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Sensitivity analysis in life prediction of extrusion dies

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Analysis and modelling

ABSTRACT

Purpose: Building up on the fracture mechanics (Paris law for crack propagation) based fatigue life prediction model developed earlier by the authors, Monte Carlo simulation has been performed to evaluate sensitivity of die life related to important geometrical and material parameters. Stochastic nature of various fatigue-related die parameters is used to reflect their variability.

Design/methodology/approach: Life of the die is one of the most important factors affecting productivity and profitability in hot extrusion of metals. It has been reported in earlier works by the authors that extrusion dies most often fail by fatigue fracture. Experimental studies have shown that cracks preexist in dies due to various factors including heat treatment, machining, and surface hardening. High levels of repeated mechanical and thermal loads result in crack propagation leading to ultimate fracture failure.

Findings: Findings of the sensitivity analysis are that fracture life of an extrusion die is very sensitive to initial crack size, section thickness, profile outer diameter and billet length; moderately sensitive to Paris constant and extrusion ratio; and only slightly sensitive to fracture toughness and ram speed.

Practical implications: The study can be of direct utility in extrusion die design improvement, formulation of an optimum die replacement strategy, etc.

Originality/value: The paper provides basis for a deeper understanding of the factors responsible for fracture failure of an extrusion die exposed to thermo-mechanical fatigue environment.

Keywords: Sensitivity analysis; Die life; Fracture failure; Extrusion die; Monte Carlo simulation

1. Introduction

Life of an extrusion die is of prime importance in a commercial setup. Failure of dies and affiliated tooling is a complex phenomenon, and is not easy to control or contain. However, optimum die replacement and thus reduction in tool warehousing can be achieved through good *failure prediction*. In a previous study conducted by the authors [1], it was found that the most common mode of extrusion die failure is fatigue fracture. Repeated cycles of billet by billet loading-deformation-unloading in extrusion create an environment of thermo-mechanical fatigue. Also, most of the commercial extrusion profiles have complex geometries, giving rise to stress concentrations. Fracture is thus the principal failure mode for extrusion dies and tooling.

In another work [2], the authors developed a life prediction model for extrusion dies based on fracture mechanics. The stochastic nature of fatigue-related die parameters was correlated to die life, and Monte Carlo simulation was used to predict the life distribution of a die under given manufacturing conditions and mechanical properties. Various standard probability distributions were then fitted to the simulated values of die life, Weibull fit being the most accurate. Compared with actual life data from the industry, the simulated life was found to be quite realistic.

Building up on the previous study, the current paper reports results of a sensitivity analysis when using the Paris-Erdogan crack-growth model for die life prediction. The knowledge of how sensitive die failure is to various material properties and geometrical features can be of great interest and direct value to die designers and manufacturers, and to commercial extrusion practitioners.

Actual fracture failure data for a tube die (simple hollow profile) were collected from a commercial aluminum extrusion setup. Geometry of this die profile is described by an outer dia (D_o) of 25.4 ± (0.2, 0.1) mm and thickness (t) of 16 ± (0.15, 0.1) mm. Die material was heat treated and surface hardened H13 steel, while billet material was aluminum alloy Al-6063. Average extrusion temperature was around 460°C, and ram speed was 5 mm/s. The average die life or mean time to failure (*MTTF*) was 704 extrusion cycles (billets extruded).

2. Crack initiation and growth

Typically, there are preexisting cracks in extrusion dies and tooling due to manufacturing operations such as spark erosion. Crack depths of such cracks are typically about 0.01 mm. Later, during surface hardening (such as nitriding) of the bearing area, small cracks of the order of 0.05 mm may also be formed in H13 tool steels [3]. These existing flaws usually get enlarged during heat treatment sequences (austenitizing, tempering, quenching, etc) carried out to achieve optimum hardness and toughness in the die [4]. When such dies are used for extrusion, these initial cracks (depth of 0.05 to 0.1 mm) start to grow in a fracture mechanics pattern, and failure often takes place due to crack propagation to a critical value.

In fracture mechanics, crack growth rate (*da/dN*) is plotted on a log-log scale against applied stress intensity range (ΔK), a being the crack size and N being the number of cycles to fracture failure. The first part of this curve is the *crack initiation* or *Region-I* crack growth. During *Region-II crack growth*, a crack grows gradually and subcritically. Finally, when the fatigue crack has grown to a critical size, it starts to grow in an unstable fashion, quickly terminating in fracture failure; *Region-III growth*. During *Region-II*, fatigue crack growth data plot as a straight line over some interval (*Stage-II*) for most metallic materials. The famous Paris law (proposed by Paris and Erdogan) can be written as:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

Values of the Paris constants C and m for a number of metals has been reported by Sanford [5] and Osgood [6].

3. Fatigue life (cycles to failure)

Neglecting the finite size factor $f\,(a/W)$ for simplicity, the applied stress intensity range can be expressed as

$$\Delta K = \alpha \Delta\sigma \sqrt{\pi a}\,, \tag{2}$$

where α is the geometry factor for the crack type [5]. Since each extrusion cycle starts from a minimum load of zero, $\Delta\sigma = \sigma_{max} - \sigma_{\min} = \sigma_{max.}$ Substituting equation (2) into (1), rearranging, and integrating, we can obtain the fatigue life (number of cycles to failure):

$$N_f = \frac{(a_0)^{1-m/2} - (a_c)^{1-m/2}}{C(m/2-1)\alpha^m \pi^{m/2} {\sigma_{\max}}^m}\,. \tag{3}$$

As H13 tool steel falls under the ultrahigh strength steel category, values of C and m can be found for such steels from standard references. Size of preexisting cracks (a_0) in heat treated and surface hardened H13 steel (extrusion dies), as mentioned above, is generally in the 0.05-0.1 mm range. Assuming the crack type to be an edge crack, value of the geometry factor (α) is 1.12.

Using the standard definition of mode-I stress intensity factor, neglecting the finite-size factor $f\,(a/W)$, and knowing that the crack becomes unstable ($a = a_c$) when $K_I = K_{IC}$, we get

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{\alpha\sigma_{\max}}\right)^2. \tag{4}$$

To find the maximum stress for the simple tube die, we treat it as a thick-walled cylinder with internal pressure [7]:

$$\sigma_{\max} = p\left(\frac{{r_o}^2 + {r_i}^2}{{r_o}^2 - {r_i}^2}\right). \tag{5}$$

Here, r_o and r_i are the outer and inner radii of the tube. To simplify the expression for extrusion pressure (p), we take into consideration friction at the billet-container interface but neglect the relatively small billet-die friction [8]:

$$p = \overline{Y}_f\left(\varepsilon + \frac{2L}{D_b}\right). \tag{6}$$

Here, L is the billet length remaining to be extruded, D_b is the billet diameter, and $\overline{Y}_f$ is the average flow stress of the billet material. True strain is given by

$$\varepsilon = \ln R\,. \tag{7}$$

Extrusion ration R for a multi-cavity die (n_1 being the number of cavities) can be expressed as

$$R = \frac{A_b}{n_1 A_s} = \frac{{D_b}^2}{n_1({d_o}^2 - {d_i}^2)}\,. \tag{8}$$

Of course, maximum pressure occurs at the beginning of the extrusion process, when the whole billet length remains to be extruded ($L = L$, $p = p_{max}$ in equation (6)).

Flow stress of the material can be evaluated from

$$\overline{Y}_f = \overline{\sigma} = \overline{\sigma}_0\left(\frac{\dot{\overline{\varepsilon}}}{\dot{\varepsilon}_0}\right)^{m*}, \tag{9}$$

where $\overline{\sigma}_0$ is the known flow stress at a known average strain rate $\dot{\bar{\varepsilon}}_0$. A typical value of the exponent m^* at 500°C for Al-Mg-Si alloy (the category to which 6xxx alloys of aluminum belong) is 0.125 [9]. At an average strain rate of 50 s^{-1}, the average flow stress for Al-6063 was found to be around 40 MPa using the graph developed by Sheppard et al. [10]. The average strain rate can be found from

$$\dot{\bar{\varepsilon}} = \frac{6V}{D_b}\ln R\,, \tag{10}$$

where V is the ram speed during extrusion [11].

4. Material properties; geometric data

As the geometric parameters and material properties of an extrusion die are random variables, die life is largely probabilistic in character. In this study, mean (μ) and standard deviation (σ) values for profile dimensions are derived from tolerances specified on manufacturer's drawing, and those for initial crack size are based on studies about preexisting cracks due to spark erosion in tool steels [3]. Using typical statistical process control (SPC) guidelines, data is assumed to be spread within ± 3σ limits of the mean. Values of μ and σ and the Weibull parameters m and θ for plane strain fracture toughness are based on variation in K_{IC} values of tool steels as reported in references such as ASM [12] and tool steel manufacturers. Optimum heat treatment (to obtain the best combination of hardness and toughness) for H13 steel being used for hot work dies is tempering to around 550°C [4, 12]. As H13 steels are optimally heat treated to yield the desired combination of hardness and toughness, use of room-temperature values of K_{IC} is not advisable. In this paper, therefore, the correlation developed earlier by the authors [13] is used for determination of K_{IC} value of H13 steels at a given operating temperature, in terms of hot hardness $HRC(T)$ and hot impact energy $CVN(T)$:

$$\frac{K_{IC}(T)}{HRC(T)} = 2.39\left(\frac{CVN(T)}{HRC(T)}\right) + 0.17\,. \quad (\text{MPa}\sqrt{\text{m}},\ \text{J}). \tag{11}$$

Mean, variance, and lognormal parameter (y_0 and ω) values of the Paris constant C of the die material are estimated from spread of crack growth data for tool steels [14]. The other quantities (Paris constant m, geometry factor α, flow stress exponent m^*, ram speed V, number of cavities n_1) have been treated as constants. Information about the distribution type, average values, standard deviations, etc of all the variables is listed in Table-1

5. Monte Carlo simulation

In the previous paper [2], Monte Carlo simulation was used to generate 10,000 instances of fracture life of an extrusion die, and then to determine the average simulated die life and its probability distribution. Independent random numbers were generated and transformed into the required statistical distributions through appropriate transformations for each of the independent variables, such as a_0, D_b, t, L, C, and K_{IC}. The derived variables (a_c, σ_{max}, p, ε, etc) were calculated for all the 10,000 instances. Finally, cycles to failure (number of billets) due to fatigue fracture were calculated using equation (3). Compared to an actual *MTTF* of 704 billets, the simulated average die life came out to be 773 cycles (less than 10% error), verifying the validity of the approach.

Table 1.
Geometrical and material variables, their distributions and values

Variable	Distribution	Mean Value	Std Deviation
Billet dia D_b	Normal	184 mm	0.5 mm
Billet length L	Normal	660 mm	4 mm
Die outer dia d_o	Normal	25.4 mm	0.1 mm
Die thickness t	Normal	1.6 mm	0.05 mm
Paris constant C	Lognormal	1.6 x 10^{-12} (y_0 = 1.493 x 10^{-12})	12% of mean (ω = 0.229)
Paris exponent m	Constant	2.85	-
Fracture toughness K_{IC}	Weibull	83.6 MPa√m θ = 89.6 MPa√m	15% of mean m = 6.67
Ram speed V	Constant	5 mm/s	-
Initial crack size a_0	Normal	0.01 mm	10% of mean
Geometry factor α	Constant	1.12	-
Number of cavities n_1	Constant	4	-

6. Sensitivity analysis

Though stringent control on material quality is targeted, material properties of the billet and die show significant variation in real industrial practice. Similarly, even in the face of very tight tolerance control on profile dimensions, routine practices in die maintenance and die correction result in notable parameter variation. Also, a medium-sized extrusion plant would customarily warehouse around 10,000 different die profiles. It can offer great insight into the study of die failure and die design improvement if we can estimate how the various material and geometrical factors affect die life. Based on the strategy of die life simulation outlined above, a sensitivity analysis was therefore conducted. Variation of die life was calculated for a ± 10% variation around the mean value of each variable listed in Table-1. Results of this sensitivity exercise are shown graphically in Fig-1 and Fig-2. A numerical value of sensitivity of die life to each parameter was also calculated, using the formula:

$$Sensitivity = \frac{\Delta N_f / \overline{N}_f}{\Delta x / \overline{x}}\,. \tag{12}$$

Here, Δx is the total variation in the parameter of interest, $\overline{x}$ is the mean value for that parameter (as given in Table-1), ΔN_f

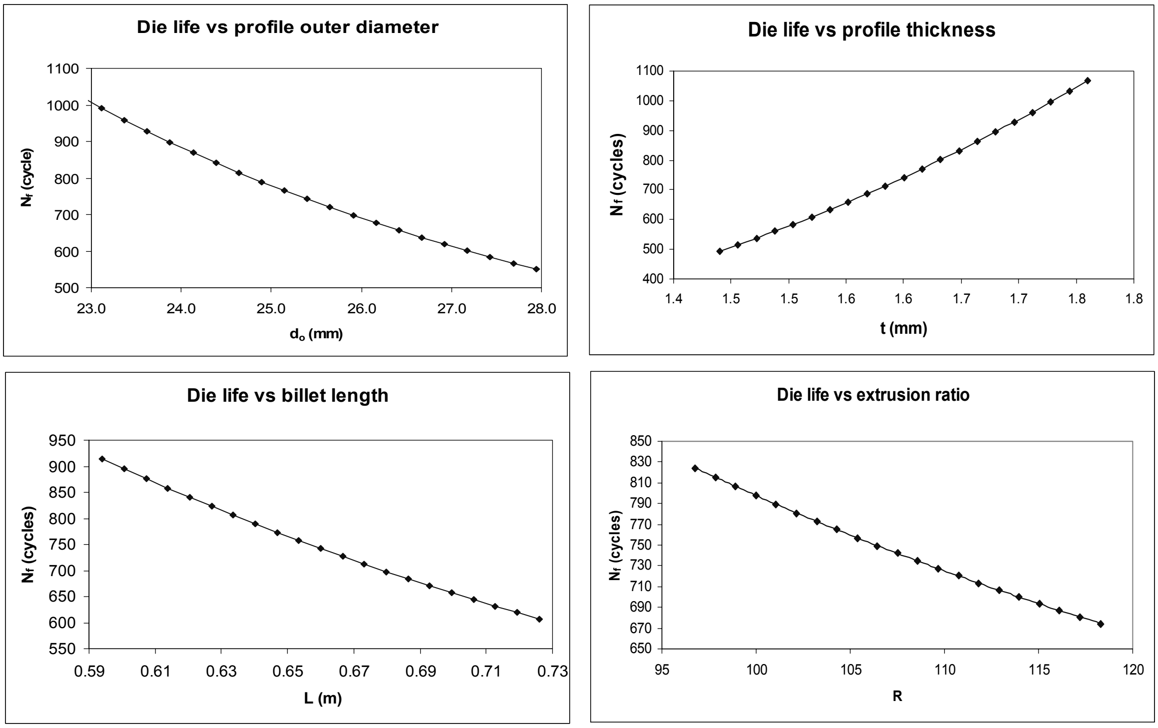

Fig. 1. Variation of die life against geometrical parameters of the die profile

is the variation in die life obtained from the simulation, and $\overline{N}_f$ is the predicted average die life when all parameters take on their mean values. For instance, die life sensitivity for variation in the initial crack size will be given by

$$Sensitivity = \frac{\Delta N_f / \overline{N}_f}{\Delta a_0 / \overline{a}_o}\ . \qquad (13)$$

Linear, exponential, logarithmic, and power-law curve fittings were also carried out for these graphs. The consistently best fit for all graphs was the one based on the power-law model. This is only natural, as the Paris law assumes a power-law relationship between crack growth (leading to fracture failure) and applied stress intensity.

Using the approach described above, die life sensitivity values against the different parameters of interest are listed below:

D_o	T	L	R	K_{IC}	a_0	C	V
- 3.2	3.87	-2.07	-1.01	0.54	-4	-1.01	-0.42

Some really interesting observations can be made from these graphs and calculated results. Apart from profile thickness and plane-strain fracture toughness of the die material, all other variables show an inverse relationship with die life. This is borne out by theory as well as years of industrial extrusion practice. Extrusion pressure rapidly increases as *wall thickness* (t) of the extruded profile decreases, higher pressures causing earlier failures; thus the higher die life for larger profile thickness. *Fracture toughness* (K_{IC}) of the material is the direct indication of its resistance to fracture failure, so a higher toughness value for the die material would indicate longer die life. Extrusion pressure is directly proportional to the *circumscribing circle diameter* (*CCD*) of the die profile. In the case of a tube die, *CCD* is the same as the outer diameter of the die (D_o); larger *CCD* (or D_o), larger pressure, earlier die fracture. The longer the billet used for extrusion, the higher is the work required to overcome billet-container friction; thus larger *billet length* would lead to shorter die life. *Extrusion ratio* (R) is the ration of the billet area to the cross-sectional area of the extruded profile. Larger extrusion ratio requires larger extrusion pressure, thus the decreased die life. Fracture failure occurs because a crack of an existing size (a_0) grows to an unstable (or critical) size under fatigue loading. Larger *initial crack size* would obviously lead to earlier fracture. *Paris constant* (C) of the die material is the slope of the da/dN – ΔK line (on a log-log scale). Larger values of C mean a sharper slope of this line, leading to a faster crack-growth rate, and thus a shorter fracture life. Higher *ram speed* in the extrusion press means a higher strain rate, creating higher pressures, and therefore leading to earlier die failure.

Another significant observation will be the slopes of the sensitivity graphs, or the sensitivity values calculated. Fracture life of the die appears to be most sensitive to changes in initial crack size (sensitivity value of 4) and section thickness of the profile (value of 3.87). The first observation highlights the importance of material quality and stringent control of the manufacturing and subsequent surface and heat treatment of the die, as these are the factors that directly affect how large the initial cracks in the die will be. The second observation provides an important guideline to die designers; very thin sections in the die profile should be avoided as much as possible.

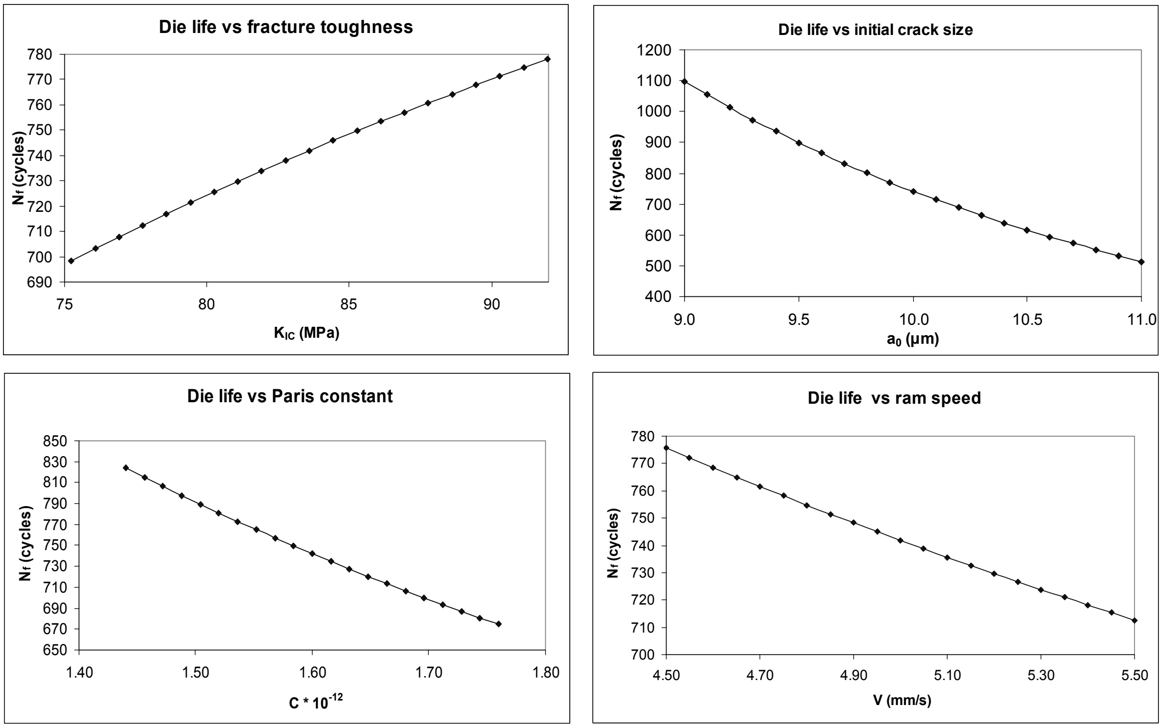

Fig. 2 Variation of die life against fracture properties of die material and ram speed

Die life also seems to be quite sensitive to profile outer diameter (3.2) and billet length (2.07). Profile outer diameter comes straight from the section size requirement of a particular application, and so there is not much the designer can do about it. On the other hand, billet length can be definitely controlled, and the issue is quite interesting. As indicated by this study, smaller billet lengths will lead to longer die life. However, smaller billets mean more billets will be required for a certain extruded length, and this in turn will require more billet-on-billet loading in the press. This may decrease productivity (due to increased downtime) and increase the chances for billet-to-billet weld defects. A judicious approach (often tempered by real industrial experience) is thus required to decide an optimal billet length.

Sensitivity of die life to Paris constant (1.01) and extrusion ratio (1.01) shows an almost inverse linear relationship. For a given die profile, extrusion ratio can be reduced by using billets of a smaller diameter. This can be done by using a press with a smaller container size (to hold the smaller-dia billets). However, this may not be practical if other profiles to be extruded are of a larger dia. Another method of reducing the extrusion ratio, and a much more practical and economical one (followed by most of the commercial extrusion facilities) is to have several cavities (of the same profile) in a single die. This way, we get a far lower extrusion ratio, and obtain a several-fold increase in productivity at the same time. Paris constant of the die material may be a contentious issue. Almost no studies can be found in published literature on Paris-law based crack growth of tool and die steels. The most comprehensive work on crack growth of various steels (including ultra-high strength steels) is the program NASA/NASGRO, reported by Sanford [5]. Unfortunately, it uses a four-parameter fatigue crack growth model that is somewhat different from the Paris model, and the *C*-value in the two models is not the same.

The lowest sensitivity that die life exhibits is to fracture toughness (0.54) of the die material and ram speed (0.42) of the extrusion press. These two factors definitely affect fracture life, but to a significantly lower level than the other parameters. In the case of fracture toughness, this observation may be misleading. It is a well-know fact that fracture toughness increases at higher temperatures. As commercial extrusion is usually done at quite high temperatures (around 460ºC in the case of aluminium alloys), fracture toughness of the die material automatically increases during the process as compared to its room-temperature value. This inhibits crack propagation to some extent, and thus the apparent lower sensitivity of die life to fracture toughness. Similarly, lower sensitivity of die failure to ram speed may wrongly encourage the engineer to arbitrarily increase the ram speed, with a view to increasing productivity without causing any serious damage to the die. The other side of the coin is that higher ram speeds may cause extrusion defects such as surface cracking and unacceptable roughness. A prudent balance between productivity and extrusion quality has to be maintained through an optimum ram speed.

7. Conclusions

Based on the Paris model of crack growth, Monte Carlo simulation of the fracture life of an extrusion die (tube profile) has been carried out. After validation of the simulation results by comparison with actual failure data (average die life) from industry, the method is used to perform a sensitivity analysis in terms of the geometrical and material parameters involved. Fracture life of the die increases with increasing profile thickness (t) and fracture toughness (K_{IC}) of the die material, while it decreases with increasing profile outer diameter (d_o), billet length (L), extrusion ratio (R), ram speed (V), initial crack size (a_0), and Paris constant (C) of the die material. Die life is highly sensitive to changes in a_0 and t, very sensitive to d_o and L, moderately sensitive to C and R, and slightly sensitive to K_{IC} and V.

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