Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 25
ISSUE 1
November
2007

Loss factor prediction for laminated plates

B. Diveyev *
Lviv National Polytechnic University, Ukraine
* Corresponding author: E-mail address: divboglviv@yahoo.com

Received 25.04.2007; published in revised form 01.11.2007

Analysis and modelling

ABSTRACT

Purpose: The study aims to predict the loss factor properties of composite laminated plates.
Design/methodology/approach: Elastic constants of laminates and damping properties have been determined by using an identification procedure based on multi-level theoretical approach.
Findings: The present paper is the first attempt at proposing a novel adaptive procedure to derive loss factor parameters for sandwich plate's vibration.
Research limitations/implications: In the future the extension of the present approach to sandwich plates with different core materials will be performed in order to test various sandwich design.
Practical implications: Structures composed of laminated materials are among the most important structures used in modern engineering and especially in the aerospace industry. Such lightweight and highly reinforced structures are also being increasingly used in civil, mechanical and transportation engineering applications.
Originality/value: The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical or approximation method.
Keywords: Computational material science; Composite materials; Laminated plates; Elastic constants

1. Introduction

The simple laminate theories [1-7] are most often incapable of determining the 3-D stress field in the lamina. Thus, the analysis of composite laminates may require the use of laminate independent theory or a 3-D elasticity theory. Exact three-dimensional solutions [8-18] have shown the fundamental role played by the continuity conditions for the displacements and the transverse stress components at the interfaces between two adjacent layers for an accurate analysis of multilayered composite thick plates. Further, these elasticity solutions demonstrated that the transverse normal stress plays a predominant role in these analyses. However, accurate solutions based on the three-dimensional elasticity theory are often intractable.

The limitations of the analysis based on the displacement formulation motivated the work [19-20] in which the mixed variation theorem for the dynamic analysis of multilayered plates was used. A semi-analytical method has been developed in to obtain the natural frequencies of vibration of simply- supported laminated composite cross-ply plates. Continuity of the transverse stresses as well as the displacements have been explicitly satisfied at the lamina interfaces in these models. Further, these models have been formulated by considering a local Cartesian co-ordinate system at the mid-surface of each individual layer. Six degrees of freedom (DOF), viz. three displacement components u, v and w (along x, y and z directions, respectively) and three transverse stress components are expressed at the bottom as well as the top surface of each individual layer. The time dependent axial and the transverse displacements along the x, y and z directions of any point can be expressed using power series expansions [8-14].

2. Some aspects of the laminated beam modeling

Exact static solutions for composite laminates in cylindrical bending. Bending by moment. Let us consider the exact solution for uniform cylindrical bending of layered symmetric composite laminates. The governing equations are:

$$\frac{\partial\sigma_{xx}}{\partial X}+\frac{\partial\tau_{xz}}{\partial Z}=0,\quad \frac{\partial\tau_{xz}}{\partial X}+\frac{\partial\sigma_{zz}}{\partial Z}=0\,. \tag{1}$$

$$\sigma_{xx}=C_{xx}\varepsilon_{xx}+C_{xz}\varepsilon_{xz},\ \sigma_{zz}=C_{xz}\varepsilon_{xx}+C_{zz}\varepsilon_{zz}\,,\quad \tau_{xz}=G\gamma_{xz}\quad . \tag{2}$$

If the stress-strain state is uniform, then these relations must hold, because τ on the face plate surfaces is equal to 0

$$\frac{\partial\tau}{\partial Z}=-\frac{\partial\sigma_{xx}}{\partial X}=0\quad and\quad \tau=0\,, \tag{3}$$

$$\frac{\partial\sigma_{zz}}{\partial Z}=-\frac{\partial\tau}{\partial X}\ =0\quad and\quad \sigma_{ZZ}=0 \tag{4}$$

If the stress σ_{xx} is not equal to 0, the following assumption must be made

$$\sigma_{xx}=S(z)\,. \tag{5}$$

From Eq.(1-5) we may obtain for the displacement

$$U=XZ,\ W=-0.5\cdot\left(\alpha Z^2+X^2\right),\quad \alpha=C_{xz}/C_{zz}$$
$$\sigma_{xx}=Z\left(C_{xx}-\alpha C_{xz}\right) \tag{6}$$

For the bending moment we obtain

$$M=\int_{-H}^{H}Z^2\cdot\left(C_{xx}-\alpha\,C_{xz}\right)dZ \tag{7}$$

Here for the material non homogeneity $C_{xx}(Z),\ \alpha(Z),\ C_{xz}(Z)$ all may be functions of the normal coordinate Z of the plate. Thus, by comparison with a uniform Tymoshenko beam of the same depth, an equation for the bending rigidity of a uniform equivalent beam may be written

$$EI_T=M\Big/\frac{d\gamma}{dx}=\int_{-H}^{H}Z^2\cdot\left(C_{XX}-\alpha\,C_{XZ}\right)dZ\,, \tag{8}$$

because $\dfrac{d\gamma}{dx}=1$.

Bending by force. In this case the primary assumptions are

$$\sigma_{xx}=x\cdot S(z),\qquad \tau=T(z) \tag{9}$$

From Hooks law we obtain

$$\varepsilon_x=\frac{\partial u}{\partial x}=xS^*,\ \varepsilon_z=\frac{\partial w}{\partial z}=-\alpha_1xS^*;$$
$$S^*=\frac{S(z)}{1-\alpha_1\alpha_2},\quad \alpha_1=\frac{C_{xz}(z)}{C_{xx}(z)},\quad \alpha_2=\frac{C_{xz}(z)}{C_{zz}(z)}\quad . \tag{10}$$

Next by integration for the displacement we obtain

$$u=\frac{x^2}{2}S^*+\varphi(z),\quad w=-x\int_0^z\alpha_1S^*dz+\psi(x)\quad . \tag{11}$$

By substituting Eq.(11) into Eq.(1) we can derive (for a symmetrically laminated plate, but the same may be made for arbitrary lamination)

$$u=\frac{Ax^2z}{2C_x}+A\int_0^z\left(\int_0^z\frac{\alpha_2z}{C_x}dz+\frac{\tau}{G}\right)dz-\Phi z\,,$$
$$w=-\frac{Ax^3}{6C_x}-Ax\int_0^z\frac{\alpha_2z}{C_x}dz+\Phi x \tag{12}$$

where for the tangential stress τ and the constant Φ we have

$$\tau=-A\int_z^H\left(1-\alpha_1\alpha_2\right)zdz\ ,\Phi=\frac{A}{H}\int_0^Hdz\left(\int_0^z\frac{\alpha_2z}{C_x}dz+\frac{\tau}{G}\right)dz \tag{13}$$

In the above equations A is an arbitrary constant number. If we equate the tangential force Q to 1, then from (12,13)

$$Q=\int_{-H}^{H}\tau\,dz=1\ \Rightarrow\quad A=\left(\int_{-H}^{H}\left(\int_z^H\left(1-\alpha_1\alpha_2\right)zdz\right)dz\right)^{-1} \tag{14}$$

Thus, by comparison with a uniform Tymoshenko beam of the same cross section, we may write an equation for the transverse rigidity of a uniform equivalent beam

$$\frac{1}{2HG_T}=-A\int_0^z\frac{\alpha_2z}{C_x}dz+\Phi=$$
$$=-A\int_0^z\frac{\alpha_2z}{C_x}dz+\frac{A}{H}\int_0^Hdz\left(\int_0^z\frac{\alpha_2z}{C_x}dz+\frac{\tau}{G}\right)dz \tag{15}$$

where τ is given by Eq.(13). It may be seen that from Eq.(12) we may obtain the same bending rigidity as in Eq.(8).

Asymptotic approach. Above (Part 1) we have obtained the exact solution for the statically loaded sandwich plate by the cylindrical bending. The equivalent Tymoshenko beam is established with the same bending and tangential rigidity. Naturally, by the dynamic loading the static approach is incorrect. It may be valid only for the thin plates and lower frequency excitation. As was said previously, many authors have maid such a work. But there assumption were submerged or to one layer model, or to hypothesis like [1-7]. Such theories based on mixed approach or another variation principles [19] are not simply proved mathematically. Thus, here we should applied Galerkin method to linear elasticity dynamic problem [20]. By the Banach fixed-point theorem [20] a) problem has a unique solution; b) we have a priory estimate; c) convergence of the Galerkin method.

Let us consider now such kinematical assumptions (for symmetrical three layered plate)

$$U_e - \begin{cases} u = \sum\limits_{i,k} u_{ik}{}^{e} z^{i} \sin k\pi x / L, & 0 < z < H, \\ w = \sum\limits_{i,k} w_{ik}{}^{e} z^{i} \cos k\pi x / L, & 0 < x < L, \end{cases} \tag{16}$$

$$U_d - \begin{cases} u = \sum\limits_{i,k} u_{ik}{}^{d} z^{i} \sin k\pi x / L, & 0 < z < H_p, \\ w = \sum\limits_{i,k} w_{ik}{}^{d} z^{i} \cos k\pi x / L & 0 < x < L. \end{cases} \tag{17}$$

By substituting Eq.(1,2) to

$$\int\limits_V (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xz}\delta\varepsilon_{xz} - \rho\frac{\partial u}{\partial t}\delta\frac{\partial u}{\partial t} - \rho\frac{\partial w}{\partial t}\delta\frac{\partial w}{\partial t})dV = 0, \tag{18}$$

and by assuming the unifrequency vibration we obtain the set of linear algebraic equations

$$[A]u = \begin{bmatrix} A_1 & A_d \\ A_d^T & A_2 \end{bmatrix} \begin{matrix} u^e \\ u^d \end{matrix} = 0 \tag{19}$$

For grater number of lamina may be wrote

$$U_d^n - \begin{cases} u = \sum\limits_{i,k} u_{ik}{}^{n} z_n{}^{i} x^{(2k-1)}, & 0 < z < H_p, \\ & 0 < x < L, \\ w = \sum\limits_{i,k} w_{ik}{}^{n} z_n{}^{i} x^{(2k-2)}, & n = 1, \dots, N-1, \end{cases} \tag{20}$$

where

$$H_p^n = H - \sum_{l=1}^{n-1} h_l \,. \tag{31}$$

Now

$$[A] = \begin{vmatrix} A_1 & A_{1d}^1 & A_{2d}^1 & \dots & A_{N-1}^1 & A_{Nd}^1 \\ A_{1d}^{1T} & A_2 & A_{1d}^2 & \dots & A_{N-2}^2 & A_{N-1}^2 \\ A_{2d} & A_{1d}^{2T} & A_2 & \dots & & \\ & & & \dots & & \\ & \dots & & & A_{N-1} & A_{1d}^{N-1} \\ & \dots & & & A_{1d}^{N-1} & A_n \end{vmatrix}. \tag{21}$$

The frequency equation should be written such

$$\left[[K] - \omega^2 [M]\right] \langle q_0 \rangle = \langle 0 \rangle \tag{22}$$

Damping. Loss factors is found by analytical solutions (1-15)

$$\eta_\Sigma = \frac{1}{\eta_1} \frac{\frac{L^3}{3} \int\limits_0^H \eta \sigma_{XX} \varepsilon_{XX} dz + L \int\limits_0^H \eta \frac{\tau^2}{G} dz}{\frac{L^3}{3} \int\limits_0^H \sigma_{XX} \varepsilon_{XX} dz + L \int\limits_0^H \frac{\tau^2}{G} dz}. \tag{23}$$

In Fig.1. the loss factor η_Σ (Eq.16) is presented for the beams with the various mechanical properties of cover layers. Nondimension mechanical parameters were: *Cxx(1)=1, Cxx(2)=10*. Another parameters *(Czz, Cxz, G)* were changed.

In Fig.2 the loss factors are shown for the three-layered beam with thin elastic interlayer (fife-layered beam). The depth of the interlayer was 0.03H. Mechanical properties of the interlayers were: *Cxx$_i$/Cxx(2)= G$_i$/G(2)= Cxz$_i$/Cxz(2)= Czz$_i$/Czz(2)=0.1.*

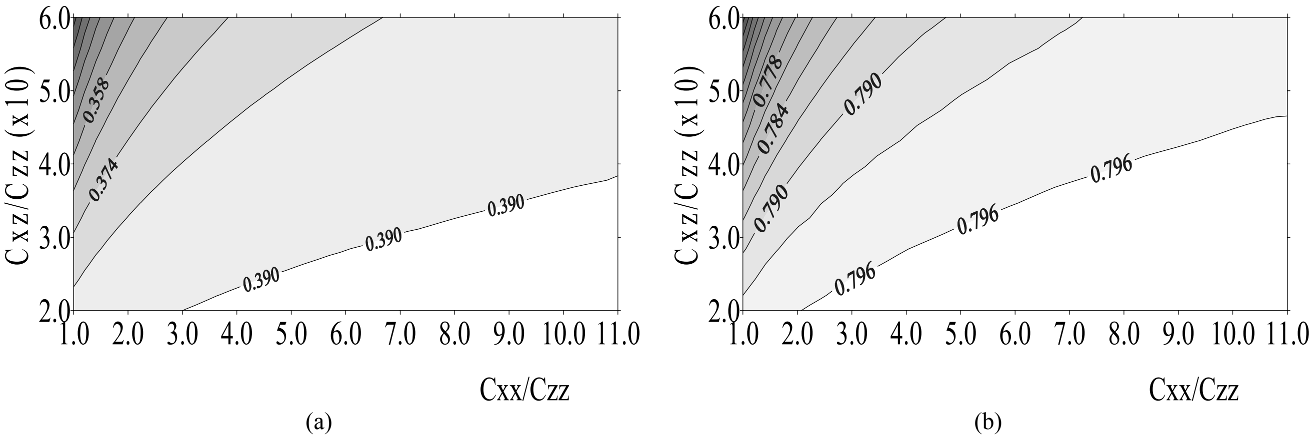
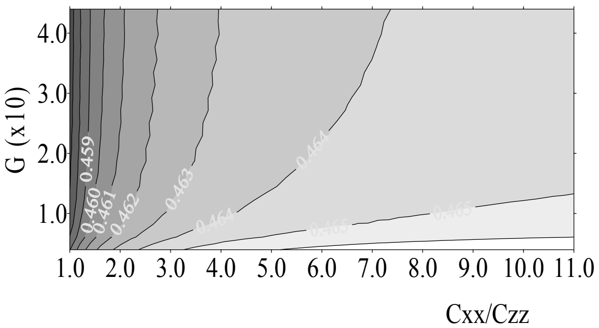
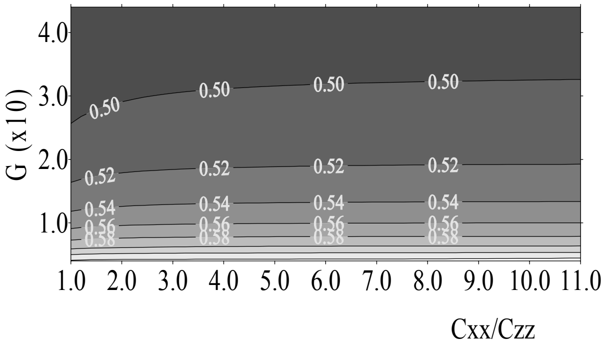


Fig.1. (a) loss factor in n a symmetrical three-layered beam (EL/H=1/30); (b) loss factor in a symmetrical three-layered beam (EL/H=1/3)



(a)



(b)

Fig.2. (a) loss factor in a symmetrical fife-layered beam (changing properties of the face; (b) loss factor in a symmetrical fife-layered beam (changing properties of the face layers,EL/H=1/30)

3. Conclusions

Various displacement models have been developed by considering combinations of displacement fields for in-plane and transverse displacements inside a layered beam to investigate the phenomenon of vibrations in laminated composite plates. Numerical evaluations are obtained for loss factor of three- an fife-layer plates.

References

[1] D. Ross, E.E. Ungar, E.M. Kerwin, Damping of plate flexural vibrations by means of viscoelastic laminate, ASME, Structural Damping (1959) 49-88.

[2] R.A. DiTaranto, Theory of vibratory bending for elastic and viscoelastic layered finite length beams, Transactions of the ASME, Journal of Applied Mechanics 32 (1965) 881-886.

[3] R.A. DiTaranto, W. Blasingame, Composite damping of vibrating sandwich beams, Journal of Engineering for Industry 89 (1967) 633-638.

[4] D.J. Mead, S. Markus, The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions. Journal of sound and vibration 10 (1969) 163-175.

[5] D.J. Mead, S. Markus, Loss Factors and Resonant Frequencies of Encastre Damped Sandwich Beams, Journal of Sound and Vibration 12 (1970) 99-112.

[6] N.J. Pagano, Exact solutions for composite laminates in cylindrical bending, Journal of Computer Materials 3 (1969) 398-411.

[7] S. Srinivas, C.V. Joga Rao, A.K. Rao, Flexural vibration of rectangular plates, Journal of Applied Mechanics 23 (1970) 430-436.

[8] M.P. Sheremetjev, B.L. Pelekh, To refined plate theory construction, Engineering Journal 4 (1964) 504-510 (in Russian).

[9] B.L. Pelekh, B.M. Diveiev, Some dynamic problems for viscoelastic anisotropic envelopes and plates. The generalized dynamic equations of the theory of stratified shells in view of boundary conditions on surfaces, Mechanics of Composite Materials 2 (1980) 277-280 (in Russian).

[10] B.L. Pelekh, B.M. Diveiev, Some dynamic problems for viscoelastic anisotropic shells and plates, An impedance of viscoelastic anisotropic shells and plates, Mechanics of Composite Materials 3 (1980) 546-548 (in Russian).

[11] B. Diveiev, One approach for calculating of laminated structures, AS USSR, Institution of Applied Problem of Mechanic and Mathematic, 1991 (in Ukrainian).

[12] B.M. Diveyev, M.M. Nykolyshyn, Refined Numerical Schemes for a Stressed-Strained State of Structural Joints of Layered Elements, Journal of Mathematical Sciences 107 (2001) 130-133.

[13] K.H. Lo, R.M. Christensen, E.M. Wu, A High-Order Theory of Plate Deformation - Part 2: Laminated Plates, Journal of Applied Mechanics 44 (1998) 669-676.

[14] S. Karczmaryk, An Analytical model of flexural vibration and the bending of plane viscoelastic composite structures, Warsaw University of Technology, Scientific Works 172 (1999) 159-163.

[15] M. Springolo, G. Van Erp, A. Khennane, Design and analysis of a composite beam for infrastructure applications, Part I, Preliminary investigation in bending International Journal of Materials and Product Technology 25 (2006) 297-312.

[16] M. Springolo, G. Van Erp, A. Khennane, Design and analysis of a composite beam for infrastructure applications, Part III, Experimental results and non-linear FE analysis International Journal of Materials and Product Technology 25 (2006) 331-346.

[17] S.H. Zhang, H.L. Chen, X.P. Wang Numerical parametric investigation of loss factor of laminated composites with interleaved viscoelastic layers, International Journal of Vehicle Noise and Vibration 2 (2006) 62-74.

[18] C. Kyriazoglou, F.J. Guild Finite element prediction of damping of composite GFRP and CFRP laminates – a hybrid formulation – vibration damping experiments and Rayleigh damping, Composites Science and Technology 66 (2006) 487-498.

[19] M.K. Rao, Y.M. Desai, Analytical solutions for vibrations of laminated and sandwich plates using mixed theory, Composite Structures 63 (2004) 361-373.

[20] I.T. Oden, I.N. Reddy, Variational Methods in Theoretical Mechanics, Springer Verlag, 1976.