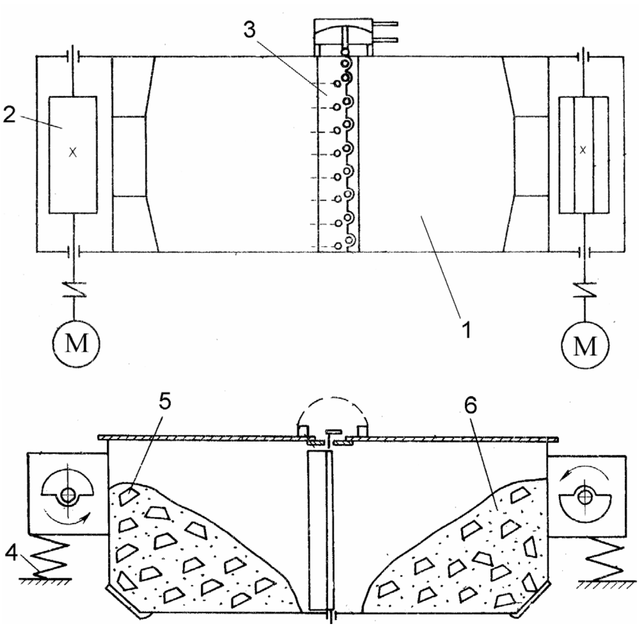

Fig. 1. Unbalanced mass vibromachine with two independently driven vibrational exciters

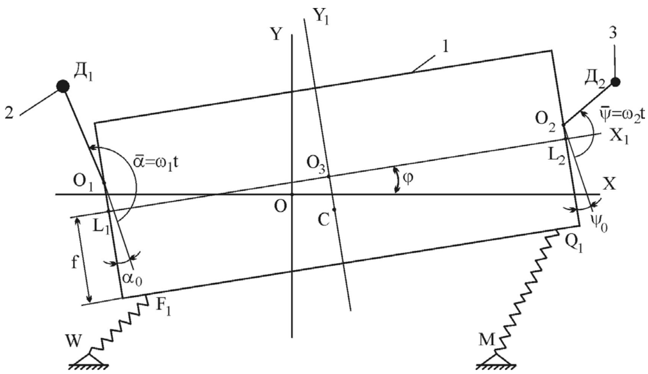

Fig. 2. Diagram of generalized two-unbalance vibromachine, 1 - working container; 2 - left unbalance (Д1); 3 - right unbalance (Д2); XOY - fixed coordinate system; X1O3Y1 - moving coordinate system connected with geometric centre and symmetry axes of container; C - mass centre of container; f - distance between geometric centre and bottom of container; O3L1 and O3L2 - displacement of centers of rotation of unbalances with respect to vertical symmetre axis of container; WF1 and MQ1 - lengthes of suspension springs at arbitrary instant of time during the motion of container

The machine consists of container 1 with two unbalance vibrational exciters. The container is divided into two compartments by partition (jalousie) 3, and is mounted on elastic suspension 4. To the butt ends of the container two identical unbalance vibrational exciters are fastened 2. The unbalances are independently driven by corresponding electric motors through elastic muffs. The axes of rotating of the unbalances are parallel. There are treating medium 6 and treated articles 5 in both compartments of the container.

For the creation of the complex model of "vibromachine - treating medium" - system, separate models describing the motion of vibromachine container and the medium were drived, and finally they are united into the complex model of the system.

To derive the vibromachine container motion model, its diagram is drown (Fig 2) where the vibromachine is depicted as plane mechanical system having several degrees of freedom, and the main geometric and kinematic parameters are given symbolically (parametrically) [2].

Using Lagrange's equations of II-nd type, the motion of the vibromachine container is described by means of the system of non-linear differential equations:

$$\begin{cases} \ddot{x}_c + \omega^2 x_c = \varepsilon f_x(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega_1 t + \alpha_0, \omega_2 t + \psi_0); \\ \ddot{y}_c + \omega^2 y_c = \varepsilon f_y(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega_1 t + \alpha_0, \omega_2 t + \psi_0); \\ \ddot{\varphi} + \omega_\varphi(t)\varphi = \varepsilon f_\varphi(\varphi, \dot{\varphi}, \ddot{x}_c, \ddot{y}_c), \end{cases} \tag{1}$$

where x_c, y_c - are coordinates of geometric centre of the container at arbitrary time instant t; φ - angle of its turn; ω - frequency of natural oscillations of the container; ε - small parameter; f_x, f_y, f_φ - functions which take into account physical and geometric non-linearity of mechanical system; $\omega_\varphi(t)$ - frequency of circular oscillations of container, unbalances taken into account; $\omega_i\,(i = 1,2)$ - angular velocities of vibrational exciters motion, $\omega_1 t + \alpha_0$ and $\omega_2 t + \psi_0$ are their phases.

The first approximation of the solutions of system, (1) is obtained by means of asymptotic methods of non-linear mechanics:

$$\begin{cases} x_{o3} = x_0 \sin(\omega t + \alpha_x) + \dfrac{\varepsilon}{\omega}\displaystyle\int_0^t f_x\left(\varphi^*, \dot{\varphi}^*, \ddot{\varphi}^*,\right. \\ \left.\omega_1 t + \alpha_0, \omega_2 t + \psi_0\right)\sin\left(\omega(t-u)\right)du; \\ y_{o3} = y_0 \sin(\omega t + \alpha_x) + \dfrac{\varepsilon}{\omega}\displaystyle\int_0^t f_y\left(\varphi^*, \dot{\varphi}^*, \ddot{\varphi}^*,\right. \\ \left.\omega_1 t + \alpha_0, \omega_2 t + \psi_0\right)\sin\left(\omega(t-u)\right)du; \\ \varphi^* = \varphi_0 \cos\left(\omega_0 t + \theta(t)\right), \end{cases} \tag{2}$$

where $\theta(t)$ is known function; ω_0, x_0, y_0, α_x and α_y are constants determined by parameters of the system and its initial condition.

An important stage in deriving models of such kind is the analysis of the stability of the solutions obtained in the form (2), i.e. the determination of the influence of parameters of this equation upon the character of its solution. Such analysis is necessary for ensuring perfect adequacy of the mathematical model to the real physical process both during designing and in the course of operating. It is necessary that the equation deseribing the model to have solution at intervals of time of any duration (the duration of treating articles in vibromachine may be considered unlimited). These laws of motion must meet the operational requirements of the system during the time interval of treating. As the result of investigation, it is found that for really existing parameters of the vibromachine the derived system of analytical expressions describes stable motion of the container.

The next stage of the work is the creation of the treating medium motion model of the vibromachine container [3,4]. For this purpose, the following hypotheses are assumed:

1. The material of the medium is continuous and homogeneous represented as stratification (multy-layer) of planar elastic-plastic beams, the thickness of which is considerably less than their length, and which in certain way contact with the container walls; in mathematical model this enables us to take into account both different kinds of interaction of working medium with the container and the motion of the container itself.
2. The medium moves layerwise and is in compound motion (in the plane of the motion of the container); the transportation motion of a layer is the motion (Fig. 1) together with the vibromachine container, relative motion of a layer of the medium is the longitudinal oscillations along layer . The schematic diagram of the compound motion of the cross-section of layer i of the medium (point M of a volume element of the medium layer) is drawn in Fig. 3. Such assumption enables us to mathematically describe the circulation of the medium in the container (volumwise) at different velocities.
3. The correlation between stress and deformation in the medium material is described by non-linear F?cht's law; the following two kinds of non-linearity are considered here:

   a) non linearity of viscous stresses

$$\sigma = E\zeta + k_0 \left(\frac{d\zeta}{dt} \right)^{\nu+1} \tag{3}$$

   b) non linearity caused by elastic properties of the medium

$$\sigma = E(\zeta)^{\nu+1} + k_0 \frac{d\zeta}{dt} \tag{4}$$

   where σ - is normal stress in a layer of the medium; E , k_0, ν are constants which characterize elastic and viscouse properties of the medium; $\zeta = \frac{\partial u}{\partial \xi}$ is its relative deformation (here $u = u(\xi, t)$ is displacement of arbitrary crose – section of medium layer along axis ξ during a small interval of time t).

4. In a layer of the medium as well as between layers, internal friction force R is determined by means of Bolotin's law:

$$R = \frac{\partial u}{\partial t}\left(B + B_0 u^2 \right), \tag{5}$$

where B, B_0 are constants determined by the kind of medium material.

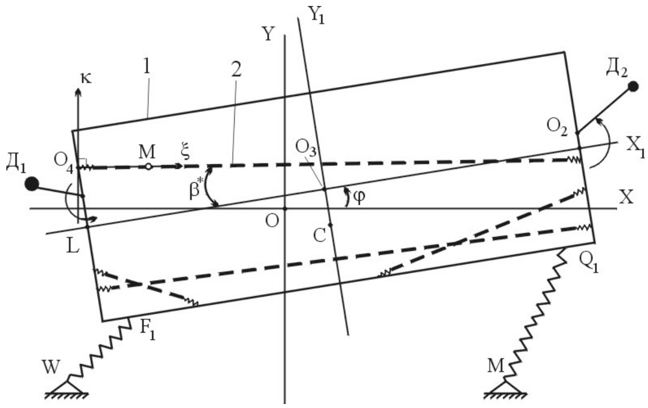

Fig. 3. Graphic representation of vibromachine medium motion model, 1 – working container; 2 – i[th] layer of the medium that represent elastic-plastic beam which elastically contacts the walls, of container; $\xi O_4 \kappa$ - coordinate system bound with i[th] layer of the medium; $u(\xi, t)$ (point M) - arbitrary cross-section of i[th] layer of the medium, in the relative motion this layer oscillates along axis ξ ; β^* - angle between coordinate system $\xi O_4 \kappa$ and moving coordinate system $Y_1O_3X_1$, which is bound with the geometric centre of the container (angle of inclination of ith layer of the medium to the bottom of the container)

Under the assumptions mentioned, the equation of longitudinal oscillations (equation of relative motion of cross-section) of the medium layer takes the following forms:

a) form (3) in the case on non-linear Föcht's law

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2 \frac{\partial^2 u}{\partial \xi^2} - \beta \left(\frac{\partial^3 u}{\partial \xi^2 \partial t} \right)^{\nu+1} = f(t) + \frac{\partial u}{\partial t}\left(\vartheta + \delta u^2 \right) \tag{6}$$

b) form (4) in the case of nonlinear-elastic properties of friable medium

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2 \frac{\partial}{\partial \xi}\left(\frac{\partial u}{\partial \xi} \right)^{\nu+1} - \beta \frac{\partial^3 u}{\partial \xi^2 \partial t} = f(t) + \frac{\partial u}{\partial t}\left(\vartheta + \delta u^2 \right) \tag{7}$$

where α , β , ϑ , δ are coefficients determined by the kind of medium material (metal balls, abrasive, etc.); $f(t)$ is external disturbance caused by the motion of drive unbalanced masses.

It is assumed that in equations (6) and (7) the forces of viscouse friction are small in comparison with non-linear-elastic (restoring) force in this layer, i.e. $\beta, \vartheta, \delta << \alpha^2$.

The model of medium layer motion with non-linear viscouse component of stress is of the form:

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial^2 u}{\partial \xi^2}=\varepsilon\left[\left(\frac{\partial^3 u}{\partial \xi^2\partial t}\right)^{\nu+1}+\left(\vartheta_1+\delta_1 u^2\right)\frac{\partial u}{\partial t}+b_1\sin\mu t\right] \quad (8)$$

where εb_1 and μ are parameters expressing the influence of external disturbance on the medium motion.

Unifrequent dynamic processes in the medium the motion of which can be described by means of (8), provided the interaction between the medium and the vibromachine container corresponds to hinge model of the contact between the medium and the working container ($u(0,t)=u(l,t)=0$), is described by the formula:

$$u_k(\xi,t)=a(t)\tilde{\Xi}_k(\xi)\cos(\overset{*}{\omega}_k t+\theta(t)) \quad (9)$$

where $\overset{*}{\omega}_k=\alpha\frac{k\pi}{l}$ (here l is geometric parameter of container); $k=1,2,\dots$.

For the boundary conditions of nonlinear-elastic contact between the medium and the container:

$$\left.\left[\alpha_1\frac{\partial u}{\partial \xi}+\beta_1 u\right]\right|_{\xi=0}=\left.\varepsilon\varphi_1(\xi,t)\right|_{\xi=0};$$
$$\left.\left[\alpha_2\frac{\partial u}{\partial \xi}+\beta_2 u\right]\right|_{\xi=l}=\left.\varepsilon\varphi_2(\xi,t)\right|_{\xi=l}, \quad (10)$$

where α_1, α_2, β_1, β_2 $\in[0,1]$ are constants; $\varphi_1(\xi,t)=h_1u+d_1u^3-b_1\sin\mu t$, $\varphi_1(\xi,t)=-\varphi_2(\xi,t)$, and h_1, d_1, h_2, d_2 are parameters of nonlinear-elastic contact between the medium and the container, for the case of non-resonance we have:

$$u(\xi,t)=a_2\Xi(\xi)\cos(\alpha\lambda_1+\mu)t,$$

where $P_1=\int_0^l\Xi^2(\xi)d\xi$, $\quad P_3=\delta_1(\alpha\lambda_1+\mu)\frac{\pi}{l}\int_0^l\Xi^4(\xi)d\xi$

$$a_2=\sqrt{\frac{\varepsilon P_2}{2\alpha\lambda_1\pi P_1P_3}},\quad P_2=\pi(\alpha\lambda_1+\mu)\left[\int_0^l\Xi(\xi)\Xi''(\xi)d\xi+\vartheta_1P_1\right],$$

$\Xi(\xi)=\sin\lambda_1\xi-\frac{\alpha_1}{\beta_1}\lambda_1\cos\lambda_1\xi$, and λ_1 is the first eigen value of the boundary problem corresponding to non-disturbed motion equations (8), (10).

The model of medium layer motion with non-linear stress component is of the form:

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial}{\partial \xi}\left(\frac{\partial u}{\partial \xi}\right)^{\nu+1}=$$
$$=\varepsilon\left[b_1\sin\mu t-(\vartheta_1+\delta_1)\frac{\partial u}{\partial t}u^2+\beta\frac{\partial^3 u}{\partial \xi^2\partial t}\right] \quad (11)$$

The solutions of differential equations are obtained by means of special Ateb-functions.

For stabilized mode of motion the equation of medium layer motion under the condition of rigid contact between the medium and container is of the form:

$$u(\xi,t)=a(\nu)sa\left(1,\frac{1}{\nu+1},\Pi_\xi\frac{\xi}{l}\right)ca(\nu+1,1,\psi) \quad (12)$$

w

$$a(\nu)=\sqrt{\frac{\left(\frac{8\pi\Gamma^2\left(1+\frac{\nu}{2}\right)\Gamma^2\left(\frac{1}{\nu+2}\right)}{(\nu+2)^2 l\Gamma^2\left(\frac{3}{2}+\frac{1}{\nu+2}\right)}-\frac{4\vartheta_1\sqrt{\pi}\,pl\Gamma\left(\frac{1}{\nu+2}\right)}{(\nu+2)\Gamma\left(\frac{3}{2}+\frac{1}{\nu+2}\right)}\right)}{\left(\frac{3\delta_1\pi\Gamma\left(\frac{3}{\nu+2}\right)\Gamma\left(\frac{\nu+1}{\nu+2}\right)}{8\Gamma\left(\frac{3}{2}+\frac{3}{\nu+2}\right)\Gamma\left(\frac{5}{2}+\frac{\nu+1}{\nu+2}\right)}\cdot\frac{l}{\Pi_\xi}\right)}};$$

$\Pi_\xi=\sqrt{\pi}\Gamma\left(\frac{\nu+1}{\nu+2}\right)\Gamma^{-1}\left(\frac{1}{2}+\frac{\nu+1}{\nu+2}\right)$; $\Gamma(\dots)$ is γ - function of corresponding argument.

Finally, the complex model of "vibromachine-treating medium"-system takes the form [5]:

$$\begin{cases} x_M(t)=x_{03}(t)+S\sin\varphi(t)-w\cos\varphi(t)- \\ -q\sin\varphi(t)+(\xi+u(\xi,t))\cos\left(\varphi(t)+\beta^*\right), \\ y_M(t)=y_{03}(t)-S\cos\varphi(t)-w\sin\varphi(t)+ \\ +q\cos\varphi(t)+(\xi+u(\xi,t))\sin\left(\varphi(t)+\beta^*\right), \end{cases} \quad (13)$$

where $S=O_3C$ is the distance between geometric centre of the container and the mass centre of the medium, $\xi=O_4M$ is a position of an element of medium layer at initial instant of time, $u(\xi,t)$ is the law of longitudinal oscillations of the layer (law of relative motion), which is described by corresponding equations for a resonance case of medium motion or non-resonance one, $w=LO_3$, $q=LO_4$.

4. Results of modelling

By means of the complex model obtained it is possible to investigate different modes of the motion of the vibromachine and the medium, to plot trajectories of motion for arbitrary points of the container and the medium, to investigate the influence of the parameters of the vibromachine and the medium on the factors of the intensity of articles treatment according to corresponding criterion of intensity.

Graphic correlations obtained by means of the complex model and the software elaborated on its basis are shown in Fig 4-8; they illustrate the influence of vibromachine parameters upon the amplitude of its container oscillations; the amplitude is the main factor of the articles treatment intensity according to such criterion of intensity as removing metal from the surface of the treated article per unit of time.

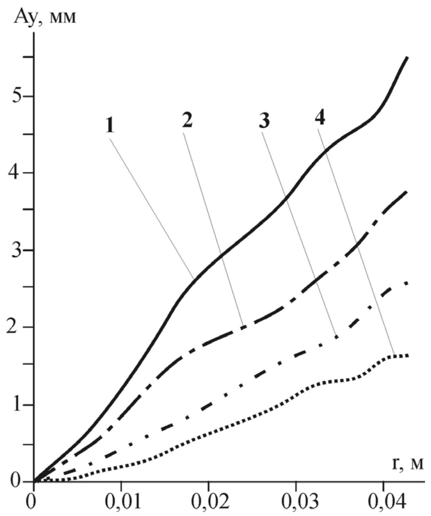

Fig. 4.Correlation between vertical component of container oscillation and radius of unbalanced masses 1. mass of 9.238 kg; 2. mass of 6.927 kg; 3. mass of 4.618 kg; 4. mass of 2.309 kg

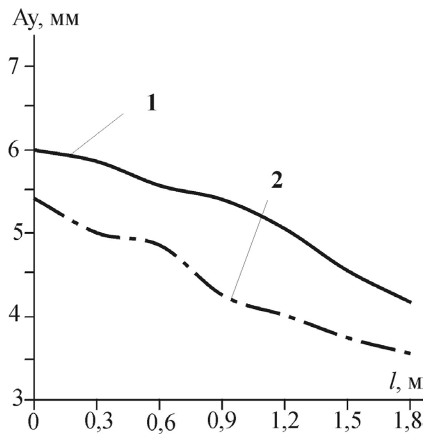

Fig. 5.Correlation between amplitude of vibromachine container and the placement of suspension attachment.1. oscillating mass 70 kg; 2. oscillating mass 84 kg

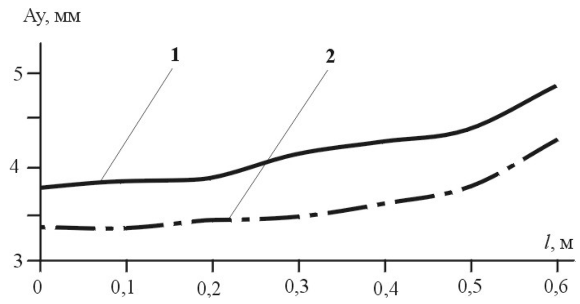

Fig. 6. Correlation between vibromachine container oscillations and the placement of unbalance units, 1. oscillating mass 70 kg; 2. oscillating mass 84 kg

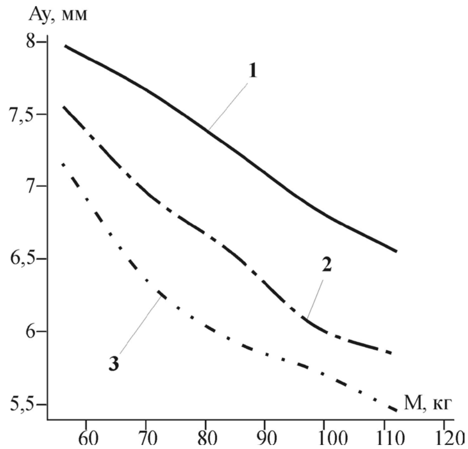

Fig. 7. Correlation between amplitude of vertical oscillation of container centre and oscillating mass at unbalance rotation angular velocity of $\omega_2 = 96c^{-1}$, 1. rigidity of suspension (left or right) 22860 N/m; 2. rigidity of suspension (left or right) 34290 N/m; 3. rigidity of suspension (left or right) 45710 N/m

Finally, on the basis of investigations carried out, parameters and the mode of work of the two-unbalanced masses vibromachine (input data: mass of container - 55kg, its length 0.59m, distance between supports - 0.77m, frequency of vibration - 22.5Hz, mass of the load - 20kg in the form of metal balls 0.005m in diameter) which ensure the highest intensity of article treatment (in this particular case-the largest removing of material from the article surface per unit of time) are the following:

a) unbalanced masses of vibromachine should rotate in the same sense, that ensure the most intensive mixing and circulation of treating medium in working container volumwise;
b) rigidity of working suspension (total) - 45720 N/m;
c) angular velocity of unbalance rotation – 141 c^{-1};
d) unbalanced mass – 9,238 kg;
e) radius of unbalanced mass - 0,0424 m (static moment of unbalanced mass 0,3921 kg·m);
f) length of vibromachine container – 0.5 m;
g) distance between attachment of unbalance units and container bottom – 0,3 m;

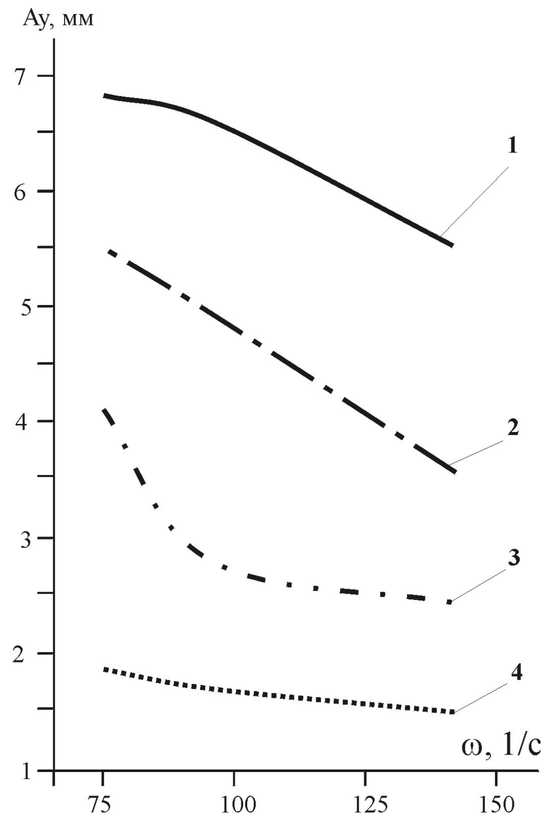

Fig. 8. Correlation between vertical component of container oscillation and angular velocity of rotation unbalanced mass, 1. mass of 9.238 kg; 2. mass of 6.927 kg; 3. mass of 4.618 kg; 4. mass of 2.309 kg

The chosen mechanical criterion of intensity - "removing of material from the article surface per init time" is proportional to the square of amplitude and to the cube of oscillation frequency of container [6,7]. It is may be taken as a criterion of optimization.

$$q = k \cdot a^2 \cdot \omega^3,$$

where q (g/s) is amount of material removed per init of time, a (mm) - amplitude of container oscillation, ω (c^{-1}) - container oscillation frequency, k (g/mm) - empirical constant taking into account the other parameters of vibrotreating (influence of vibromachine parameters of treading medium, etc.)

5. Conclusions

Thus, the obtained complex mathematical model of the system "vibromachine – trading medium" enables us to analytically determine the influence of its parameters upon the intensity of articles treatment in it according to the chosen criterion; thus it saves us from carrying out time taking cumbersome experiments both for designing a vibromachine and for selecting modes of treating articles in it. Due to their non-linearity, models of such kind are adequate to real situation of vibrotreatment, they can also be applied in investigation of dynamics of the vibromachine and its medium.

In general way the elaboration and investigation of complex non-linear mathematic models of the "vibromachine – treating medium" – system by means of creating interconnected models of the motion of working tool of machine and friable medium; and on the basis of this dynamic phenomena of the system should be investigated in order to determine the influence of machine parameters and the medium upon factors of intensivity of article treatment is the complex problem solving of which has several basic stages (Fig 9).

Analysis of certain types of vibromachines needing for solving the technological task should be performed at the first stage. Constructions of vibromachines are analyzed. The unitized construction that combines the constructional features of the each machine is chosen.

I stage	Analysis of vibromachine types. Choice of the basic type for solving given technological task. Elaboration of the unitized calculating diagram for the vibromachine
II stage	Elaboration of non-linear mathematic model of the motion of vibromachine container and the friable medium. Elaboration of the elements of the systems for automatic calculations of the vibromachine dynamics
III stage	Investigation of the parameters of vibromachine influence and the friable medium (for their different combinations) upon factors of intensity of article treatment in the machine
IV stage	Recommendations as to the choice of the parameters of the vibromachine and the treating medium for receiving highest labor productivity according to the chosen criterion of intensity

Fig. 9. Diagram of carrying out investigation of dynamics of volumetric vibration treatment: from the choice of the type of vibromachine to the determination of the highest intensity of article treatment

The elaboration of parametric model of the "vibromachine - treating medium" using the unitized construction is done at the second stage are basic. Non-linear statement of the problem and its analytic solution is accomplished successfully by means of asymptotic methods of non-linear mechanics and special Ateb-functions. Parameterization of the models enables us to take into account both the constructional (geometric), the kinematics characteristics of vibromachine and the physic-mechanical properties of different kinds of friable treating and the articles manufactured for stabilized and transient modes. It is possible to use prepared results (models of the motion) with available results obtained before. Elements of the systems for automatic calculations in order to determine the influence of machine parameters and the medium upon factors of intensity of article treatment are elaborated at this stage. Obtained analytic solutions enable to create programs on the basis of applied systems for mathematical calculations, in particular MathCad and MatLab.

Investigation of the parameters of vibromachine influence and charging (for their different combinations) upon factors of intensity of article treatment, in particular, amplitude, oscillation frequency and character of the motion of vibromachine container and its medium are performed at the third stage. Specified factors determine labor productivity. They are included in the mechanical criterions for the determination of intensity of treating: criterion of highest removing metal from the surface of the treated article per unit of time.

Finally, at the fourth stage, on the basis of investigations carried out, as to the choice of the parameters of the vibromachine and physico-mechanical properties of treating medium, which give highest labor productivity, the recommendations are given.

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