

Material parameters identification by use of hybrid GA

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ABSTRACT

Purpose: of this paper is to develop material parameters identification algorithm for yield criterion BBC2003 using global optimization techniques.

Design/methodology/approach: An algorithm proposed is based on use of error minimization function, which allows considering over-constraining. Due to strong nonlinearity of the problem considered a number of solutions is available. In order to determine global extreme two stage GA (global optimization technique) is treated.

Findings: Numerical material parameters identification algorithm is developed. An approach provided allows reducing significantly the dimension of the nonlinear system before its numerical solution. Convergence to global extreme can be expected due to global optimization technique employed.

Research limitations/implications: An analysis is done by keeping formability analysis in mind and only material parameters involved in yield criterion in space of principal stresses are considered. Thus the results can be generalized by including terms corresponding to shear stresses.

Practical implications: Advanced yield criteria like BBC2003 are still not used extensively due to the complexities accrued: increasing number of material parameters (additional tests), a complex non-linear programming problem. An algorithm proposed simplifies the material parameters identification process for considered yield criteria BBC2003. The formability analysis of the 6000 series aluminium alloy sheet AA6181-T4 is considered as a case study and used for testing the algorithm proposed.

Originality/value: In the case of posed optimization problem the dimension of the design space is reduced from six to two. Over-constraining and under-constraining are considered in algorithm (situations, where number of unknown parameters is not equal with the number of given constraints, are covered).

Keywords: Numerical analysis; BBC2003; Stochastic optimization; GA

1. Introduction

In order to describe the plastic anisotropy more accurately a large number of new anisotropic yield criteria are derived during the last decade [1-6]. Barlat et al. [3], Karafillis and Boyce [7] suggested a yield function consisting of the sum of two convex functions. Plastic anisotropy is modelled in these papers using a linear transformation of the stress tensor. The concept of linear

stress transformation was extended by Barlat et al. [2]. The BBC2002 [6] is a plane stress yield criterion developed on the basis of the Barlat and Lian criterion given in [3]. This criterion is used successfully in fitting material data if a Newton solver is utilized for computing the anisotropy coefficients. Banabic et al. [1] modified BBC2002 in order to apply an error minimization function for computing the anisotropy coefficients. A new yield criterion was improved and referred as BBC2003. Yld2003 is a

new plane stress yield function proposed by Aretz [4]. The advantage of this eight parameter plane stress yield criterion is its simplicity and compatibility in FE simulation. The yield function calibration methods for orthotropic sheet metals are proposed by Aretz [8].

Unfortunately, numerous mechanical testing procedures involving different loading modes such as directional uniaxial tensile tests and an equibiaxial tensile or bulge test are necessary for material parameters identification in the case of advanced yield criteria. Additionally, a complex nonlinear system of algebraic equations is needed to solve for material parameters identification. This is the cost of the flexibility and reason why the advanced yield criteria are still not used commonly.

In the current study the error minimization function based algorithm is employed. The nonlinear optimal design problem is formulated, which allows to consider over-constraining and under-constraining. The error minimization function based algorithm is used also for 'validation' of the new yield criteria [1]. In [1] and [7] an error function is defined by means of Gaussian square of error, the steepest descent and downhill-simplex methods are employed, respectively.

In the current study the two stage genetic algorithm (GA) is treated for material parameters identification, since traditional gradient based optimization methods have trend to converge to nearest optimum (which may appear to be local) [9,10]. The advantages of the GA over traditional gradient based techniques can be outlined as: convergence to global extreme can be expected; integer type design parameters can be used; no need for computing derivatives of objective and constraints functions.

However, there are also some disadvantages common to GA: convergence to solution close to global optimum (not exactly optimum); relatively long computing time.

In order to overcome the above mentioned drawbacks, several refined GA approaches are proposed in literature [11-13].

In the current study first the symbolic-numerical algorithm based on reduction of the number of nonlinear relations is applied. Next the search for global minimum of the error function is performed by use of genetic algorithm (global level of hybrid GA). Finally the design improvement is realized by use of gradient method (local level of hybrid GA). The symbolic-numerical procedure is implemented in MAPLE 10 code. The results of the symbolic calculation are transferred into MATLAB code and used in optimization algorithm proposed. The obtained numerical results are applied for formability analysis of the 6000 series aluminium alloy sheet AA6181-T4 [14-15].

2. Problem statement

In order to describe the plastic properties of the material more precisely a number of new 2D and 3D anisotropic yield criteria can be found in literature (reviewed by Banabic et al. [1]). In the following the one of the most recently available plane stress yield criterion – BBC2003 is considered

$$\bar{\sigma} = \left[a(\Gamma + \Psi)^{2k} + a(\Gamma - \Psi)^{2k} + (1-a)(2\Lambda)^{2k} \right]^{\frac{1}{2k}},$$

$$\Gamma = \frac{\sigma_{11} + M\sigma_{22}}{2}, \quad \Psi = \sqrt{\frac{(N\sigma_{11} - P\sigma_{22})^2}{4} + Q^2\sigma_{12}\sigma_{21}},$$

$$\Lambda = \sqrt{\frac{(R\sigma_{11} - S\sigma_{22})^2}{4} + T^2\sigma_{12}\sigma_{21}}, \quad (1)$$

where σ_{ij} are the stress components, $\bar{\sigma}$ is the equivalent stress and the exponent k is associated with the crystal structure of the sheet material (3 and 4 for BBC and FCC metals, respectively).

Let us consider that the values of anisotropy coefficients $R_0, R_{45}, R_{90}, R_b, \sigma_0, \sigma_{45}, \sigma_{90}, \sigma_b$ are known (determined from uniaxial and equi-biaxial tensile tests). Our goal is to determine the material parameters a, M, N, P, Q, R, S and T involved in yield criterion (1).

3. Nonlinear system of equations for material parameters identification

Let us proceed from concepts used in [1]. The stress components in a tensile test specimen with orientation angle α with respect to the sheet rolling direction can be written as

$$\sigma_{11} = \sigma_{\alpha} \cos^2(\alpha), \quad \sigma_{22} = \sigma_{\alpha} \sin^2(\alpha),$$

$$\sigma_{12} = \sigma_{21} = \sigma_{\alpha} \cos(\alpha) \sin(\alpha). \quad (2)$$

Substituting the stress components (2) in yield criterion (1) one obtains nonlinear equations for uniaxial yield stresses as

$$\bar{\sigma}(\sigma_{\alpha}, \bar{m}) = \sigma_{ref}, \quad \alpha \in \{0^{\circ}, 45^{\circ}, 90^{\circ}\}. \quad (3)$$

In (3) $\bar{m} = (a, M, N, P, Q, R, S, T)$ is a material parameters vector and σ_{ref} is a yield parameter, which is taken most commonly as one of the following parameters values $\sigma_0^{exp}, \sigma_{45}^{exp}, \sigma_{90}^{exp}, \sigma_b^{exp}$ (upper index exp indicates that experimental value is used) or average of the $\sigma_0^{exp}, \sigma_{45}^{exp}$ and σ_{90}^{exp} .

The expression for the biaxial yield stress σ_b can be obtained by substituting in-plane stress components (equi-biaxial tensile test) $\sigma_{11} = \sigma_{22} = \sigma_b, \sigma_{12} = \sigma_{21} = 0$ in yield criterion (1)

$$\bar{\sigma}(\sigma_b, \bar{m}) = \sigma_{ref}. \quad (4)$$

Similarly, the expressions for Lankford coefficients R_{α} can be derived as

$$R_{\alpha}(\sigma_{\alpha}, \bar{m}) = R_{\alpha}^{exp}, \quad \alpha \in \{0^{\circ}, 45^{\circ}, 90^{\circ}\},$$

$$R_b(\sigma_b, \vec{m}) = R_b^{exp} \quad (5)$$

The system (3)-(5) is presented in implicit form due to conciseness sake. In the space of principal stresses only six equations and six independent material parameters are included in system (3)-(5) (the equations corresponding to angle $\alpha = 45^\circ$ are omitted). Performing theoretical analysis and applying CAS-es (computer algebra systems) method, the nonlinear system (3)-(5) can be transformed into system of two nonlinear equations and set linear equations. The nonlinear equations contain two variables and can be solved separately

$$F_1(x, y) = 0, \quad F_2(x, y) = 0, \quad (6)$$

where new introduced variables x and y are related to material parameters as

$$x = \frac{M-P}{1+N}, \quad y = \frac{M+P}{1-N} \quad (7)$$

The set of linear equations

$$m_i = g_i(x, y), \quad i = 1, \dots, n \quad (8)$$

contains expressions for the material parameters m_i presented in terms of x and y .

4. Material parameters identification as optimal design problem

Let us proceed from the simplified form of the system (3)-(5) i.e. from the equivalent system given by equations (6)-(8). As mentioned above, instead of direct solution of the nonlinear system the error minimization function based algorithm is employed, which allows to consider over-constraining and under-constraining. The error function can be written by means of the Gaussian square of error as

$$\mathcal{E}(x, y) = [F_1(x, y)]^2 + [F_2(x, y)]^2 \rightarrow \min. \quad (9)$$

The system of linear equations (8) is considered as a set of constraints and is not included directly in error function. The design variables x and y are subjected to linear constraints

$$x - x^{upper} \leq 0, \quad -x + x^{lower} \leq 0, \quad y - y^{upper} \leq 0, \quad -y + y^{lower} \leq 0, \quad (10)$$

where the limits x^{upper} , x^{lower} , y^{upper} , y^{lower} can be obtained by substituting the upper and lower bounds of the material parameters M, P and N in (7). The solution of the constrained optimal design problem (9)-(10) is given by the stationary value of the Lagrange function

$$L = \mathcal{E}(x, y) + \lambda_1(x - x^{upper}) + \lambda_2(-x + x^{lower}) + \lambda_3(y - y^{upper}) + \lambda_4(-y + y^{lower}) + \sum_{i=1}^n \gamma_i(m_i - g_i), \quad (11)$$

where λ_i and γ_i stand for Lagrangian multipliers.

Numerical solution of the optimization problem posed in divided into two subtasks: search for global extreme in global and local level (hybrid GA). The global search is performed by use of MATLAB direct search and genetic algorithm toolbox function ga. In order to achieve higher accuracy the real coded algorithm is used. Note, that the expression of the error function can be reduced by replacing quadrates of the functions with the corresponding absolute values, since here is no differentiation needed in realization of the genetic algorithm. However, the local search is performed by use of gradient method, which needs calculation of derivatives of objective function with respect to design variables. For that reason, the objective function in form (9) is considered. The best individual (solution) of the population generated by GA is used as an initial value of the gradient method (local level search). In the cases where elite population (set of solutions obtained by fitness-based selection rule) contains individuals, which chromosomes (parameters) differ substantially it is reasonable to perform local search for all these individuals. Thus, the number local searches necessary depend on result of global search. The local search may be considered as design improvement, since the global search realized by use GA may convergence to solution close to global optimum not exactly to optimum, also the gradient method is less time consuming. The final solution is determined by comparison of the results of all local searches performed (selection is based on value of objective function).

5. Formability analysis of the 6000 series aluminium alloy sheet AA6181-T4

One of the most widely used formability criterion is the forming limit diagram (FLD). The traditional forming limit diagram is described by a curve in a plot of major strain vs. minor strain. This curve defines boundary between elastic or stable plastic deformation (below curve) and unsafe flow (above curve).

Modelling of plastic anisotropy (yield criteria used) is one of the key factors on which FLD depends (also strain hardening law and instability criteria). In the following the higher order anisotropic yield criterion BBC2003 is employed. The FLD is composed for the 6000 series aluminium alloy sheet AA6181-T4 with the following mechanical properties of the material [1] $\sigma_0 = 142$

MPa, $\sigma_{90} = 137$ MPa, $\sigma_b = 134$ MPa, $R_0 = 0.67$, $R_{90} = 0.82$,

$R_b = 0.82$ ($\sigma_{ref} = 142$ MPa). The material parameters identifica-

tion procedure described above results the material parameters a, M, N, P, R and S involved in yield criterion BBC2003 as $a = 0.46$, $M = 1.12$, $N = 1.04$, $P = 0.99$, $R = 0.96$ and $S = 1.02$. The FLD corresponding to yield criterion BBC2003 is depicted in Fig.1. For comparison the forming limits corresponding to Barlat-Lian 1989 and Hill-s quadratic anisotropic yield criteria, respectively are given in the same figure. It is seen from Figure 1 that the forming limit curves corresponding BBC2003 and Barlat-Lian

1989 yield criteria are close to each other. It is not surprising, since the yield criterion BBC2003 (Banabic et al. [1]) is modification of the BBC2002 (Paraianu et al., [8]) and BBC2002 was developed on the basis of the Barlat-Lian 1989 yield criteria.

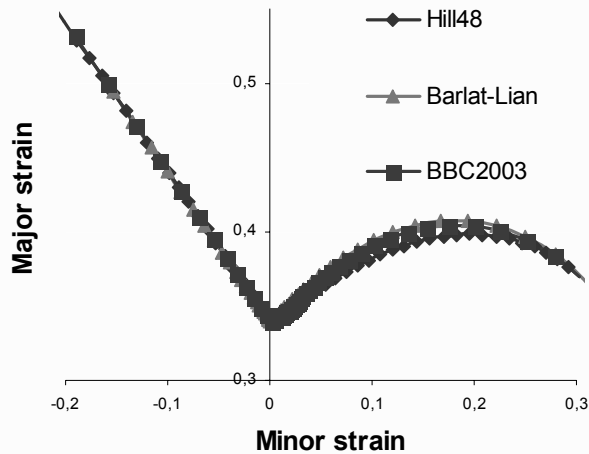


Fig. 1. Strain-based FLD of aluminum alloy sheet A6181-T4

6. Discussion

The material parameters identification procedure for advanced yield criteria (BBC2003) is developed. Current approach allows reducing significantly the dimension of the nonlinear system before its numerical solution. The results of the analysis hold good in more general case (similar yield criteria: BBC2002, BBC2000).

Some positive feedback can be outlined as:

- Dimension of the design space is reduced from six to two;
- Convergence to global extreme can be expected (by taking use GA, although not guaranteed);
- Material parameters identification problems, where number of unknown parameters is not equal with the number of given constraints, are covered. Thus over-constraining and under-constraining are considered. Such situation can be met also in the case of yield criterion BBC2003 (see [1]);
- Since, in the case of global minimum the error function is equal to zero, the accuracy of the obtained solution can be estimated relatively easily (without using necessary optimality conditions).

The problem considered is cumbersome and the following drawbacks can be observed:

- The theoretical and numeric-symbolical analysis should be performed separately for bbc ($k=3$) and fcc ($k=4$) materials;
- The expressions of the functions $F_1(x,y)$ and $F_2(x,y)$ are complicated, closed form analysis of the extremes seems not available.

Initial material parameters identification problem is simplified substantially, but further analysis is justified, since the problem posed is most commonly in use as a subtask in more general application (formability analysis, different kind of sheet metal forming process, FEA applications, etc.). The algorithm proposed is tested on 6000 series aluminium alloy sheet AA6181-T4. The obtained results are found to be in agreement with the results given in [1].

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