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Dynamical flexibility of rod and beam systems in transportation

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Analysis and modelling

ABSTRACT

Purpose: Purpose of this paper is to present a mathematical model of rod and beam systems in transportation. The mathematical model is presented in the form of dynamical flexibility of systems. Systems are considered as flexible rotating rods and flexible rotating beams. In solution there was took into account the interaction between the main motion and local vibrations of elements.

Design/methodology/approach: The dynamical flexibility was derived by the approximate method, the Galerkin's method. The example dynamical characteristics were presented in form of attenuation-frequency characteristics. The dynamical flexibility was derived on the basis of known equations of motion derived in other publications.

Findings: There can be observed so called the transportation effect. This effect consist in that when analyzed system rotates with some angular velocity in the characteristic of dynamical flexibility we can notice additional poles and after increasing angular velocity it is noticeable that created modes are symmetrically propagated from the original mode. It is also palpable fact, instead of the original mode there is created zero.

Research limitations/implications: Analyzed systems are simple linear type beams and rods in rotational motion. Motion was restricted to plane motion. Future research ought to consider complex systems, damped models and also nonlinearity.

Practical implications: Practical implications of derived mathematical models of beam and rod systems both the free-free ones and fixed ones is a possibility of derivation of the stability zones of analyzed systems and derivation of eigenfrequencies and zeros in the way of changing the value of angular velocity.

Originality/value: Presented models apply to rotating flexible rod and beam systems with taking into consideration the transportation effect. It is a new approach of analyzing rod and beam systems and can be put to use in modelling and analyzing machines and mechanisms in rotational transportation.

Keywords: Applied mechanics; Numerical techniques; Vibrations; Transportation effect

1. Introduction

There are many technical problems connected with vibrations in contemporary technical sciences and there are many methods of analyzing vibrations. One of the most popular method is a method of dynamical flexibility. This method can be used both to discrete and continuous systems. Thank to this method we can very easily assign resonance zones and find the amplitude of vibrations of the analyzed element, find the zeros of dynamical characteristics where the vibrations are minimally.

In this thesis considered problems apply to rotational beam and rod systems. The rotation is treated as transportation movement. In the literature [2-6, 12-13] there are publications connected with the subject area of vibrating systems in transportation as distinguished from ones connected with the stationary systems [1, 7-11, 14-19]. There are derived mathematical model in form of dynamical flexibility of the system. The dynamical flexibilities were derived on the basis of formerly derived equations of motion.

There are many ways of minimizing amplitudes of vibrations for example in the way of changing forces acting into the systems, changing framework of system or changing the geometrical or physical parameters of the system.

2. Dynamical flexibility of rod systems

In this section there was presented the dynamical flexibilities of rod systems both in form of mathematical model and the dynamical characteristics on the chart (Fig. 1). The equation (1) applies to the dynamical flexibility of stationary free-free system derived by the Galerkin's method.

$$Y(\Omega)=\frac{2}{\rho\cdot A\cdot l}\cdot\sum_{n=0}^{\infty}\frac{\cos(n\pi)\cdot\cos\left(\frac{n\pi x}{l}\right)}{a^2\cdot\left(\frac{n\pi}{l}\right)^2-\Omega^2}, \tag{1}$$

where:
$Y(\Omega)$ – the dynamical flexibility in function of frequency of extorted force,
A – the cross-section of rod,
l – length of the rod,
ρ – mass density of the rod,
n – mode of vibrations of rod,
a – velocity of the wave propagation in the rod,
Ω – frequency of vibrations,
x – the position of analyzed section.

The equation (2) presents the dynamical flexibility of the stationary fixed rod.

$$Y(\Omega)=\frac{2}{\rho\cdot A\cdot l}\cdot\sum_{n=0}^{\infty}\frac{\sin\left[\left(n+\frac{1}{2}\right)\pi\right]\cdot\sin\left[\frac{(2n+1)\pi x}{2l}\right]}{a^2\cdot\left[\frac{(2\cdot n+1)\cdot\pi}{2\cdot l}\right]^2-\Omega^2}. \tag{2}$$

The equation (3) is a dynamical flexibility of free-free rod system rotating with the angular velocity ω.

$$Y=\sum_{n=0}^{\infty}\frac{2\cdot\cos(n\pi)\cdot\left(a^2\cdot n^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right)\cdot\cos\left(n\pi\frac{x}{l}\right)}{\rho\cdot A\cdot l\cdot\left[\left(a^2\cdot n^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right)^2-4\cdot\omega^2\cdot\Omega^2\right]}. \tag{3}$$

The equation (4) is a dynamical flexibility of fixed rod system rotating with the angular velocity ω.

$$Y=\sum_{n=0}^{\infty}\frac{2\cdot\sin\left(\frac{2n+1}{2}\pi\right)\cdot\left[a^2\cdot\left(\frac{2\cdot n+1}{2}\right)^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right]\cdot\sin\left(\frac{2n+1}{2}\pi\frac{x}{l}\right)}{\rho\cdot A\cdot l\cdot\left\{\left[\left(a^2\cdot\left(\frac{2\cdot n+1}{2}\right)^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right)\right]^2-4\cdot\omega^2\cdot\Omega^2\right\}}. \tag{4}$$

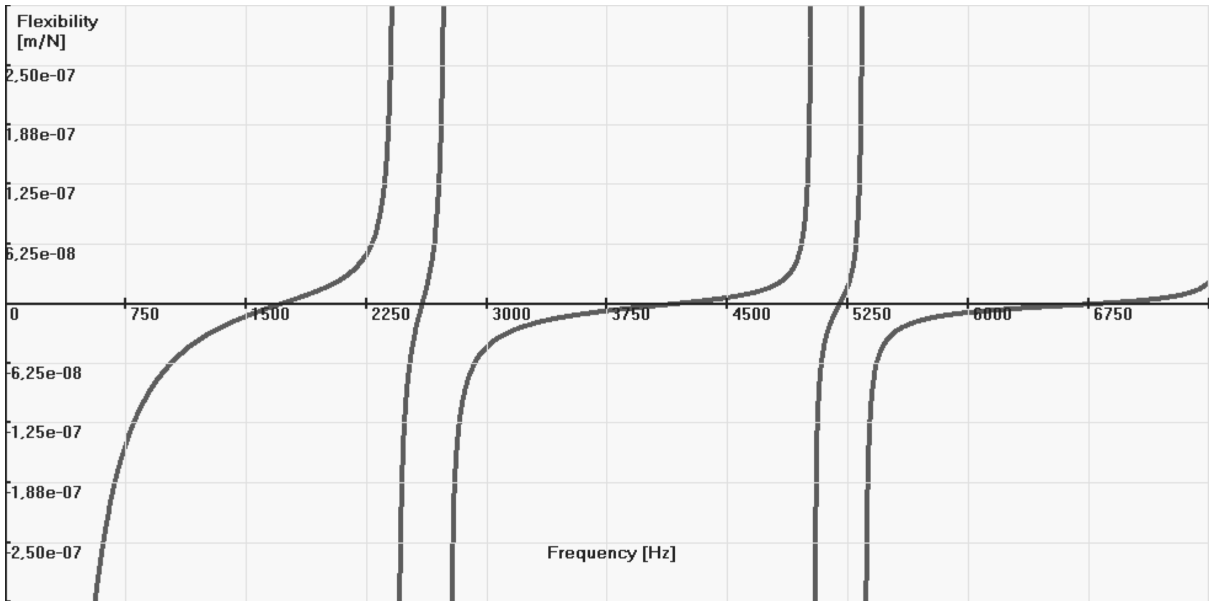

Fig. 1. The dynamical flexibility of free-free rod rotating with angular velocity equals 1000 rad/s

3. Dynamical flexibility of beam systems

This section presents the dynamical flexibilities of beam systems both the stationary ones and in transportation as well. The results are presented as mathematical models of analyzing systems derived by the Galerkin's method and as dynamical characteristics on charts presented onto Figures (Fig. 2 and Fig. 3).

The equation (5) presents dynamical flexibility of free-free stationary beam systems derived on base of known equations of motion of this beam.

$$Y(\Omega)=\frac{1}{\rho\cdot A\cdot\gamma_n^2}\cdot\sum_{n=1}^{\infty}\frac{X(l)\cdot X(x)}{c^2\cdot\left(\frac{2n-1}{2\cdot l}\pi\right)^4-\Omega^2}, \quad (5)$$

where:
I_Z – geometric momentum of inertia,
E – Young modulus,
c – the formula (6):

$$c=\sqrt{\frac{E\cdot I_Z}{\rho\cdot A}}, \quad (6)$$

$$X(x)=\sin(kx)+\frac{\cos(kl)-\cosh(kl)}{\sin(kl)+\sinh(kl)}\cdot\cos(kx)+\sinh(kx)+ +\frac{\cos(kl)-\cosh(kl)}{\sin(kl)+\sinh(kl)}\cdot\cosh(kx) \quad (7)$$

and

$$k\approx\frac{2\cdot n+1}{2\cdot l}\cdot\pi,\quad n=0\Rightarrow k=0. \quad (8)$$

The equation (8) presents dynamical flexibility of fixed stationary beam systems derived on base of known equations of motion of this beam.

$$Y(\Omega)=\frac{1}{\rho\cdot A\cdot\gamma_n^2}\cdot\left[\frac{X(l)\cdot X(x_{k=0})}{\Omega^2}+\sum_{n=1}^{\infty}\frac{X(l)\cdot X(x)}{c^2\cdot\left(\frac{2n+1}{2\cdot l}\pi\right)^4-\Omega^2}\right], \quad (9)$$

where:

$$X(x)=\sin(kx)+\frac{\cos(kl)+\cosh(kl)}{\sin(kl)-\sinh(kl)}\cdot\cos(kx)+ -\sinh(kx)-\frac{\cos(kl)+\cosh(kl)}{\sin(kl)-\sinh(kl)}\cdot\cosh(kx), \quad (10)$$

$$k\approx\frac{2\cdot n-1}{2\cdot l}\cdot\pi. \quad (11)$$

The dynamical flexibility of rotating free-free beam with angular velocity signed as ω with the same *X(x)* as in the equation (7).

$$Y=\frac{-X(l)\cdot X(x_{k=0})\cdot(\Omega^2+\omega^2)}{\rho\cdot A\cdot\gamma_n^2\cdot(\Omega^2-\omega^2)^2}+ +\sum_{n=1}^{\infty}\frac{-\left[c^2\cdot\left(\frac{2\cdot n+1}{2\cdot l}\cdot\pi\right)^4-\Omega^2-\omega^2\right]\cdot X(l)\cdot X(x)}{\rho\cdot A\cdot\gamma_n^2\cdot\left[\left(c^2\cdot\left(\frac{2\cdot n+1}{2\cdot l}\cdot\pi\right)^4-\Omega^2-\omega^2\right)^2-4\cdot\omega^2\cdot\Omega^2\right]}. \quad (12)$$

$$Y=\sum_{n=1}^{\infty}\frac{\left[c^2\cdot\left(\frac{2\cdot n-1}{2}\cdot\frac{\pi}{l}\right)^4-\Omega^2-\omega^2\right]\cdot X(l)\cdot X(x)}{\rho\cdot A\cdot\gamma_n^2\cdot\left\{\left[c^2\cdot\left(\frac{2\cdot n-1}{2}\cdot\frac{\pi}{l}\right)^4-\Omega^2-\omega^2\right]^2-4\cdot\omega^2\cdot\Omega^2\right\}}, \quad (13)$$

where:

$$\gamma_n^2=\int_0^l X^2(x)\,dx. \quad (14)$$

In the Figure 2 the dynamical flexibility of the free-free beam rotating with angular velocity equal 100 rad/s was presented.

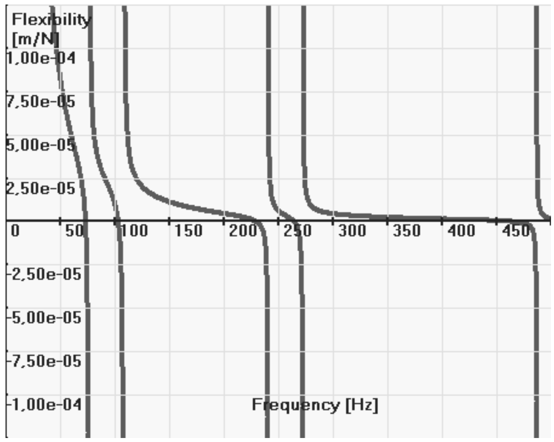

Fig. 2. Dynamical flexibility of the free-free beam system rotating with angular velocity equal 100 rad/s

In the Figure 3 there was presented the dynamical flexibility of fixed beam rotating with angular velocity equal 100 rad/s.

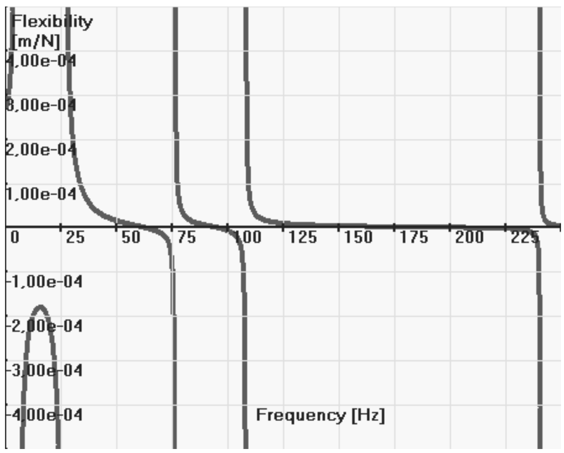

Fig. 3. Dynamical flexibility of the fixed beam system rotating with angular velocity equal 100 rad/s

4. Conclusions

The comfortable method of analyzing vibrations is a method of dynamical flexibility presented in this thesis. The numerical application Modyfit was used to generate the dynamical characteristics shown in this work. Working motion treated here as transportation changes the dynamical characteristics and moves the zeros and modes of dynamical flexibility together with increase of angular velocity both rods and beams. The so called transportation effect has more affect onto beam systems than rod ones.

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