

## Comparison of methods of reduction of vibrations

**K. Białas \***

Institute of Engineering Processes Automation and Integrated Manufacturing Systems, Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland  
\* Corresponding author: E-mail address: katarzyna.zurek@polsl.pl

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### Analysis and modelling

#### ABSTRACT

**Purpose:** In this work there are presented basic methods of reverse task of active and passive mechanical systems realization. The principal aim of the research taken is to work out a method of structure and parameters searching i.e. structural and parametric synthesis of discrete model of mechanical system on the base of desired requirements. The requirements refer to dynamic features of the system, particularly their frequency spectrum. Purpose of work is also comparison of active and passive method reduction of vibrations.

**Design/methodology/approach:** In this work used unclassical method of polar graphs and their relationship with algebra of structural numbers. This method enables analysis without limitations depending on kind and number of elements of complex mechanical system using electronic calculation technique. Using method makes also possible designing systems including active or passive elements reducing of vibrations.

**Findings:** Introduced in this paper approach adopted makes it possible to undertake actions aiming at the elimination of phenomena resulting in the unwanted operation of machinery or generation of hazardous situations in the machinery environment.

**Research limitations/implications:**

**Practical implications:** The results represented this work in form of polar graphs extend the tasks of synthesis to other spheres of science e.g. electric systems. The practical realization of the reverse task of dynamics introduced in this work can find uses in designing of machines with active and passive elements with the required frequency spectrum.

**Originality/value:** Thank to the approach, unclassical method of polar graphs and their relationship with algebra of structural numbers, can be conducted as early as during the designing of future functions of the system as well as during the construction of the system. Using method and obtained results can be value for designers of mechanical systems.

**Keywords:** Process systems design; Polar graphs; Structural numbers; Reduction of vibrations

### 1. Introduction

There are many methods of reducing undesirable vibration of mechanisms. The implementation of active elements into the disposal of vibration offers the possibility to overcome the restrictions of the methods of passive disposal of vibration, such as, in particular, the impossibility to reduce the vibration of specific parts of machinery and low efficiency in case of low-frequency vibration. The major partition is that into passive and

active methods of reducing vibration and active and passive forms of their execution. The term of passive methods of reducing vibration of machinery pertains to such additional constructional elements of vibroisolation systems which do not constitute integral elements of a machine structure but are implemented additionally by way of dissemination of mechanical vibration signals such as mechanical filters of these signals. The low-frequency character of vibration may result in the imperfection of passive vibroisolation to ensure efficient reduction of vibration or

may even lead to the enhancement of vibration and that is why in such chance, the active reduction of vibration often replaces the passive one. The term of active methods of reducing vibration usually refers to all these subsystems, whose purpose is to reduce or eliminate reasons for formation of machinery vibration i.e. the subsystems interfering in the sources of generation of such vibration. A characteristic feature of the active vibration reduction is the fact, that vibration is compensated by interaction from additional sources. Active vibroisolation systems are controlled by introduce function. Passive and active measures of vibration reduction may take passive and active forms [1÷6].

In the methods of active reduction of vibration are distinguished controlling or adjusting processes of mechanical vibration. The controlling of the motion of an object pertains to a situation when a command signal is supplied to the system from the outside and this signal does not depend on the current condition of the object but is previously designed. The adjusting processes of mechanical vibration consists in the adjustment of an object motion and in such a case a command signal depends on the current condition of the object and it is necessary to implement additional items such as output sensors, a control unit and executive devices [1].

## 2. The system of the research

In the case of the problem of reducing the vibration of mechanical system, it is necessary to execute the synthesis or identification of a system [6÷14,17]. Depending, on a structure, parameters and input functions affecting the system, to determine the structure of a system containing active or passive elements (fig. 1, fig.2).

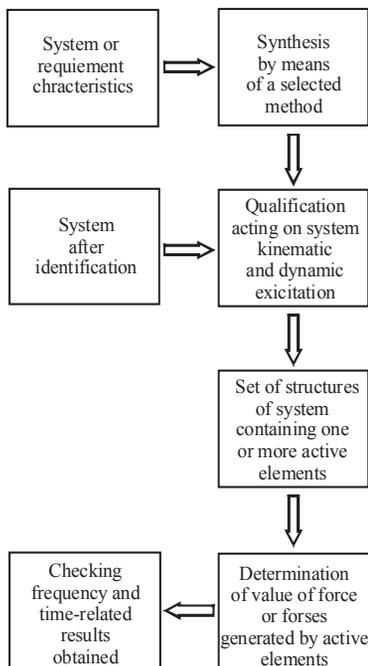


Fig. 1. Idea of synthesis of active mechanical systems

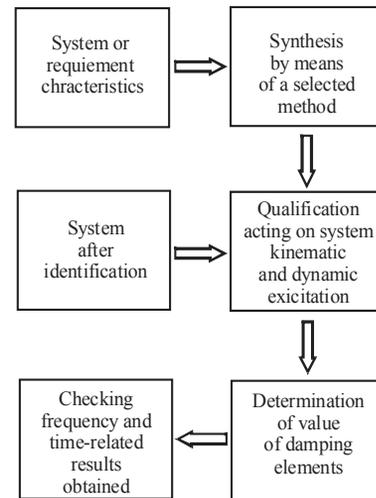


Fig. 2. Idea of synthesis of mechanical systems with damping elements

### 2.1. Synthesis by means of a selected method

The required frequency spectrum:

$$\begin{cases} \omega_1 = 6 \frac{rad}{s}, & \omega_3 = 12 \frac{rad}{s}, & \omega_5 = 18 \frac{rad}{s}, \\ \omega_0 = 0 \frac{rad}{s}, & \omega_2 = 9 \frac{rad}{s}, & \omega_4 = 15 \frac{rad}{s}. \end{cases}$$

Because system in result of synthesis has to be received from excitation kinematic then function subjected synthesis should be as follows:

$$V(s) = H \left( \frac{s(s^2 + \omega_2^2)(s^2 + \omega_4^2)}{(s^2 + \omega_1^2)(s^2 + \omega_3^2)(s^2 + \omega_5^2)} \right) \tag{1}$$

The structure and parameters of system in this case of schedule of function on chain fraction were received.

$$V(s) \frac{1}{H} = \frac{s}{c_1} + \frac{1}{m_1s + \frac{1}{\frac{s}{c_2} + \frac{1}{m_2s + \frac{1}{\frac{s}{c_3} + \frac{1}{m_3s}}}}} \tag{2}$$

$$V(s) \frac{1}{H} = \frac{s}{198} + \frac{1}{1s + \frac{1}{\frac{s}{77} + \frac{1}{0.75s + \frac{1}{\frac{s}{32} + \frac{1}{0.38s}}}}} \tag{3}$$

On basis of expansion of function receives the polar graph (fig.3).

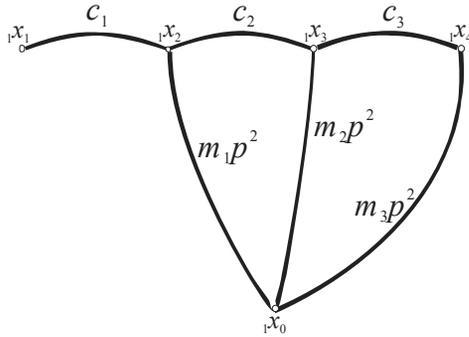


Fig. 3. Polar graph

Excitation kinematic:

$$F_k = c_1 y \sin \omega t \tag{4}$$

If is kinematic excitation then graph accepts form how in figure 4.

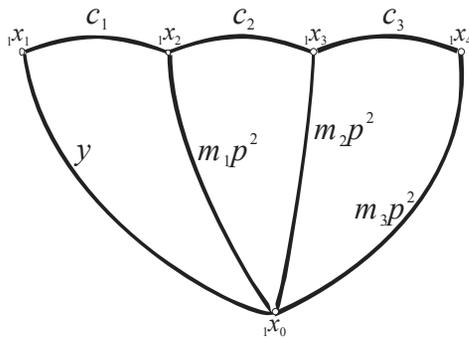


Fig. 4. Polar graph of the systems with excitation kinematic

After representation system looks as follows (fig.5):

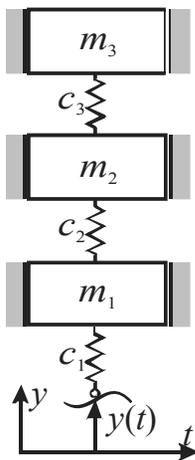


Fig. 5. The model of the system

## 2.2. Analysis of system

In order to determine the amplitudes of vibration of the system in question it is necessary to use the interdependence between graphs and structural numbers [6,15-20]. The amplitudes of vibrations are introduced in fig.6-8. In the said case they are as follows:

$$A_1 = \frac{\frac{\partial D(\omega)}{\partial [1]} [7] [4]}{D(\omega)} \tag{5}$$

$$A_1 = \frac{(m_2 m_3 \omega^4 - \omega^2 (m_2 c_3 + m_3 c_2 + m_3 c_3) + c_2 c_3) F_k}{-m_1 m_2 m_3 + \omega^2 (m_1 m_2 c_3 + m_1 m_3 c_2 + m_1 m_3 c_3 + m_2 m_3 c_1 + m_2 m_3 c_2) - \omega^2 (m_1 c_2 c_3 + m_2 c_1 c_3 + m_2 c_2 c_3 + m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3) + c_1 c_2 c_3} \tag{6}$$

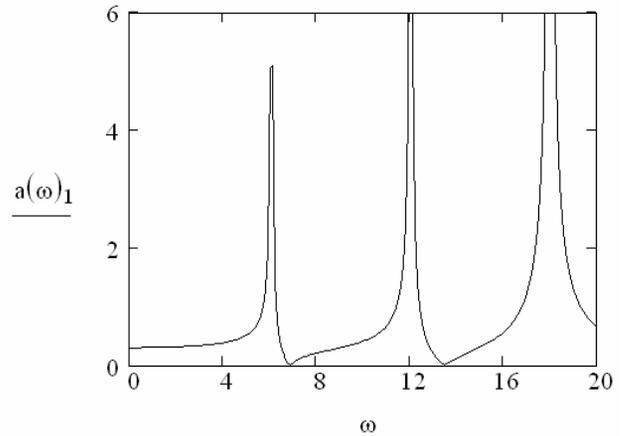


Fig. 6. Diagram of A<sub>1</sub> amplitude

$$A_2 = \frac{Sim \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [2]} \right) [7] [4]}{D(\omega)} \tag{7}$$

$$A_2 = \frac{(-m_3 c_2 \omega^2 + c_2 c_3) F_k}{-m_1 m_2 m_3 + \omega^2 (m_1 m_2 c_3 + m_1 m_3 c_2 + m_1 m_3 c_3 + m_2 m_3 c_1 + m_2 m_3 c_2) - \omega^2 (m_1 c_2 c_3 + m_2 c_1 c_3 + m_2 c_2 c_3 + m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3) + c_1 c_2 c_3} \tag{8}$$

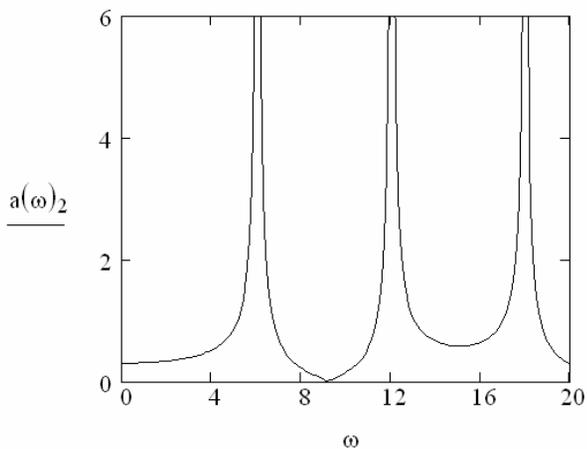


Fig. 7. Diagram of  $A_2$  amplitude

$$A_3 = \frac{Sim_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [3]} \right) [7] [4]}{D(\omega)} \quad (9)$$

$$A_3 = \frac{(c_2 c_3) F_k}{-m_1 m_2 m_3 + \omega^2 (m_1 m_2 c_3 + m_1 m_3 c_2 + m_1 m_3 c_3 + m_2 m_3 c_1 + m_2 m_3 c_2) - \omega^2 (m_1 c_2 c_3 + m_2 c_1 c_3 + m_2 c_2 c_3 + m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3) + c_1 c_2 c_3} \quad (10)$$

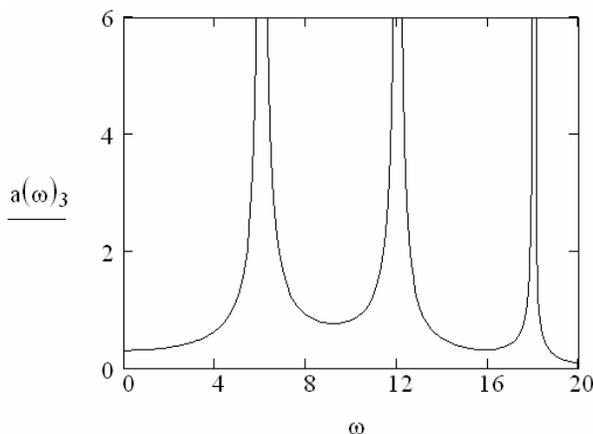


Fig. 8. Diagram of  $A_3$  amplitude

### 3. The model of the research

Discrete vibratory mechanical system is being considered, with dynamic and kinematical input function. The model considered (fig.5) is created from the following elements:

$m = 3$  inertial elements

$c = 3$  elastic elements

$F_k = 1$  kinematic excitation.

In the case of the problem of reducing the vibration of selected parts of a system it is necessary to apply active elements by „locating” them in optionally selected places of the system. That specify such elements, it is necessary to perform the synthesis or identification of a passive system and then, depending on a structure, parameters and input functions affecting the system, to determine the structure of a system containing active elements.

In conformity with the theory of polar graphs and their relation to structural numbers [2], it is possible to determine the values of amplitudes of forces generated by active elements.

A general formula for amplitude value is as follows:

$$A_n = \frac{\left( Sim_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [n]} \right) (F_1 + F_{k1} + G_1) \right) + \left( Sim_z \left( \frac{\partial D(\omega)}{\partial [2]}, \frac{\partial D(\omega)}{\partial [n]} \right) (F_2 + F_{k2} + G_2) \right) + \dots + \left( \left( \frac{\partial D(\omega)}{\partial [n]} \right) (F_w + F_{kl} + G_g) \right)}{D(\omega)} \quad (11)$$

where:

$D(\omega)$  - characteristic equation,

$\frac{\partial D(\omega)}{\partial [1]}$  - derivative of structural number the in relation to of edge [1],

$Sim_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [2]} \right)$  - function of simultaneousness of structural number,

$F_{k1}, F_{k2}, \dots, F_{kl}$  - kinematic excitation,

$F_1, F_2, \dots, F_w$  - dynamic excitation,

$G_1, G_2, \dots, G_g$  - forces generated through active elements.

Systems with active elements reducing vibrations they be introduced in figure 9 (polar graph in fig.10):

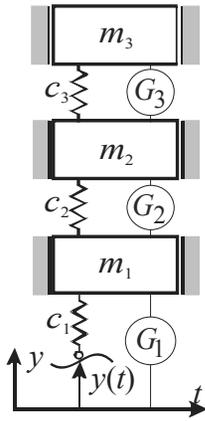


Fig. 9. The models of the system with active elements

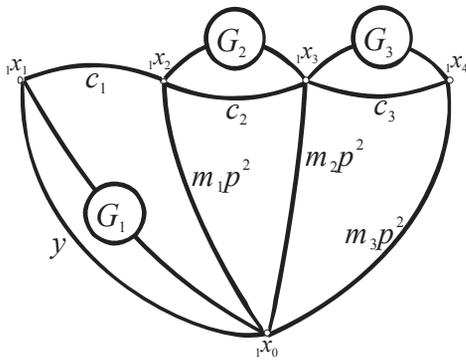


Fig. 10. Polar graph of the systems (fig.9)

$$A_1 = \frac{\frac{\partial D(\omega)}{\partial [1]} [7[4] + \text{Sim}_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [2]} \right) ([9] + [10])}{D(\omega)}}{\text{Sim}_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [3]} \right) ([10])} \quad (12)$$

$$A_1 = \frac{(m_2 m_3 \omega^4 - \omega^2 (m_2 c_3 + m_3 c_2 + m_3 c_3) + c_2 c_3)(F_k + G_1 + G_2) + (-m_1 m_2 m_3 + \omega^2 (m_1 m_2 c_3 + m_1 m_3 c_2 + m_1 m_3 c_3) - m_3 c_2 \omega^2 + c_2 c_3)(G_2 + G_3) + (c_2 c_3)(G_3)}{m_2 m_3 c_1 + m_2 m_3 c_2 - \omega^2 (m_1 c_2 c_3 + m_2 c_1 c_3 + m_2 c_2 c_3) + m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3 + c_1 c_2 c_3} \quad (13)$$

$$A_2 = \frac{\text{Sim}_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [2]} \right) ([7[4] + [8] + [9]) + \frac{\partial D(\omega)}{\partial [2]} ([9] + [10]) + \text{Sim}_z \left( \frac{\partial D(\omega)}{\partial [2]}, \frac{\partial D(\omega)}{\partial [3]} \right) ([10])}{D(\omega)} \quad (14)$$

$$A_2 = \frac{(-m_3 c_2 \omega^2 + c_2 c_3)(F_k + G_1 + G_2)}{-m_1 m_2 m_3 + \omega^2 (m_1 m_2 c_3 + m_1 m_3 c_2 + m_1 m_3 c_3) + (m_1 m_2 \omega^4 - \omega^2 (m_1 c_3 + m_3 c_1 + m_3 c_2) + c_1 c_3 + c_2 c_3)(G_2 + G_3)} \quad (15)$$

$$A_3 = \frac{\text{Sim}_z \left( \frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [3]} \right) ([7[4] + [8] + [9]) + \text{Sim}_z \left( \frac{\partial D(\omega)}{\partial [2]}, \frac{\partial D(\omega)}{\partial [3]} \right) ([9] + [10]) + \frac{\partial D(\omega)}{\partial [3]} ([10])}{D(\omega)} \quad (16)$$

$$A_3 = \frac{(c_2 c_3)(F_k + G_1 + G_2) + (-m_1 c_3 \omega^2 + c_1 c_3 + c_2 c_3)(G_2 + G_3) + (m_1 m_2 \omega^4 - \omega^2 (m_1 c_2 + m_1 c_3 + m_2 c_1 + m_2 c_2) + c_1 c_2 + c_1 c_3 + c_2 c_3)(G_3)}{m_2 m_3 c_1 + m_2 m_3 c_2 - \omega^2 (m_1 c_2 c_3 + m_2 c_1 c_3 + m_2 c_2 c_3) + m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3 + c_1 c_2 c_3} \quad (17)$$

Solving a system of equations (13,15,17) leads to the obtaining of values of individual amplitudes generated by active elements.

At  $\omega = \omega_1$  the values are as follows:

$$G_1 = 359.09 \text{ N},$$

$$G_2 = -45.82 \text{ N},$$

$$G_3 = -131.54 \text{ N}.$$

The amplitudes of vibrations are introduced in fig.11-13.

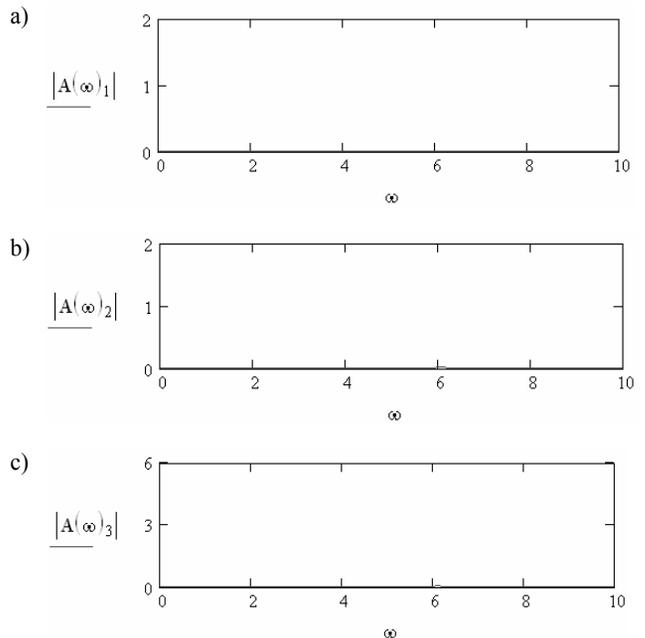


Fig. 11. Diagram of  $A_1, A_2, A_3$  amplitude

At  $\omega=\omega_2$  the values are as follows:

$$G_1 = 897.43 \text{ N},$$

$$G_2 = -973.59 \text{ N},$$

$$G_3 = -541.53 \text{ N}.$$

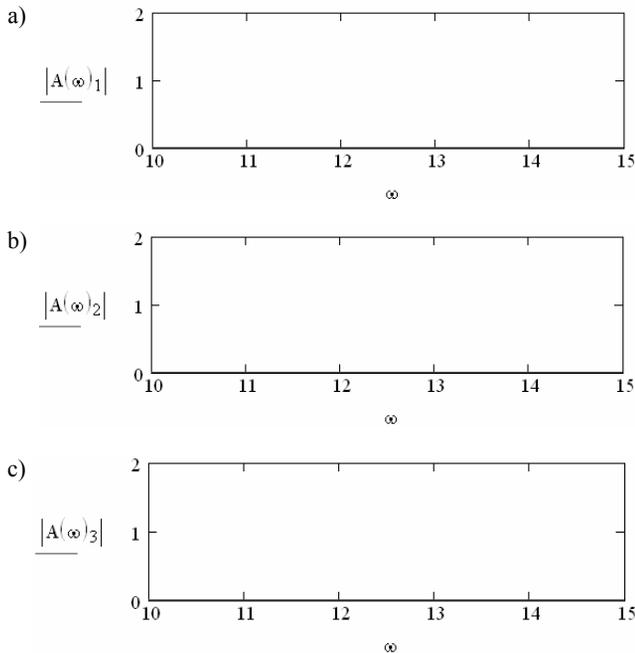


Fig. 12. Diagram of  $A_1, A_2, A_3$  amplitude

At  $\omega=\omega_3$  the values are as follows:

$$G_1 = -3245.09 \text{ N},$$

$$G_2 = -2300.64 \text{ N},$$

$$G_3 = 1075.8 \text{ N}.$$

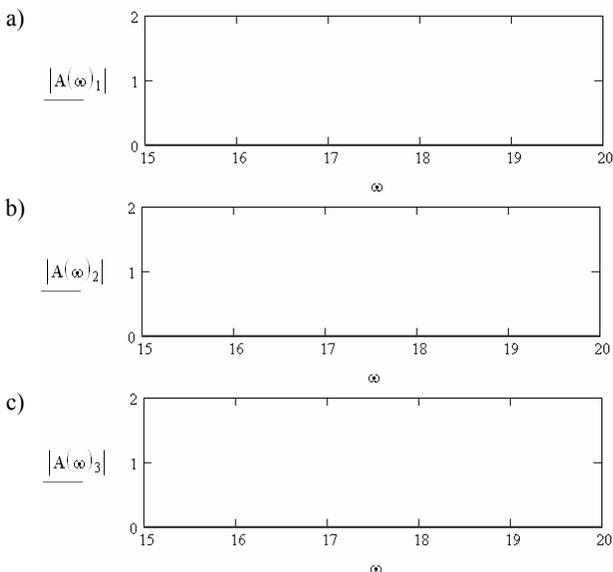


Fig. 13. Diagram of  $A_1, A_2, A_3$  amplitude

In the case of the problem of reducing the vibration of system it is possible to implement passive elements. In order to specify such elements, it is necessary to perform the synthesis or identification of system and then, depending on a structure, parameters and input functions affecting the system, to determine the structure of a system containing passive elements.

A general formula for value of damping [15÷17], when damping is proportional to elastic element, is as follows:

$$b_i = \lambda c_i \tag{18}$$

where:

- $b_i$  - damping elements
- $\lambda$  - modulus of proportionality
- $c_i$  - elastic elements

$$\lambda = 0.01$$

$$b_1 = 1.98 \frac{Ns}{m}$$

$$b_2 = 0.77 \frac{Ns}{m}$$

$$b_3 = 0.32 \frac{Ns}{m}$$

Systems with passive elements reducing vibrations they be introduced in figure 14 (polar graph in fig.15):

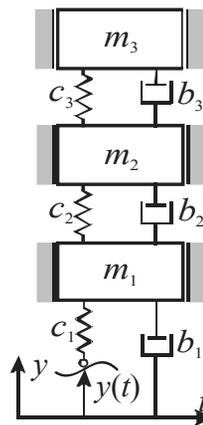


Fig. 14. The models of the system with passive elements

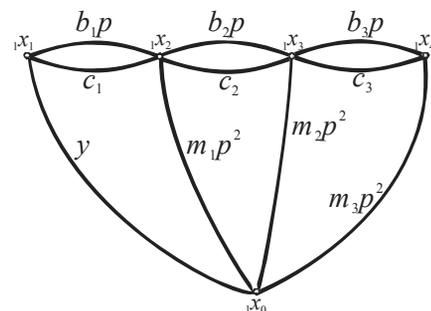


Fig. 15. Polar graph of the systems (fig.14)

$$A_1 = \frac{\frac{\partial D(\omega)}{\partial [1]} [7] [4]}{D(\omega)} \quad (19)$$

$$A_1 = \frac{((c_2 + c_3 - m_2\omega^2 + b_2 + b_3)(c_3 - m_3\omega^2 + b_3)(c_3 - b_3))F_k}{D(\omega)} \quad (20)$$

$$A_2 = \frac{Sim_z\left(\frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [2]}\right) [7] [4]}{D(\omega)} \quad (21)$$

$$A_2 = \frac{((-c_2 - b_2)(c_3 - m_3\omega^2 + b_3))F_k}{D(\omega)} \quad (22)$$

$$A_3 = \frac{Sim_z\left(\frac{\partial D(\omega)}{\partial [1]}, \frac{\partial D(\omega)}{\partial [3]}\right) [7] [4]}{D(\omega)} \quad (23)$$

$$A_3 = \frac{((-c_2 - b_2)(-c_3 - b_3))F_k}{D(\omega)} \quad (24)$$

The amplitudes of vibrations are introduced in fig.16-18.

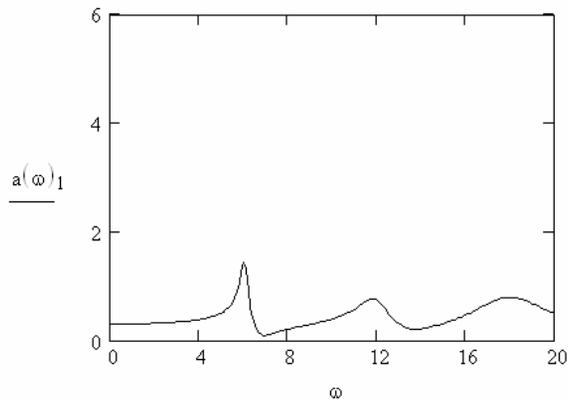


Fig. 16. Diagram of A<sub>1</sub> amplitude

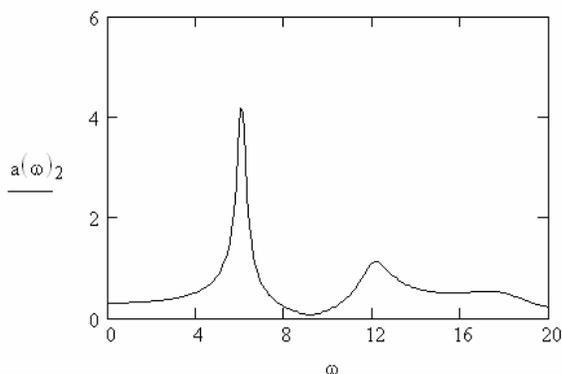


Fig. 17. Diagram of A<sub>2</sub> amplitude

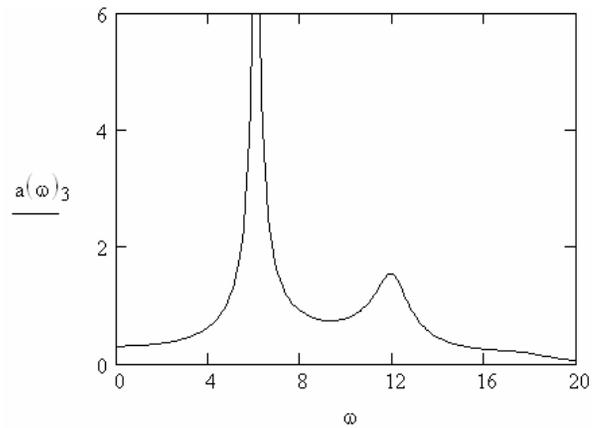


Fig. 18. Diagram of A<sub>3</sub> amplitude

## 4. Conclusions

In this work, the method of polar graphs and their relationships with structural numbers were used in order to derive equations determining the values of amplitudes of forces generated by active elements. The use of such a method enables the automation of calculation during the determination of dynamic characteristics of a system and the algorithmisation of calculations.

Introduced in this work approach adopted makes it possible to undertake actions aiming at the elimination of phenomena resulting in the unwanted operation of machinery or generation of hazardous situations in the machinery environment. Thank to the approach, the aforementioned preventive activities can be conducted as early as during the designing of future functions of the system as well as during the construction of the system in question.

Comparison of passive and active reduction of vibrations of mechanical systems was shown, that active elements give better results than passive elements. On diagrams of amplitudes is visible that the using to reduction of vibration active elements gives completely effects however using passive partially.

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## References

- [1] S. Michałowski, Active systems in machines construction. Cracow University of Technology Press, Monograph 171, Cracow, 1994, (in Polish).
- [2] A. Buchacz, K. Żurek, Reverse task of active mechanical systems depicted in form of graphs and structural numbers. Monograph 81, Silesian University of Technology Press, Gliwice, 2005, (in Polish).

- [3] A. Buchacz, K. Żurek, Selection of active elements reducing vibrations, Proceedings of 8<sup>th</sup> Conference on Dynamical Systems Theory and Applications, Łódź, 2005, 863-868.
- [4] K. Żurek, Design of reducing vibration mechatronical systems. Comment Worldwide Congress on Materials and Manufacturing Engineering and Technology, Computer Integrated Manufacturing, (2005) 292-297.
- [5] K. Białas, Comparison of passive and active reduction of vibrations of mechanical systems. Journal of Achievements in Materials and Manufacturing Engineering 18/1-2 (2006) 455-458.
- [6] K. Białas, Synthesis of mechanical systems including passive or active elements reducing of vibrations, Journal of Achievements in Materials and Manufacturing Engineering 20/1-2 (2007) 323-326.
- [7] A. Buchacz, J. Świder (red.) in., Computer support CAD CAM. Support for construction of systems reducing vibration and machine noise, WNT, Warsaw, 2001, (in Polish).
- [8] A. Buchacz, Hypergraphs and their subgraphs in modelling and investigation of robots. Journal of Materials Processing Technology 157-158 (2004) 37-44.
- [9] A. Buchacz, The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs. Journal of Materials Processing Technology 164-165 (2005) 1277-1280.
- [10] A. Buchacz, Modifications of cascade structures in computer aided design of mechanical continuous vibration bar systems represented by graphs and structural numbers. Journal of Materials Processing Technology 157-158 (2005) 45-54.
- [11] A. Buchacz, Influence of piezoelectric on characteristics of vibrating mechatronical system. Journal of Achievements in Materials and Manufacturing Engineering 17 (2006) 229-232.
- [12] A. Buchacz, Sensitivity of mechatronical systems represented by polar graphs and structural numbers as models of discrete systems. Journal of Materials Processing Technology 175 (2006) 55-62
- [13] A. Buchacz, A. Wróbel, Piezoelectric layer modelling by equivalent circuit and graph method, Journal of Achievements in Materials and Manufacturing Engineering 20 (2007) 299-302.
- [14] A. Buchacz, S. Żółkiewski, Dynamic analysis of the mechanical systems vibrating transversally in transportation, Journal of Achievements in Materials and Manufacturing Engineering 20 (2007) 331-334.
- [15] A. Dymarek, Reverse task of damping mechanical systems depicted in form of graphs and structural numbers. Doctoral thesis, Silesian University of Technology, Gliwice, 2000.
- [16] A. Dymarek, T. Dzitkowski, Modelling and synthesis of discrete-continuous subsystems of machines with damping, Journal of Materials Processing Technology 164-165 (2005) 1317-1326.
- [17] T. Dzitkowski, A. Dymarek, Synthesis and sensitivity of machine driving systems. Journal of Achievements in Materials and Manufacturing Engineering 20 (2007) 359-362
- [18] A. Sękała, J. Świder, Hybrid graphs in modelling and analysis of discrete-continuous mechanical systems, Journal of Materials Processing Technology 164-165 (2005) 1436-1443.
- [19] G. Wszolek, Modelling of mechanical systems vibrations by utilisation of GRAFSIM software, Journal of Materials Processing Technology 164-165 (2005) 1466-1471.
- [20] J. Świder, G. Wszolek, K. Foit, P. Michalski, S. Jendrysik, Example of the analysis of mechanical system vibrations in GRAFSIM and CATGEN software, Journal of Achievements in Materials and Manufacturing Engineering 20 (2007) 3919-394.