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Synthesis of mechanical systems including passive or active elements

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Analysis and modelling

ABSTRACT

Purpose: In this paper there are presented basic methods of synthesis of mechanical systems including active and passive elements. The major aim of the research is to work out a method of structure and parameters searching i.e. structural and parametric synthesis of a discrete model of mechanical system on the base of desired requirements. The requirements pertain to dynamic features of the system, particularly its frequency spectrum. The purpose of this paper is also comparison of reduction of vibrations of mechanical systems by use the passive or active elements.

Design/methodology/approach: In this article was used unclassical method of polar graphs and their relationship with the algebra of structural numbers. This method enables analysis without limitations depending on kind and number of elements of complex mechanical system using electronic calculation technique.

Findings: Use of active elements into the elimination of vibration offers the possibility to overcome the limitations of the methods of passive elimination of vibration, such as, in particular, low efficiency in case of low-frequency vibration.

Research limitations/implications: The scope of discussion is synthesis of passive and active mechanical systems, but for this type of systems, such approach is sufficient.

Practical implications: The results represented this work in form of polar graphs extend the tasks of synthesis to other spheres of science e.g. electric systems. The practical realization of the reverse task of dynamics introduced in this work can find uses in designing of machines with active and passive elements with the required frequency spectrum.

Originality/value: Thank to use an unclassic method of polar graphs and their relationship with the algebra of structural numbers, can be possible correcting systems as early as during the designing of future functions of the system as well as during the construction of the system.

Keywords: Process systems design; Polar graphs; Structural numbers; Synthesis

1. Introduction

Implementation the condition of vibration reduction into the set of constructional criteria substantially extends the scope of knowledge and qualifications required from designers and constructors. Designers, manufacturers and users have to face problems of preventing undesired effects in the operation of newly designed machinery or adapting already manufactured and operating machines to meet requirements resulting from current knowledge of hazards caused by machinery [1-6].

Many methods exist to preventing excessive vibration of machinery elements. It is possible to divide them into passive and active measures of reducing vibration and active and passive forms of their execution. The application of active elements to eliminate vibration enables overcoming limitations which occur if passive elements are used. One of the most important limitations is low efficiency in case of low-frequency vibration and inability to reduce vibration of selected parts of the system as well as inability of passive systems to respond to changes occurring in the system. The low-frequency character of vibration may result in the failure of passive vibroisolation to ensure efficient reduction of vibration or may even lead to the increase of vibration and that is why in such cases, the active reduction of vibration often replaces the passive one. A characteristic feature of the active vibration reduction is the fact, that vibration is compensated by interaction from additional sources. The methods of active reduction of vibration are divided into controlling or adjusting processes of mechanical vibration [1].

2. Synthesis of mechanical systems

To solve the problem of reducing the vibration of mechanical system, it is necessary to execute the synthesis or identification of a system. Depending, on a structure and parameters as well as input functions affecting the system, to appoint the structure of a system containing active or passive elements (Fig. 1).

Mechanical systems can be described at using dynamic characteristics in form of dynamic slowness and mobility [2,7-12, 15-17], about following figures:

$$U(s) = H\frac{d_l s^l + d_{l-1}s^{l-2} + \ldots + d_1 s}{c_k s^k + c_{k-1}s^{k-2} + \ldots + c_0} \quad (1)$$

$$V(s) = H\frac{c_k s^k + c_{k-1}s^{k-2} + \ldots + c_0}{d_l s^l + d_{l-1}s^{l-2} + \ldots + d_1 s} \quad (2)$$

3. Synthesis of mechanical system by means of continued fraction expansion method

The required frequency spectrum:

$$\begin{cases} \omega_1 = 6\dfrac{rad}{s}, & \omega_3 = 19\dfrac{rad}{s}, & \omega_5 = 31\dfrac{rad}{s}, \\ \omega_0 = 0\dfrac{rad}{s}, & \omega_2 = 13\dfrac{rad}{s}, & \omega_4 = 25\dfrac{rad}{s}. \end{cases}$$

The structures of systems after the synthesis (distribution of characteristic function into partial fraction or continued fraction expansion) was introduced in table 1.

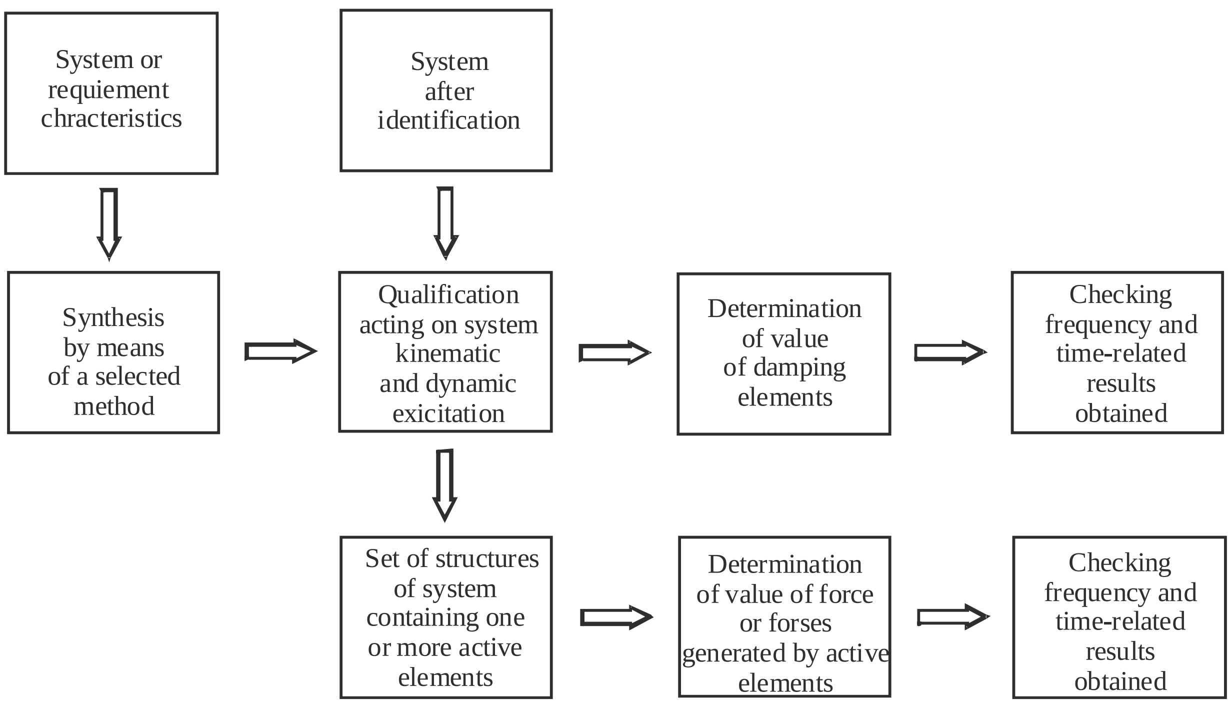
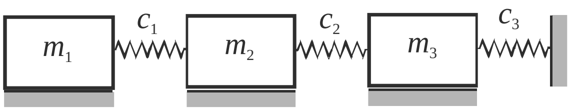
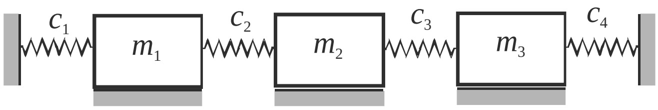
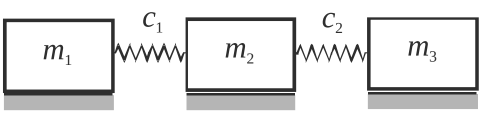
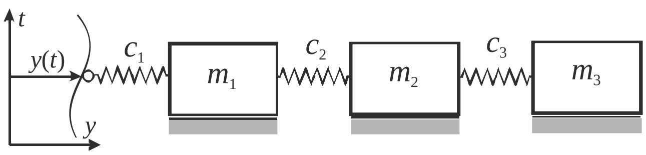


Fig. 1. Idea of synthesis of mechanical systems

Table 1.
The structures of systems after accomplishment the synthesis

No	FUNCTION	STRUCTURE
1	$U(s)=\frac{s^6+1358s^4+394513s^2+12489156}{s^5+794s^3+105625s}=$ $=s+\cfrac{1}{\cfrac{s}{564}+\cfrac{1}{2s+\cfrac{1}{\cfrac{s}{500}+\cfrac{1}{2.08s}+\cfrac{1}{\cfrac{s}{213}}}}}$	
2	$U(s)=\frac{s^6+1358s^4+394513s^2+12489156}{s^5+794s^3+105625s}=$ $=\frac{80}{s}+s+\cfrac{1}{\cfrac{s}{484}+\cfrac{1}{1.474s+\cfrac{1}{\cfrac{s}{250}+\cfrac{1}{1.01s}+\cfrac{1}{\cfrac{s}{50}}}}}$	
3	$U(s)=\frac{s^5+1322s^3+346921s}{s^4+794s^2+105625}=$ $=s+\cfrac{1}{\cfrac{s}{528}+\cfrac{1}{1.56s+\cfrac{1}{\cfrac{s}{227}+\cfrac{1}{1.72s}}}}$	
4	$V(s)=s+\frac{564s^4+288888s^2+12489156}{s^5+749s^3+105625s}=$ $=\frac{564}{s}+\cfrac{1}{s+\cfrac{s}{333}+\cfrac{1}{1.08s+\cfrac{1}{\cfrac{s}{250}+\cfrac{1}{2.68s}}}}$	

3.1. Qualification acting on system dynamic excitation

Structure of system number 1 (from table 1) was selected to more far research. This system was weighted one dynamic excitation F (Fig.2). Polar graph of the system was introduced in figure 3.

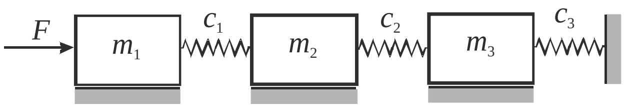


Fig. 2. Model of system with dynamic excitation

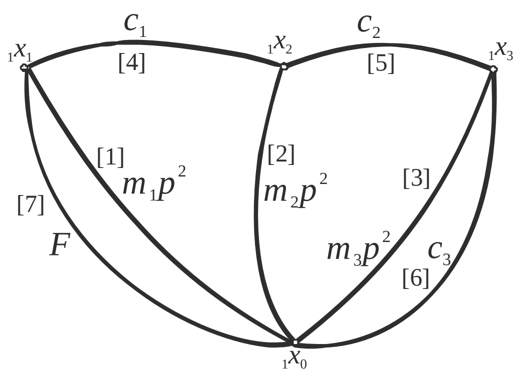


Fig. 3. Polar graph of the system with dynamic excitation

The above elements of polar graph are numbered according to the following standard:

$[1]-m_1p^2 \rightarrow m_1 = 1\,kg \Rightarrow$ inertial element,

$[2]-m_2p^2 \rightarrow m_2 = 2\,kg \Rightarrow$ inertial element,

$[3]-m_3p^2 \rightarrow m_3 = 2.23\,kg \Rightarrow$ inertial element,

$[4]-c_1 \rightarrow c_1 = 564\,\frac{N}{m} \Rightarrow$ elastic element,

$[5]-c_2 \rightarrow c_2 = 432\,\frac{N}{m} \Rightarrow$ elastic element,

$[6]-c_3 \rightarrow c_3 = 229\,\frac{N}{m} \Rightarrow$ elastic element,

$[7]-F \rightarrow F = 30\sin\omega t\;N \Rightarrow$ dynamic excitation.

A general formula for amplitude value is as follows:

$$A_n = \frac{\left(\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[n]}\right)(F_1+F_{k1}+G_1)\right)+}{} \quad \frac{+\left(\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[2]};\frac{\partial D(\omega)}{\partial[n]}\right)(F_2+F_{k2}+G_2)\right)+}{D(\omega)} \quad \frac{+\ldots+\left(\left(\frac{\partial D(\omega)}{\partial[n]}\right)(F_w+F_{kl}+G_g)\right)}{} \tag{3}$$

where:

$D(\omega)$ - characteristic equation,

$\frac{\partial D(\omega)}{\partial[1]}$ - derivative of structural number the in relation to of edge [1],

$\underset{Z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[2]}\right)$ - function of simultaneousness of structural number,

$F_{k1}, F_{k2}, \ldots, F_{kl}$ - kinematic excitation,

$F_1, F_2, \ldots, F_w$ - dynamic excitation,

$G_1, G_2, \ldots, G_g$ - forces generated through active elements.

The amplitudes of vibrations are introduced in Figs.4-6.

$$A_1 = \frac{\left(m_2m_3\omega^4-\omega^2(m_2c_3+m_3c_2+m_3c_3)+c_2c_3\right)F}{-m_1m_2m_3+\omega^2(m_1m_2c_3+m_1m_3c_2+m_1m_3c_3+} \quad \frac{}{m_2m_3c_1+m_2m_3c_2)-\omega^2(m_1c_2c_3+m_2c_1c_3+m_2c_2c_3+} \quad \frac{}{m_3c_1c_2+m_3c_1c_3+m_3c_2c_3)+c_1c_2c_3} \tag{4}$$

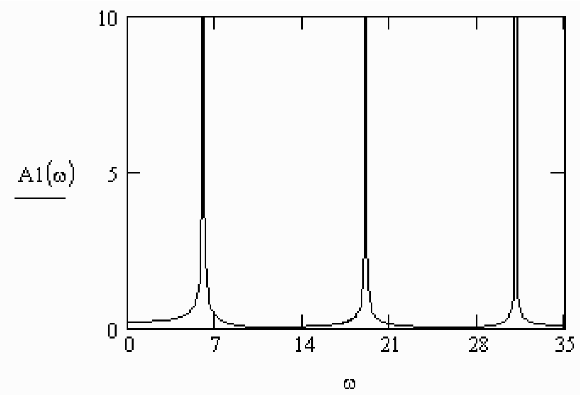


Fig. 4. Diagram of A_1 amplitude

$$A_2 = \frac{\left(-m_3c_2\omega^2+c_2c_3\right)F}{-m_1m_2m_3+\omega^2(m_1m_2c_3+m_1m_3c_2+m_1m_3c_3+} \quad \frac{}{m_2m_3c_1+m_2m_3c_2)-\omega^2(m_1c_2c_3+m_2c_1c_3+m_2c_2c_3+} \quad \frac{}{m_3c_1c_2+m_3c_1c_3+m_3c_2c_3)+c_1c_2c_3} \tag{5}$$

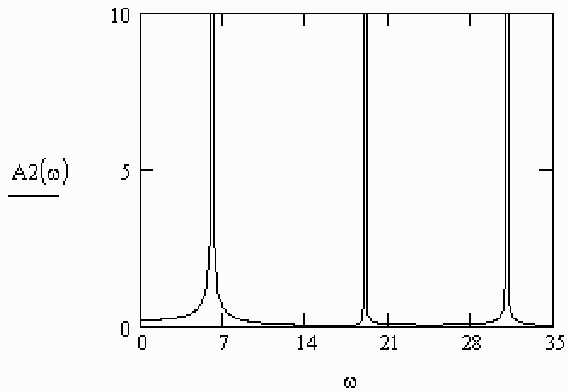


Fig. 5. Diagram of A_2 amplitude

$$A_3 = \frac{(c_2c_3)F}{-m_1m_2m_3+\omega^2(m_1m_2c_3+m_1m_3c_2+m_1m_3c_3+} \quad \frac{}{m_2m_3c_1+m_2m_3c_2)-\omega^2(m_1c_2c_3+m_2c_1c_3+m_2c_2c_3+} \quad \frac{}{m_3c_1c_2+m_3c_1c_3+m_3c_2c_3)+c_1c_2c_3} \tag{6}$$

3.2. Determination of value of damping elements

Implementation of passive elements make possible to solve the problem of reducing the vibration of system.

A general formula for value of damping [15], when damping is proportional to elastic element, is as follows:

$$b_i = \lambda c_i \tag{7}$$

where:
b_i - damping elements
λ - modulus of proportionality $\left(0 < \lambda < \frac{2}{\omega_n}\right)$
ω_n - the largest value of frequency
c_i - elastic elements

$\lambda = 0.01$

$b_1 = 5.64 \frac{Ns}{m}$

$b_2 = 4.32 \frac{Ns}{m}$

$b_3 = 2.29 \frac{Ns}{m}$

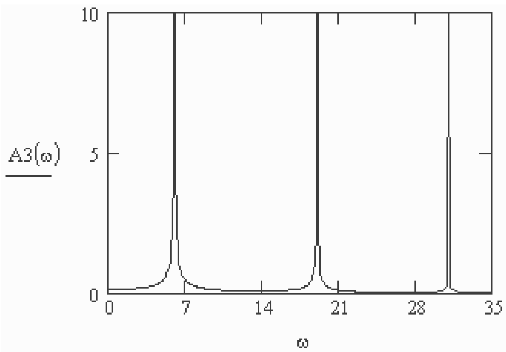


Fig. 6. Diagram of A_3 amplitude

Systems with passive elements reducing vibrations they be introduced in figure 7 (polar graph in fig.8):

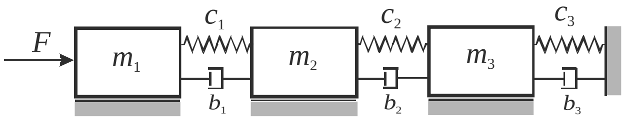


Fig. 7. The models of the system with passive elements

Amplitudes of system were determined using formula (3):

$$A_1 = \frac{\left(\left(c_2 + c_3 - m_2\omega^2 + b_2 + b_3\right)\left(c_3 - m_3\omega^2 + b_3\right)\left(c_3 - b_3\right)\right)F}{D(\omega)} \tag{8}$$

$$A_2 = \frac{\left(\left(-c_2 - b_2\right)\left(c_3 - m_3\omega^2 + b_3\right)\right)F}{D(\omega)} \tag{9}$$

$$A_3 = \frac{\left(\left(-c_2 - b_2\right)\left(-c_3 - b_3\right)\right)F}{D(\omega)} \tag{10}$$

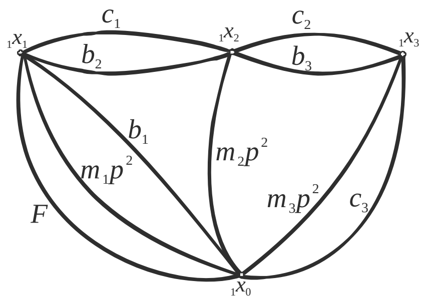


Fig. 8. Polar graph of the systems from Fig. 7

3.3. Determination of value of forces generated by active elements

Implementation of active elements make possible to solve the problem of reducing the vibration of selected parts of a system.

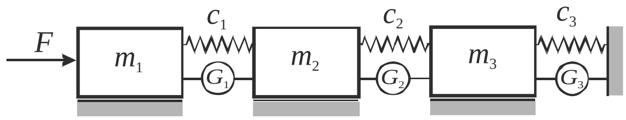


Fig.9. The models of the system with active elements

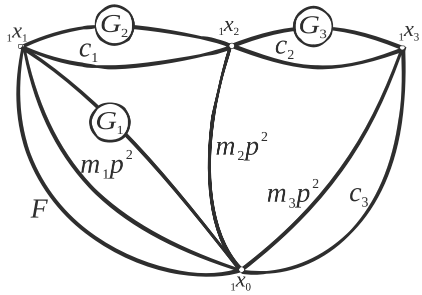


Fig. 10. Polar graph of the systems (Fig.9)

Using the theory of polar graphs and their relation to structural numbers [2, 10÷14, 18÷20], it is possible to determine the values of amplitudes of forces generated by active elements.

Systems with active elements reducing vibrations they be introduced in figure 9 (polar graph in Fig.10):

Amplitudes of system were determined using formula (3):

$$A_1 = \frac{\left(m_2 m_3 \omega^4 - \omega^2\left(m_2 c_3 + m_3 c_2 + m_3 c_3\right) + c_2 c_3\right)\left(F + G_1 + G_2\right) +}{-m_1 m_2 m_3 + \omega^2\left(m_1 m_2 c_3 + m_1 m_3 c_2 + m_1 m_3 c_3 +\right.}$$
$$\frac{\left(-m_3 c_2 \omega^2 + c_2 c_3\right)\left(G_2 + G_3\right) +}{\left. m_2 m_3 c_1 + m_2 m_3 c_2\right) - \omega^2\left(m_1 c_2 c_3 + m_2 c_1 c_3 + m_2 c_2 c_3 +\right.} \tag{11}$$
$$\frac{\left(c_2 c_3\right)\left(G_3\right)}{\left. m_3 c_1 c_2 + m_3 c_1 c_3 + m_3 c_2 c_3\right) + c_1 c_2 c_3}$$

$$A_2 = \frac{\left(-m_3c_2\omega^2 + c_2c_3\right)\left(F + G_1 + G_2\right)}{-m_1m_2m_3 + \omega^2\left(m_1m_2c_3 + m_1m_3c_2 + m_1m_3c_3 + \right.}$$
$$\frac{\left(m_1m_2\omega^4 - \omega^2\left(m_1c_3 + m_3c_1 + m_3c_2\right) + c_1c_3 + c_2c_3\right)\left(G_2 + G_3\right)}{\left. m_2m_3c_1 + m_2m_3c_2\right) - \omega^2\left(m_1c_2c_3 + m_2c_1c_3 + m_2c_2c_3 + \right.}$$
$$\frac{\left(-m_1c_3\omega^2 + c_1c_3 + c_2c_3\right)\left(G_3\right)}{\left. m_3c_1c_2 + m_3c_1c_3 + m_3c_2c_3\right) + c_1c_2c_3} \tag{12}$$

$$A_3 = \frac{\left(c_2c_3\right)\left(F + G_1 + G_2\right) +}{-m_1m_2m_3 + \omega^2\left(m_1m_2c_3 + m_1m_3c_2 + m_1m_3c_3 + \right.}$$
$$\frac{\left(-m_1c_3\omega^2 + c_1c_3 + c_2c_3\right)\left(G_2 + G_3\right) +}{\left. m_2m_3c_1 + m_2m_3c_2\right) - \omega^2\left(m_1c_2c_3 + m_2c_1c_3 + m_2c_2c_3 + \right.}$$
$$\frac{\left(m_1m_2\omega^4 - \omega^2\left(m_1c_2 + m_1c_3 + m_2c_1 + m_2c_2\right) + c_1c_2 + c_1c_3 + c_2c_3\right)\left(G_3\right)}{\left. m_3c_1c_2 + m_3c_1c_3 + m_3c_2c_3\right) + c_1c_2c_3} \tag{13}$$

Solving a system of equations, when $A_1 \div A_3$ was equated into zero (11-13) leads to the obtaining of values of individual amplitudes generated by active elements.

The comparison of amplitudes of vibrations is introduced in Fig.11-19.

Symbols in Fig. 11-19:

A1(ω), A2(ω), A3(ω) – amplitudes of system without reduction
Ap1(ω), Ap2(ω), Ap3(ω) – amplitudes of system with passive reduction
Aa1(ω), Aa2(ω), Aa3(ω) – amplitudes of system with active reduction

At $\omega=\omega_1$ the values are as follows:

G_1= -16.92 *N*,
G_2= -13.07 *N*,
G_3= 7.50 *N*.

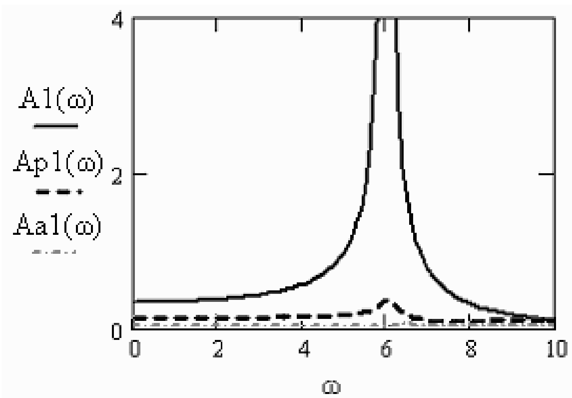


Fig. 11. Diagram of A_1 amplitude at $\omega=\omega_1$

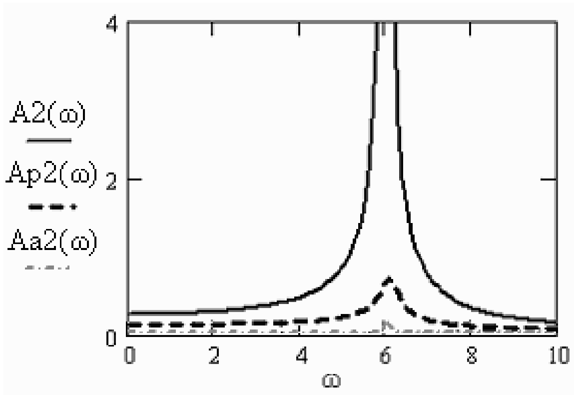


Fig. 12. Diagram of A_2 amplitude at $\omega=\omega_1$

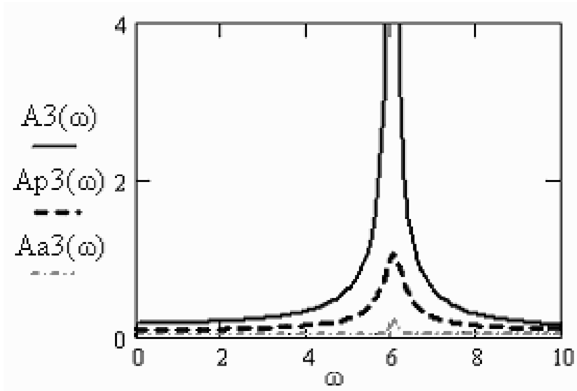


Fig. 13. Diagram of A_3 amplitude at $\omega=\omega_1$

At $\omega=\omega_2$ the values are as follows:

G_1= 27.03 *N*,
G_2= -57.1 *N*,
G_3= -28.72 *N*.

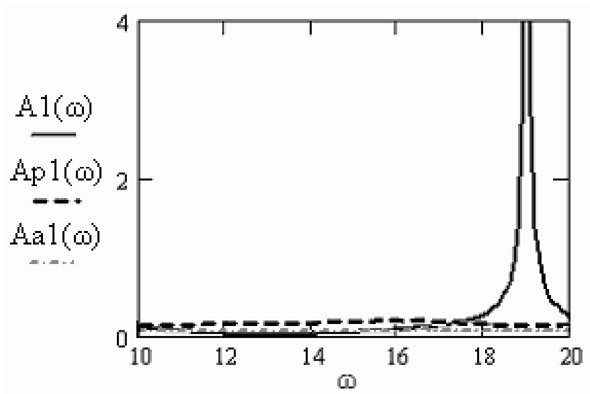


Fig. 14. Diagram of A_1 amplitude at $\omega=\omega_2$

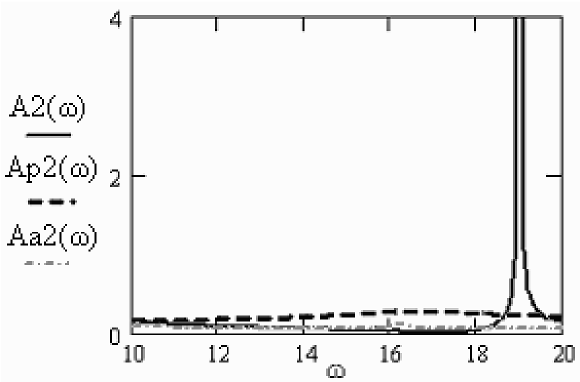


Fig. 15. Diagram of A_2 amplitude at $\omega=\omega_2$

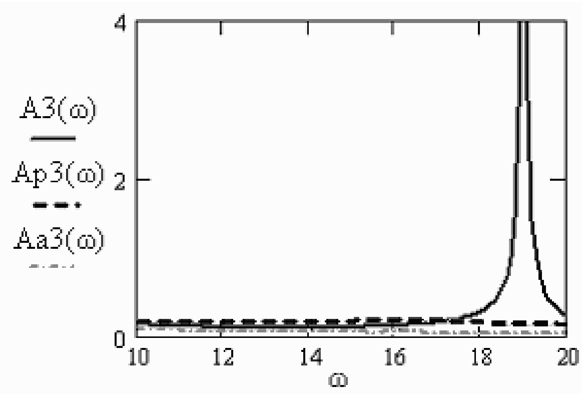


Fig. 16. Diagram of A_3 amplitude at $\omega=\omega_2$

At $\omega=\omega_3$ the values are as follows:

G_1= -164.44 *N*,
G_2= 132.9 *N*,
G_3= -190.52 *N*.

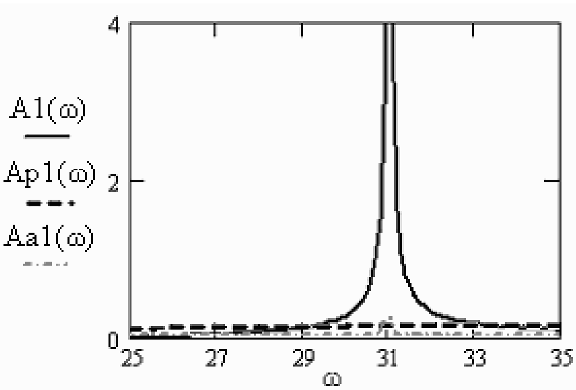


Fig. 17. Diagram of A_1 amplitude at $\omega=\omega_3$

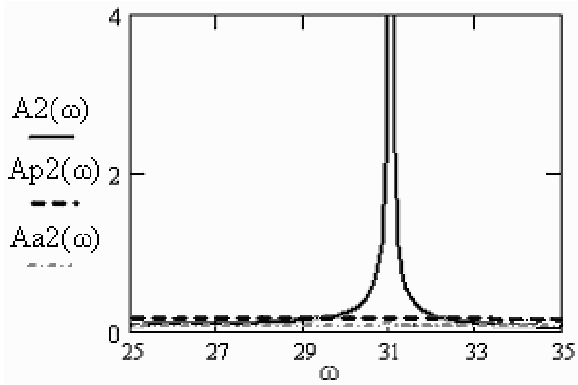


Fig. 18. Diagram of A_2 amplitude at $\omega=\omega_3$

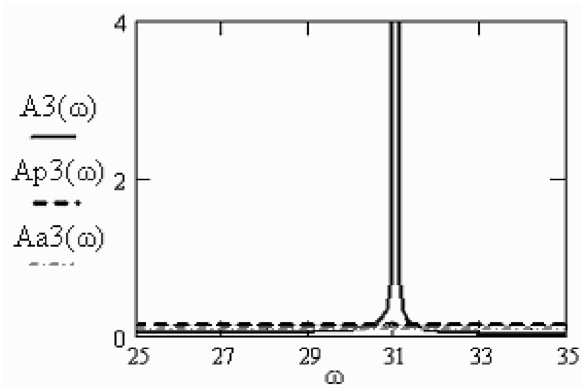


Fig. 19. Diagram of A_3 amplitude at $\omega=\omega_3$

4. Conclusions

In this work was introduced synthesis of mechanical systems including passive or active elements reducing of vibrations. One of received systems was weighted dynamic excitation. Then it was accomplished analysis of this system. Afterwards were used active and passive elements to reducing of vibrations. Comparison of both methods of reducing of vibrations of mechanical systems was shown, that active elements give better results than passive elements in the case of low frequencies. On diagrams of amplitudes is visible that the using to reduction of vibration active elements gives completely effects however using passive partially. In case of higher frequencies passive as well as active give similar results.

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