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Process description of piercing when using degenerated model

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Analysis and modelling

ABSTRACT

Purpose: The material piercing process is an extremely complex phenomenon for the theoretical analysis. This is connected with the fact that the materials used mainly for ballistic shields may not be treated as isotropic media. The description of piercing of ballistic shields made of light weight composite materials, on the basis of the Hooke's model and the Young's module reflects the too low result effectiveness for practical applications. The purpose of this article is the mathematical description of the impact phenomenon of a bullet of the speed ca. 400 m/s, with the use of a degenerated model.

Design/methodology/approach: In the study, an attempt has been made to apply an untypical model for the piercing phenomenon analysis. Basing on the model, the theoretical analysis of the piercing phenomenon in quasistatic and dynamic load conditions, in the impact load form, has been carried out.

Findings: This analysis enabled derivation of significant conclusions useful in the design process of effective ballistic shields.

Research limitations/implications: In the study, the concept has been assumed that a dynamic model simple as possible, that may be analyzed not only by numerical methods but also (at least approximately) with the mathematical analysis methods, may provide significant directions concerning material piercing.

Practical implications: The use of so called degenerated model allows to describe the phenomenon in more detail at various piercing speeds what extends the possibilities in the sphere of designing the optimum ballistic shields and of identification of the properties of materials applied for construction of shields.

Originality/value: The proposed method of identification of material properties in the piercing process, within the relation: force – deformation, is a novel one since the essence of the identification is the standard rheological model in an adequate plastic component describing the viscous attenuation with dry friction.

Keywords: Computational mechanics; Impact load; Impact; Composites

1. Introduction

To create more and more effective protection for human or vehicles, there is necessary to create very effective and light ballistic shields (e.g. composite materials). For military application such materials as sandwich panels with reinforcing of plastic fibers or structures made of laminates are more and more popular. These materials have high resistance characteristics for impact loads. However, the he strict mathematical analysis of the piercing process for the light weight material structures is very complex and difficult to conduct because of the fact that the damage process is not usually univocal but it consists of several stages. According to Greaves [1-2] destruction occurring during the dynamical penetration should be divided into two phases. He claims that the first phase with indentation and shear is predominating and the most energy consuming (the biggest part of the energy is absorbed in this phase). To similarly conclusions came Zhu, Goldsmith and Dharan dividing penetration laminates made out of Kevlar into three stages: indentation, perforation and final stage. However, the experimental data with light weight composite materials put through quasi-static piercing conducted by the authors of study [3-4] points out that in final stage the different theoretical model focusing on the first and second phase should by taken. Description of this process, however, is done solely on the based on determination of substitute rigidity values deriving from the general elasticity theory for isotropic bodies. [5-6]. Also non-classical ways for modeling, synthesis, analyzing and testing fragility of models can be found in paper [7-17]. In the present work, the author suggests another strategy wherein not only pure elastic interactions are assumed in the theoretical model, but also dissipative components in an appropriate (non parallel) configuration with the elastic components. Such model allows a more precise description of the piercing process in which the relation of the characteristic curves „force – deformation" on the speed of the deformations occurring is observed often, what does not happen for the pure elastic liner model case [23].

2. Description of the approach

In the present work, the premises as follow have been used for taking the piercing model:

1) the predominating role in the piercing process is played by the material sphere directly adhering to the piercing material (bullet),
2) for the analysis, it has been assumed that the bullet does not subject to deformation,
3) shield vibration after the impact are on irrelevant level for the bullet movement in the shield (the wave propagation speed in the shield is slow with relation to the bullet movement),
4) the material damage process within the impact scope has been divided into two stages:
   - stage I, wherein the resilient deformation (that does not destroy the material) occurs,
   - stage II, wherein the irreversible deformation (the material is destroyed) occurs.

   It has been assumed for the stage I and, first of all, in the stage II, energy losses have been taken into consideration (the shield material reaction forces on the bullet have been assumed as proportional to the deformation speeds. In addition, during the stage II, the dry friction component has been introduced),
5) it has been accepted that the joint of shield and an inertia-type impact system has been made in an ideally elastic way.

It has been intended that the character of the acting force changes considerably after the force exceeds the value of *h* as dry friction during movement of the bullet in the material being pierced in the stage II.

The diagram of the model accepted a priori has been presented in Fig. 1, where:

- x – a constant describing the bullet movement in the shield,
- x_0 – a constant describing the shield shift,
- u – a variable describing the I stage of deformation,
- ξ – a variable describing the deformation stage II,
- c_1 – a constant describing the static rigidity of the material of stage I,
- c_0 – a constant describing the dynamic rigidity of the material of the stage I,
- c_z – a constant describing the shield fixing,
- k_0 – attenuation of the deformation stage I,
- h – dry friction of the deformation stage II,
- k – attenuation of the deformation stage II.

The bullet movement with respect to the inertia-type (immovable) reference system is defined with a variable $x(t)$ being the sum of the shift of the shield x_0, the reversible deformation u of the shield and the irreversible deformation ξ describing the values of the shield damage, i.e. in accordance with the relation (1).

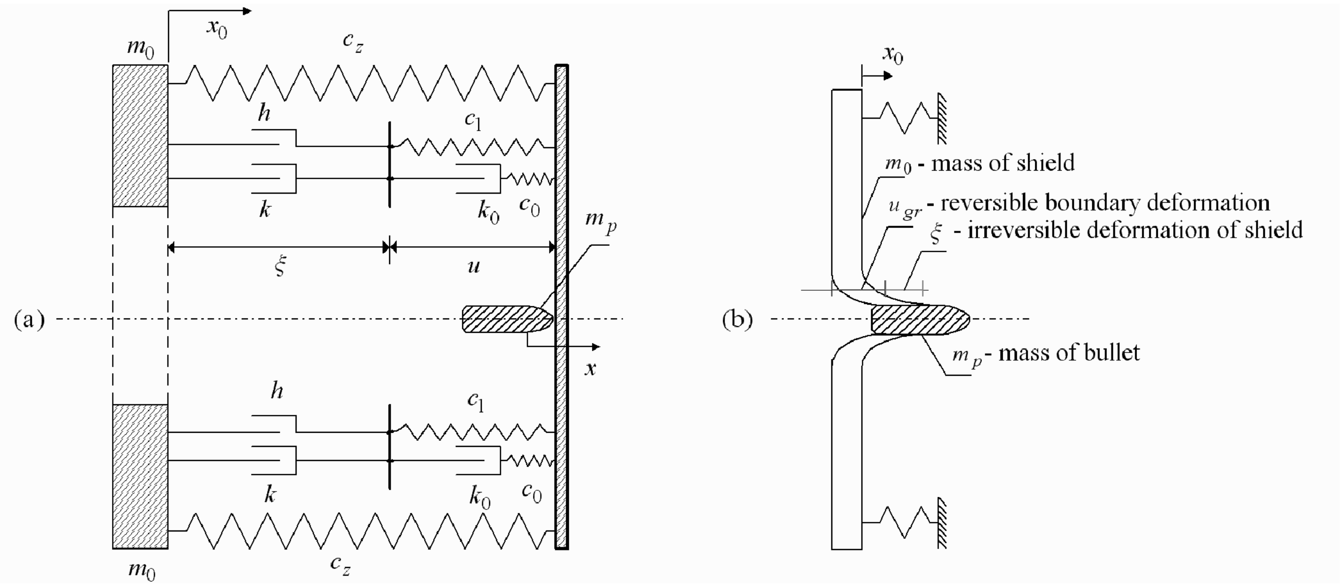

Fig. 1. The diagram of the model assumed (a) in the piercing process (b)

$$x = x_0 + u + \xi \tag{1}$$

As shown above, within the scope of reversible deformations, the standard rheologic model has been assumed (so called Zener Model [18]). It includes the Maxwell component characterized with the constants k_0, c_0, in a parallel connection with the Hooke's component of the constant c_1. Let us notice that at a rigid shield fixing, it may be accepted: $c_z = \infty$. What is more, the assumed model allows also an account of simpler material models, as follows:

- ideally elastic material (it should be assumed that $c_0 = 0$),
- ideally plastic material (assume: $c_1 = 0$, $c_0 = \infty$),
- elastic / plastic material ($c_0 = \infty$),
- material of a constant (independent of the deformation speed) plasticity limit (assume $k = 0$).

In the assumed model all constants: c_z, h, k, c_1, k_0, c_0 may be determined experimentally, e.g.: at static loads, at quasistatic piercing tests (at given deformation speeds) as well as at dynamic load conditions, applying adequate identification methods for degenerated models [19-21].

3. Description of achieved results

3.1. Static load

In the event of a constant piercing force, $S(t) = S_0$ = const, the movement differential equations are as follow:

$$m_0\ddot{x}_0 + c_z x_0 = S_0 \tag{2}$$

$$hSgn\dot{\xi} + k\dot{\xi} = S_0 \ \text{ for } \ S_0 > h \tag{3}$$

$$\dot{\xi} = 0 \ \text{ for } \ S_0 \le h \tag{4}$$

$$c_1 u + c_0(u - z) = S_0 \tag{5}$$

$$k_0 \dot{z} = c_0(u - z) \tag{6}$$

From the equations written above, it follows:

$$x_0(t) = \frac{S_0}{c_z} - \frac{S_0}{c_z}\cos\omega_0 t \ , \quad \omega_0 = \sqrt{\frac{c_z}{m}} \tag{7}$$

$$\xi(t) = \xi_0 = const \quad \text{for } \ S_0 \le h \tag{8}$$

$$\xi(t) = \xi_0 + \left(\frac{S_0 - h}{k}\right)t \quad \text{for } \ S_0 > h \tag{9}$$

ξ_0 - the plastic strain at the initial instant, i.e. at the moment the force S_0 is applied. For the system unloaded earlier $\xi_0 = 0$. If the force $S_0 > h$, the plastic deformation ξ increases in time at a constant speed $\dot{\xi} = \frac{S_0 - h}{k}$. However, if the force $S_0 \le h$, then the plastic deformation is constant and equal to the initial value ξ_0. Eliminating of variable „z" from (5) and (6) we get following equation:

$$k_z \dot{u} + c_1 u = S_0 \tag{10}$$

Where k_z is constant:

$$k_z = k_0 \frac{c_1 + c_0}{c_0} = k_0\left(1 + \frac{c_1}{c_0}\right) \tag{11}$$

We'll call k_z substitute attenuation. In the event of constant force S_0 degenerated element works identical with Kelvin – Voigt model employing substitute attenuation. Solution of the equation (10) for zero starting condition is as follows:

$$u(t) = \frac{S_0}{c_1}\left[1 - e^{-\frac{c_1}{k_z}t}\right] \tag{12}$$

The plot of the function $u(t)$ in the form (12) is presented in Fig. 2.

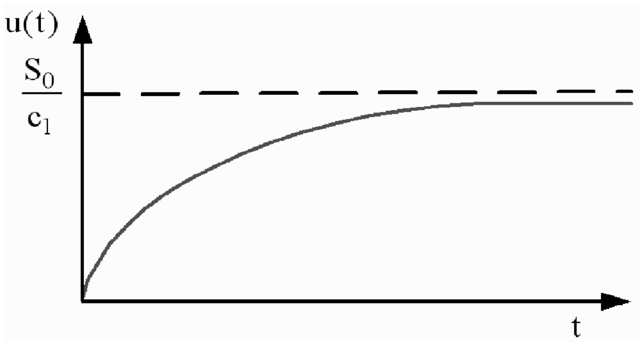

Fig. 2. The graphic form of the deformation of a component degenerated under influence of a constant load S_0

We can observe when $c_0 = \infty$ (Kelvina – Voigt model) changes only value of coefficient k_z to value k and character of dependence $u(t)$ remains unchanged. However in case, when $k_0 = 0$ (model Hooke'a) $k_z = 0$ for any values c_0, c_1. Than

$$u(t) = \lim_{k_z \to 0}\left[\frac{S_0}{c_1} - \frac{S_0}{c_1}e^{-\frac{c_1}{k_z}t}\right] = \frac{S_0}{c_1} \tag{13}$$

means, that deformation is immediate from zero value to constant $\frac{S_0}{c_1}$. Observing, cases when load of real sample is slow changing, same as with static loads, simplified Hooke's model is justified because time interval to value determine $\frac{S_0}{c_1}$ can be omitted. The situation is, however, quite different in the case of short-term dynamic loads, wherein the momentary behavior of the material may be of the deciding influence upon the system motion. This happens in particular in the case of penetration of the shield by the bullet. In a case of $c_z = \propto$ and the piercing force lower that some constant h, $\xi = 0$ is obtained. Then $x = u$ while the relation $x(t)$ is described by the plot depicted in Fig. 3 function $x(t) = u(t)$ in the form (13). However, for $S_0 > h$, we have:

$$\xi = \frac{S_0 - h}{k} \cdot t = v_p \cdot t \tag{14}$$

where the constant $v_p = \frac{(S_0 - h)}{k}$ defines the permanent deformation speed for the material in the static load conditions. Thus, in the case, the total deformation x will be equal to:

$$x = u + \xi = \frac{S_0}{c_1}\left(1 - e^{-\frac{c_1}{k_z} \cdot t}\right) + v_p t \tag{15}$$

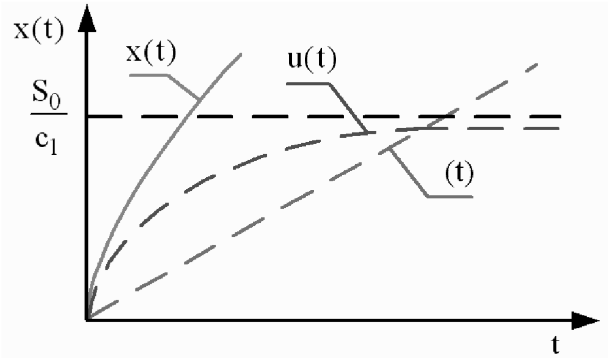

Fig. 3. Material behavior at rigid fixing under influence of the piercing force $S_0 > h$

3.2. Quasistatic load

The piercing force is usually instability in time in quasi-static loads. For the analysis, the constant piercing force v_0 is assumed. As a result of that test are often the relations $S(x)$. For rigid fixing ($c_z = \infty$), the movement equation will be as written below:

$$\dot{\xi} = 0 \quad \text{for } S \le h \tag{16}$$

$$c_1 u + c_0 (u - z) = S \tag{17}$$

$$k_0 \dot{z} = c_0 (u - z) \tag{18}$$

Eliminating of variable:

$$S = \frac{k_0}{c_0}\left[(c_1 + c_0) \cdot \dot{u} - \dot{S}\right] + c_1 u \tag{19}$$

In case $S \le h$ is $\dot{\xi}_0 = 0$ and when $\dot{x} = \dot{u}$. If piercing has constant speed v_0 than

$$u = v_0 t \tag{20}$$

Substituting (20) to (19) we get differential equation of force S as below:

$$\dot{S} + \frac{c_0}{k_0} S = (c_0 + c_1) v_0 + \frac{c_1 c_0}{k_0} v_0 t \tag{21}$$

It's function is as follows:

$$S(u) = k_0 v_0 + c_1 u - k_0 v_0 e^{-\frac{c_0}{k_0 v_0} u} \tag{22}$$

Is easy to see, that dependence $S(u)$ for standard model is different from dependence $S(u)$ for Kelvin's – Voigt model. When $c_0 \to \propto$ is

$$S(u) = k_0 v_0 + c_1 u \tag{23}$$

And if $k = 0$ (Hooke's model) is easy to prove, for any value of v_0:

$$\lim_{k_0 \to 0} S(u) = c_1 u \tag{24}$$

For selected models, the relation $S(u)$ has been presented in Fig. 4 in graphic form. It can be observed that, all relations of $S(u)$ come more and more similar to the relation for the Hooke's model, when the value v_0 comes nearer zero (slow piercing). What is more, for $v_0 = 0$, all of them are exactly equal to $S(u) = c_1 u$. At high piercing speeds ($v_0 >> 0$), what happens in cases that the shield is shut through by the bullet, however, significant differences occur. (Fig. 5).

Therefore, it should be noticed that, by applying the Hooke's model an apparent growth of the material rigidity (the Young's module increase) is observed, both for the case (a) and (b). That is combined with an increase the piercing speed, what occurs in the material examinations. It is not the change the Young's module, however, but the reason is in that the constant k_0 is not taken into consideration in the constitutive interconnections.

3.3. Dynamic loads

After replacing $S_0 = -m\ddot{x}$ to the equations (2-6), the differential equations of the final mathematical form of the model accepted are obtained:

$$m_0 \ddot{x}_0 + c_z x_0 = -m\ddot{x} \tag{25}$$

$$hSgn\dot{\xi} + k\dot{\xi} = -m\ddot{x} \quad \text{for } (-m\ddot{x}) > h \tag{26}$$

$$\dot{\xi} = 0 \text{ dla } (-m\ddot{x}) \le h \tag{27}$$

$$c_1 u + c_0 (u - z) = -m\ddot{x} \tag{28}$$

$$k_0 \dot{z} = c_0 (u - z) \tag{29}$$

Eliminating of variable „z" force of material resistance on projectile is as follows:

$$S = c_1 u + \frac{k_0}{c_0}\left[(c_1 + c_0) \cdot \dot{u} + m\dddot{x}\right] \tag{30}$$

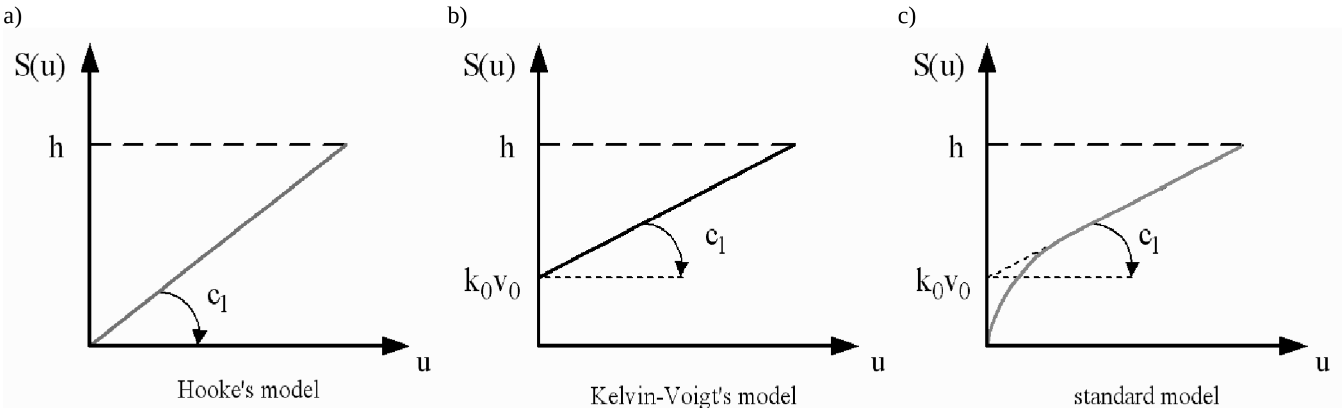

Fig. 4. The relation $S(u)$ of quasistatic piercing for the models under consideration

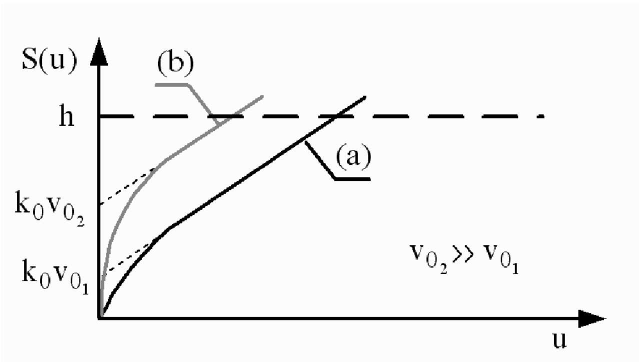

Fig. 5. A comparison of the relation $S(u)$ for the standard model in a case of the piercing speed values v_0: small (a) and big (b)

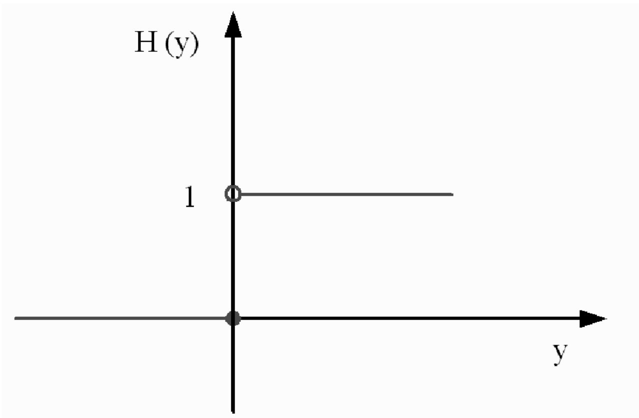

Fig. 6. Plot of the function $H(y)$

Finally mathematical model takes, following chain of differential equations:

$$m_0\ddot{x}_0 + c_z x_0 = -m\ddot{x} \tag{31}$$

$$\left(hSgn\dot{\xi} + k\dot{\xi}\right) + m\ddot{x} \cdot H\left(-m\ddot{x} - h\right) = 0 \tag{32}$$

$$m\ddot{x} + c_1 u + \frac{k_0}{c_0}\left[\left(c_1 + c_0\right)\cdot \dot{u} + m\dddot{x}\right] = 0 \tag{33}$$

where H is a function as below (Fig. 6):

$$H(y) = \begin{Bmatrix} 0 & for\ y \le 0 \\ 1 & for\ y > 0 \end{Bmatrix} \tag{34}$$

Based on the mathematical analysis of degenerated standard model following interdependences were calculated:

- dependence of cover deformation when solid fixed ($c_z = \infty$) as follows:

$$\xi_{gr} = \frac{mv_0^2}{2h} - \frac{h}{2c_1} \tag{35}$$

- dependence on piercing time (in case of solid fixed) is as follows:

$$t_{gr} = \frac{m}{h}\sqrt{v_0^2 - \frac{h^2}{mc_1}} \tag{36}$$

- dependence of cover deformation in case when c_z is end value and mass m_0 can be omitted because of minimal value:

$$\xi_{gr} = \frac{mv_0^2}{2h} - \frac{h}{2\tilde{c}} \tag{37}$$

Depending of (37) value $\tilde{c}$ is a substitute stiffness as follows:

$$\tilde{c} = \frac{c_1 c_z}{c_1 + c_z} \tag{38}$$

and marginal time t_{gr} is in accordance with dependence (36), in which instead of value c_1 substitute stiffness $\tilde{c}$ is inputted. Derived dependence $\xi_{gr}(c_1)$ or $\xi_{gr}(\tilde{c})$ has a hyperbolic character, therefore along [18] significant influence of these constants is observed for small stiffness and for large stiffness this influence can be insignificant.

The equation system for the high impact speeds has been solved by the computer simulation technique, making use of Simulink network [22]. Exemplary piercing process results for the given constant values of the model (m_0 = 10 kg, c_z = 90000 N/m, m_p = 0,008 kg, c_1 = 8610 N/m, c_0 = 2500 N/m, k_0 = 2,5 Ns/m, k = 10 Ns/m, h = 10000 N) have been depicted in Fig. 7. However main goal of the simulation was to define influence of particular parameters on deformation ξ as breaking distance of projectile after going over of plasticity borderline h (Fig. 8).

4. Conclusions

Applying the degenerated model to the piercing analysis enabled:

- estimating the relation the piercing speed on the force in the quasistatic tests (Fig. 5),
- estimating the impact of the characteristics defining the system behavior within the scope of reversible deformations (parameters c_0, k_0, c_1) on the permanent deformation (Fig. 7).

In addition, from the analysis carried out, it follows that the effectiveness of the shield increases (Fig. 8) when:

1. the shield weight decreases,
2. the shield rigidity decreases,
3. the shield fixing rigidity decreases,
4. the attenuation at shield deformation increases,
5. the yield point of the shield material increases.

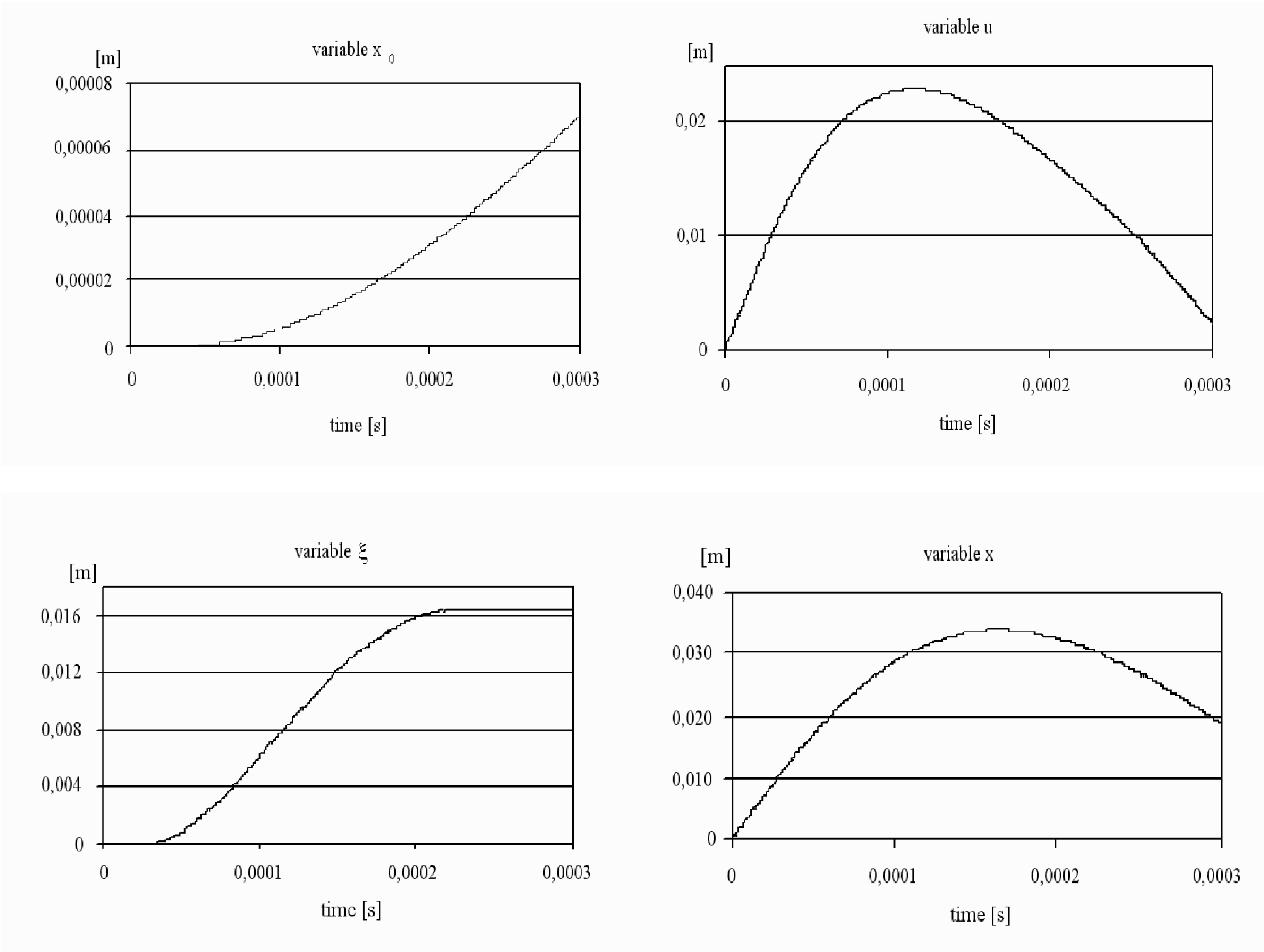

Fig. 7. Exemplary computer simulation results for given model values

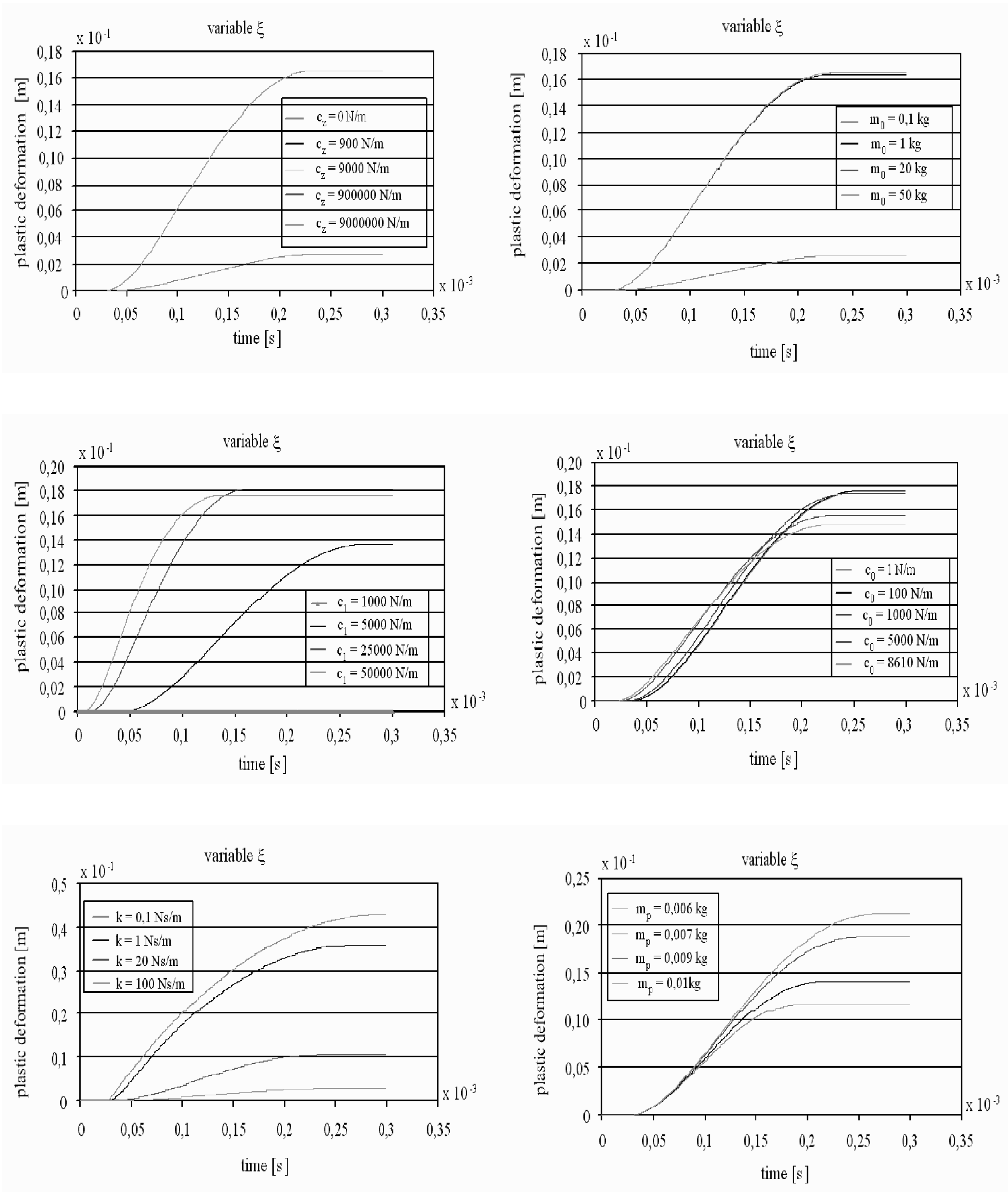

Fig. 8. Influence of model parameters for piercing distance obtained during computer simulation

What is more, it has been found that applying the degenerated model, there happen some differences in the piercing process for various ratios m/v_0, although the kinetic energy of the impact is the same. The research, however, within this scope are in progress and the problem has not been discussed in the present study.

References

[1] L.J. Greaves, Failure Mechanisms in Glass Fibre-Reinforced Plastic Armour, Unpublisched UK DRA Report, 1992.
[2] L.J. Greaves, Progress in modelling the perforation of GFRP by ballistic projectiles, Unpublisched UK DRA Report, 1994.
[3] W. Goldsmith, J.L. Sackman, An experimental study of energy absorption in impact on sandwich plates, International Journal of Impact Engineering 12 (1992) 241-261.
[4] K. Moriarty, W. Goldsmith, Dynamic energy absorption characteristics of sandwich shells, International Journal of Impact Engineering 13/2 (1993) 293-317.
[5] P. Kosza, M.B. Sayir, failure patterns in the core of sandwich structures under impact loading, International Journal of Impact Engineering 15/4 (1994) 501-517.
[6] M.L. Bernard, P.A Lagace, Impact resistance of composite sandwich panels, Journal of Reinforced Plastic Composites 8 (1989) 432-445.
[7] J.K. Kim, T.X. Yu, Impact response and dynamic failure of composites and laminate materials, Part 2: Strain-Rate Effect, Energy Absorption and Modelling, Trans Tech Publications Ltd. Switzerland 1998.
[8] P.D. Smith, J.G. Hetherington, Blast and Ballistic Loading of Structures, Butterworth-Heinemann, Oxford, 2003.
[9] E. Rusiński, J. Karliński, K. Jamroziak, The chosen aspects from researech of ballistic shields, 22 nd DANUBIA-ADRIA Symposium on Experimental Methods in Solid Mechanics, DAS, Parma, Italy, 2005, 22-23.
[10] A. Buchacz, Calculation of characterisics of torsionally vibrating mechatronic system, Journal of Achievements in Materials and Manufacturing Engineering 20 (2007) 327-330.
[11] A. Buchacz, Sensitivity of mechatronical systems represented by polar graphs and structural numbers as models of discrete systems, Journal of Materials Processing Technology 175 (2006) 55-62.
[12] A. Buchacz, Influence of a piezolectric on characteristics of vibrating mechatronical system, Journal of Achevements in Materials and Manufacturing Engineering 17 (2006) 229-232.
[13] A. Buchacz, The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs, Journal of Materials Processing Technology 164-165 (2005) 1277-1280.
[14] A. Buchacz, Hypergrphs and their subgraphs in modelling and investigation of robots, Journal of Materials Processing Technology 157-158 (2004) 37-44.
[15] A. Buchacz, Modifications of cascade structures in computer aided design of mechanical continuous vibration bar systems represented by graphs and structural numbers, Journal of Materials Processing Technology 157-158 (2004) 45-54.
[16] A. Buchacz, A. Dymarek, T. Dzitkowski, Design and examining of sensitivity of continuous and discrete-continuous mechanical systems with required frequency spectrum represented by graphs and structural numbers, Monograph No. 88. Silesian University of Technology Press, Gliwice 2005 (in Polish).
[17] L.A. Dobrzański, R. Honysz, S.D. Fassois, On the identification of composite beam dynamics based upon experimental data, Journal of Achievements in Materials and Manufacturing Engineering 16 (2006) 114-123.
[18] W. Derski, S. Ziemba, The analissys of rheology models, PWN, Warsaw, 1968 (in Polish).
[19] M. Kulisiewicz, Modeling and identification of nonlinear mechanical systems under dynamic complex loads, Monograph, University of Technology, Wroclaw, 2005 (in Polish).
[20] M. Kulisiewicz, S. Piesiak, M. Bocian, Identification of nonlinear damping using energy balance method with random pulse excitation, Journal of Vibration and Control 7 (2001) 699-710.
[21] K. Jamroziak, M. Kulisiewicz, The calculating models used to identify light weight ballistic shields subject to impact loads, Wroclaw WSOWL Report, 2003 (in Polish).
[22] MATLAB, The Language of Technical Computing, Using Matlab Version 5, The MATH WORKS Inc., December 1996.
[23] K. Jamroziak, Analysis of a degenerated standard model in the piercing process, Journal of Achievements in Materials and Manufacturing Engineering 22/1 (2007) 49-52.