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Characteristics of discrete-continuous flexibly vibrating mechatronic system

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Analysis and modelling

ABSTRACT

Purpose: The purpose of this paper is application of approximate method of solving the task of assignment the frequency-modal analysis and characteristics of flexibly vibrating mechatronic system.
Design/methodology/approach: The main approach of the subject was to formulate and solve the problem in the form of set of differential equation of motion and state equation of considered mechatronic model of object. Galerkin's solving method has been used. The considered flexibly vibrating mechanical system is a continuous beam, clamped at one of its end. Integral part of mechatronic system is a transducer, extorted by harmonic voltage excitation, to be perfectly bonded to the beam surface.
Findings: The parameters of the transducer have important influence on values of natural frequencies and on form of characteristics of the discussed mechatronic system.
Research limitations/implications: In the paper the linear mechanical subsystem and linear electric subsystem of mechatronic system has been considered, however for this kind of systems the approach is sufficient.
Practical implications: The methods of analysis and obtained results can be base on design and investigation for this type of mechatronic systems.
Originality/value: The mechatronic system formed from mechanical and electric subsystems with electromechanical bondage has been considered. This approach is different from those considered so far.
Keywords: Applied mechanics; Beam and piezotransducer; Galerkin's method; Dynamical characteristic

1. Introduction

The main interest of industry and scientists during machine design process is to give the attention to their energy conversion efficiency and reliability. Many industry branches focus on the problem of miniaturizing the existing systems and also on reducing their energy absorption. The crucial thin in this matter is to search the new solutions, which will be able to reduce the moving elements and complicated and long kinematical chains. Therefore during the last years there is a specific development within market especially when new technologies which base on the piezoelectricity electro and magnetostriction phenomenon are concerned [5,7,8,10-12,14,15].

The first attempt to solve this problem, that is, to determine the dynamical characteristic of a longitudinally and torsionally vibrating continuous bar system and various classes of discrete mechanical systems in view of the frequency spectrum, by means of graphs and structural numbers methods, and other diverse problems have been modelled by different kind of methods and were examined and analysed in the Gliwice Research Centre in [1,2,13]. The torsionally vibrating mechatronic systems have been considered in the papers [3,4,9].

In this paper the dependences on dynamic characteristics of flexibly vibrating continuous mechanic system combined with piezoelectric converter into the mechatronic system, to examine the piezoelectric influence on the whole complex system. This kind of approach may be an assumption to the examination of flexibly vibrating mechatronic systems, which purpose will be to generate vibrations with assumed parameters.

2. Mechanic system exited with voltage

The subject of deliberation is the homogeneous beam with a full section of area A and with area moment of inertia I, unchanged on the whole length l (Fig. 1). The beam was made of material with Young Modulus E and with mass density ρ. The beam was loaded with harmonic electrical voltage. The piezoelectric converter have been attached in an ideal way to the beam surface. The harmonic excitation with voltage which engenders the harmonic influence on the beam is attached to the clips of piezoelectric actuator. In the analyzed mechatronic system the electrical resistance has been taken under consideration.

The mechatronic system considered in this paper is being treated with bending stress. Therefore the model of piezoelectric converter model considered as a bending actuator is examined. The essential equation of the piezoelectric with its changes stiffness under the influence of electric pole is given as follows [5,7,10-12, 14-15]

$$\sigma_{11} = C_{11}\varepsilon_{11} - d_{31}\frac{U}{h_p} = C_{11}\frac{h_b}{2}\frac{\partial^2 y(x,t)}{\partial x^2} - U\frac{d_{31}}{h_p}, \tag{1}$$

where C_{11}—elastic module of piezoelectric converter measured with certain value of excited voltage d_{31}- piezoelectric converter constance.

Moment caused by transverse electrical loading is described by expression [5,15]

$$M = \frac{\sigma_{11} I_a}{\frac{h_b}{2} + \frac{h_p}{2}} \tag{2}$$

Area moment of inertia of piezoelectric converter after simplifications equals

$$I_a = \frac{b_p h_p}{2}\left(\frac{h_b}{2} + \frac{h_p}{2}\right)^2. \tag{3}$$

Submiting (1) and (4) to (2), after the transformation is obtained

$$M = \left(C_{11}\frac{h_b}{2}\frac{\partial^2 y(x,t)}{\partial x^2} - U\frac{d_{31}}{h_p}\right)\cdot\frac{b_p h_p}{2}\left(\frac{h_b}{2} + \frac{h_p}{2}\right). \tag{4}$$

Moment's extortion is combined with a piezoelectric converter and it is also determined within a range $x \in (z_1, z_2)$. Entering to expression Heaviside's function, influence of the flexible moment is eliminated outside the range, that means

$$M = \left(C_{11}\frac{h_b}{2}\frac{\partial^2 y(x,t)}{\partial x^2} - U\frac{d_{31}}{h_p}\right)\cdot\frac{b_p h_p}{2}\left(\frac{h_b}{2} + \frac{h_p}{2}\right)\left[H(x-z_2) - H(x-z_1)\right]. \tag{5}$$

In case of the mechatronic system (Fig.. 1), the equation of flexible vibration beam is given as

$$\rho A\frac{\partial^2 y(x,t)}{\partial t^2} + EI\frac{\partial^4 y(x,t)}{\partial x^4} = \frac{\partial^2 M}{\partial x^2} \tag{6}$$

and after submitting (6) takes form

$$\frac{\partial^2 y}{\partial t^2} = -a^2\frac{\partial^4 y}{\partial x^4} + c\frac{\partial^4 y}{\partial x^4}H(\cdot) - dU\delta'(\cdot), \tag{7}$$

where: $a = \sqrt{\frac{EI}{\rho A}}$, $c = \frac{C_{11}h_p^2 b_p}{4\rho A}\left(\frac{h_b}{2} + \frac{h_p}{2}\right)$, $d = \frac{d_{31}}{\rho A h_p}\left(\frac{h_b}{2} + \frac{h_p}{2}\right)$,
b_p - wide of beam, $H(\cdot) = [H(x-x_2) - H(x-x_1)]$ - Heaviside's function, $\delta'(\cdot) = \frac{\mathrm{d}\delta(\cdot)}{\mathrm{d}x} = [\delta'(x-x_2) - \delta'(x-x_1)]$ - Dirac's function derivative, $\delta(\cdot)$ - Dirac's function.

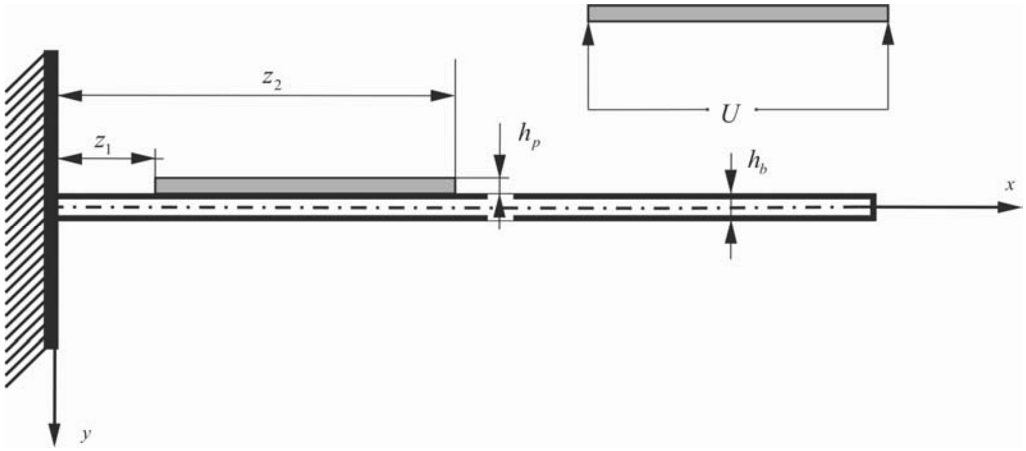

Fig. 1. Mechatronic system with electrical excitation

The piezoelectric converter equation is given as

$$\dot{U}+\frac{1}{2}d_{31}C_{11}b_p h_p\frac{\partial \dot{y}}{\partial x}(l_p,t)=U(t)\ , \tag{8}$$

Considered mechatronic system (Fig. 1) is described by following set of equation

$$\begin{cases}\ddot{y}=-a^2 y_{,xxxx}-cy_{,xxxx}H(\cdot),\\ \dot{U}+\alpha\dot{y}_{,x}(l_p,t)=U(t),\end{cases} \tag{9}$$

where: $\alpha=\frac{1}{2}d_{31}C_{11}b_p h_p$, $U(t)=U_0\cos\omega t$.

3. Determination of dynamical characteristics of flexibly vibrating mechatronic system

According to Galerkin's method the solution is searched in form of the own functions sum, which are the function of time and generalized coordinates, that means

$$y(x,t)=A\sum_{n=1}^{\infty}\sin kx\cdot\cos\omega t\ , \tag{10}$$

where: $k=(2n-1)\frac{\pi}{2l}$, $\omega=\sqrt{\frac{EI}{\rho A}}\cdot\left(\frac{(2n-1)\pi}{2l}\right)^2$.

The solution (10) must obey the boundary conditions in form

$$\begin{cases}y(0,t)=0,\\ y_{,x}(0,t)=0,\\ y_{,xx}(l,t)=0,\\ EIy_{,xxx}(l,t)=0.\end{cases} \tag{11}$$

If the excitation has a harmonic shape then the voltage generated on the piezoelectric clips will have the same character, that means

$$U=B\cos\omega t\ . \tag{12}$$

After nominating certain derivatives regarding time and generalized coordinate and after submitting them to the equations describing vibrating and state of mechatronic system, the set of equation (9) is given as

$$\begin{cases}A\{-\omega^2+a^2k^4+ck^4H(\cdot)\sin kx\cos\omega t+Bd\cos\omega t\delta'(\cdot)=0,\\ -B\omega\sin\omega t-A\alpha\omega k\cos kl_p\sin\omega t=U(t)\end{cases} \tag{13}$$

or in matrix shape

$$\begin{bmatrix}-\omega^2+a^2k^4+ck^4H(\cdot) & d\delta'(\cdot)\\ -\alpha\omega k\cdot\cos kl_p & -\omega\end{bmatrix}\begin{bmatrix}A\\ B\end{bmatrix}=\begin{bmatrix}0\\ U_0\end{bmatrix}, \tag{14}$$

that is

$$\mathbf{W}\ \mathbf{A}=\mathbf{F}\ . \tag{15}$$

The main determinant of equation set (14) is equal

$$|\mathbf{W}|=-\omega\{-\omega^2+a^2k^4+ck^4H(\cdot)\}+\alpha\omega k\cdot\cos kl_p d\delta'(\cdot). \tag{16}$$

Submitting in (14) the first column with another free word column we get the determinant

$$|\mathbf{W}_A|=\begin{vmatrix}0 & d\delta'(\cdot)\\ U_0 & -\omega\end{vmatrix}=dU_0\delta'(\cdot). \tag{17}$$

The amplitude A is nominated as

$$A=\frac{|\mathbf{W}_A|}{|\mathbf{W}|}, \tag{18}$$

therefore

$$A=-\frac{dU_0\delta'(\cdot)}{\omega\{-\omega^2+a^2k^4+ck^4H(\cdot)+\alpha k\cos kl_p d\delta'(\cdot)\}}, \tag{19}$$

and finally the dynamic characteristic-dislocation as the function of voltage is in form

$$Y=-\frac{d\delta'(\cdot)\sin kx}{\omega\{\omega^2-a^2k^4-ck^4H(\cdot)+\alpha k\cos kl_p d\delta'(\cdot)\}}. \tag{20}$$

Absolute value of dynamic characteristic $\alpha_Y=|Y|$, therefore

$$\alpha_Y=\left|\frac{d\delta'(\cdot)\sin kx}{\omega\{\omega^2-a^2k^4-ck^4H(\cdot)+\alpha k\cos kl_p d(\cdot)\}}\right|. \tag{21}$$

On the base of expression (21) the influence of material value changes which directly depend on the sort of piezoelement and its geometrical sizes on characteristics process, vibrations of mechatronic system type and especially its "activation" can be examined. Moreover presented approach determined source to further researches.

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