The reverse engineering to optimise the dimensional conical spur gear by CAD

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Analysis and modelling

ABSTRACT

Purpose: This paper propose a method to optimise the module of cutting conical spur gear, after being worn or broken, with the aid of Computer-Aided Design (CAD). The suggested method allows determining the geometric features of a pair of conical spur gear after worn.

Design/methodology/approach: The approach consists of drawing real tooth model, with dimensioning characteristics by CAD; extracting a database of tooth drawing that permits the determination of the real volume by mass destruction. It also allows creating a virtual model, by theoretical geometric characteristics, to calculate the volume.

Findings: This approach allows giving an approximation of the conditions of the machine function. However, the optimisation of the module is only a link one chain, for other parameter influences on the good working of pairs; we could mention the condition of assembly and the tribological aspects.

Practical implications: A simulation package, R2000, was used and an "AutoCAD" software has been developed to accomplish the drawing of 2D wheel conical spur gear, the verification of the system assembly and the drawing of a 3D volume pattern.

Originality/value: The numerical program gives, solution of optimization problem which consist of searching an extreme by minimising a constrained, unstrained function $f(x)$ such that Min $f(x)$ = -Max $f(x)$.

Keywords: Optimisation; CAD; Conical spur gear; Spherical tooth; Octoid toot

1. Introduction

The reverse engineering is a process, which is used, in applied research to increase the performances of a mechanism and in maintenance to establish the working drawings of definitions for their manufacturing processes. These processes rely on to the techniques of the measure and their reliabilities as to the numerical techniques of computational optimization and the CAD tools.

According to the work of Lefevre [1], the correct function of gearing is bound (connected) to the geometrical definition [2] of their respective position in the space. The variation of the distance between centres by wear of the support modifies considerably the conditions of functioning [3], and forces the most fragile set of teeth to a more or less fast destruction [4].

The work that has been given to us consists to establish the working drawing for the pairs of conical spur gear, after worn [5].

This paper presents an approach that consists of using the destruction mass phenomena [2, 4] to optimise the module of cutting, at the, least of material since there is a dimensional relation between the module of cutting and the tooth wheel.

We are using for this optimisation a CAD tools, which permits two types of programming. The parametric programming [2] is used to draw the tooth, and extract their geometric as well as mechanical data base characteristics. The Numerical programming [6] gives solutions to an optimisation problem.

2. Geometrical characteristics of the conical spur gear

In the literature, two types of tooth are considered [7, 8]:
- Spherical shape in involutes tooth contour: This contour is generated by spherical trigonometric theory [2, 8]. It is difficult to make manufacture with standard machine tools but by specific machine, because tooth presents an inflexion point, Figure 1.
- The tooth contour application, which consists of a pyramidal or octoid tooth. Therefore, we have tooth of first kind and second kind [2, 7, 8], the general Equation of thickness at any point is given by following Equation

$$e_m = R \sin \delta \left[\frac{e}{r_p} + 2(\theta_p - \theta) - 2 \left[\arccos \left(\frac{tg\delta_b}{tg\delta_p} \right) - \arccos \left(\frac{tg\delta_b}{tg\delta} \right) \right] \right] \quad (1)$$

where

$$\theta = \frac{B}{\sin \delta_b} \quad \text{as shown in Fig. 1.}$$

The thickness at the base is given by boundary condition:
$$\delta = \delta_b \quad \text{at} \quad \theta = 0,$$

Therefore, it can be written as the following form:

$$e_b = R \sin \delta \left[\frac{e}{r_p} + 2 \left[\theta_p - \arccos \left(\frac{tg\delta_p}{tg\delta_b} \right) \right] \right] \quad (2)$$

where

$$e = \frac{\pi . m}{2} \quad \text{and} \quad r_p = \frac{m.z}{2}$$

(P): spherical involutes contour
(C): basis circle who passes by M
(Cp): basis cone of addendum
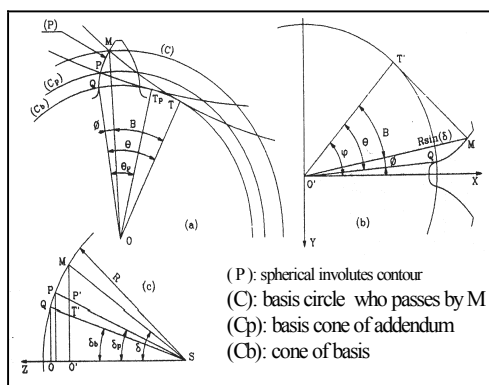(Cb): cone of basis

Fig. 1. Spherical involutes tooth contour [5]

3. Optimisation approach

There exists the mass destruction, due to the matter transfer between the wheels, when two gears are working together. This different destruction is one of the causes of a progressive worn or broken to be under sliding [1, 2, 4].

According to Shipley [9], gear fails when it can no longer efficiently do the job, which is being designed. Cause of failure may be caused by excessive wear to a complete breakage as given in Figs. 2a and b) that studied in the present paper.
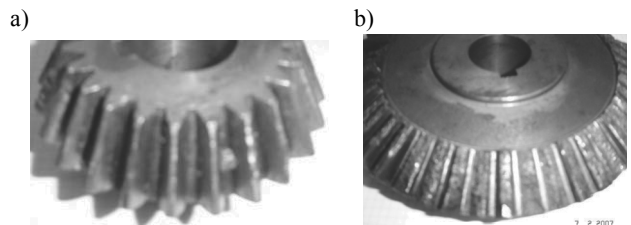
a)   b)

Fig. 2. The pinion after worn a) and the wheel after worn b)

Some authors among them Lefevre [1], Henriot [8], Shipley [9], Borsof [10], classified the failure of gearings in two categories which are wear and fracture:
- Wear: a surface phenomenon in which layers of metal are removed, (worn away), more or less uniformly from the contacting surfaces of the gear teeth .The authors classified wear to moderate wear, excessive wear, abrasive wear corrosive wear, the Pitting and the Scoring,
- Fracture caused by breakage of whole tooth or a substantial portion of tooth; this can result from overload or more commonly, by cyclic stressing of the gear tooth beyond the endurance limit of the material.

The approach is based on the use the wear and fracture phenomenon to optimise the tooth volume. The phenomena of mass destruction, allows us to propose an algorithmic procedure that consists of:
- drawing real tooth after worn, with dimensioning characteristic by ACAD 2000;
- creating the virtual model by theoretical geometric characteristic for calculating the volume;
- optimizing, the cutting module, which includes:
  - dimensional constraint,
  - mechanical resistance.

The problem is to maximize the volume with a minimal module of cutting.

Knowing the objective function subjected to a set of inequality constraints. Our approach consists of determining an extreme value of the module of cutting using a gradient method.

3.1. CAD drawing real tooth after tooth worn

In this part, the algorithmic sequence is as fellows:
- enter the geometric data of the measured pinion,

- draw the tooth by parametric programming,
  - if the tooth is a spherical model: calculate the inflexion point,
  - if the tooth is octoid model: select the type called octoid form,
- calculate the thickness at any point'
- generate the tooth shape,
- calculate the area of the tooth,
- determine the real volume after tooth worn.

Determination of real volume after tooth worn

The optimisation procedure is to calculate the volume after tooth worn that is established by the CAD system. The sequence of computation is given below:

- generate the tooth, in two-dimension drawing,
- determination of the area S_1, of the tooth and the coefficient k_n

$$K_1 = Ra - Rf,$$

$$K_2 = Rai - Rfi,$$

$$K_n = \frac{K_1}{K_2}$$

- determine the area S_2 and the supplement area (S_m):

$$S_2 = \frac{S_1}{K_n}$$

$$S_m = \frac{S_1 - S_2}{2}$$

- determine the total volume (V_T):

$$V_{T:} = S_1 . b$$

- determine the supplement volume V_1 and V_2:

$$V_1 = S_m \frac{b}{\cos\theta_a}$$

$$V_2 = S_m \frac{b}{\cos\theta_f}$$

- determine the real volume V_r:

$$V_r = V_T - (V_1 - V_2)$$

3.2. Creating a theoretical model

The algorithm for the virtual volume follows the steps:
- enter the machine characteristics P, N, Z_1, Z;
- introduce the geometric characteristics of the measured pinion;
- calculate the module of cutting using the ISO method [6] for the pinion and the wheel;

$$m_1 = \frac{da_1}{\left(Z_1 + 2\cos\delta\right)} = \frac{df_1}{\left(Z_1 - 2.5\cos\delta\right)}$$

$$m_2 = \frac{da_2}{\left(Z_2 + 2\cos\delta\right)} = \frac{df_2}{\left(Z_2 - 2.5\cos\delta\right)}$$

- calculate the average module of cutting;

$$m = \frac{m_1 + m_2}{2}$$

- select the normalised module of cutting by default and, select the number of iteration n;
- calculate the difference and the ratio;

$$dif = Ra - Rf$$

$$a = \frac{dif}{n}$$

- calculate the base ray Rb and the curvature ray of the tooth

for $1 \le i \le n$
p = 1
i= i + p
for i= 1

$$R_1 = Rb = \left(\frac{m.Z_1}{2}\right)\cos\alpha$$

$$R_2 = (R_1 + a)$$
$$R_3 = (R_2 + a)$$
$$R_i = (R_{(i-1)} + a)$$

$$\cos\alpha_i = \frac{Rp}{Ri}\cos\alpha$$

- calculate the tooth thickness at each point;
- calculate the area variation of the tooth :

$$\Sigma\Delta S = \Sigma\Delta R.\Delta e$$

$$\Sigma\Delta R = (R_2 - R_1) + (R_3 - R_2) + \ldots\ldots + (R_i - R_{i-1})$$

$$\Sigma\Delta S = (R_2 - R_1).e_1 + (R_3 - R_2).e_2 + \ldots\ldots + (R_i - R_{i-1}).e_n$$

- calculate the volume:

$$\Sigma\Delta V = \Sigma\Delta S.\Sigma\Delta b_i$$

in which

$$b_i = \frac{3R_i\cos\delta_f}{\sin\chi}$$

where

$$\widehat{\chi} = (\delta_f + var)$$

and

$$var = \frac{\delta_a - \delta_f}{n}$$

3.3. Optimisation constraints

a. Dimensional constraints:
- the volume of the tooth after worn must be less than that volume of the tooth model;

- the second constraint is that the head diameter should not exceed the diameter area of site.

The validation of sampling requires the knowledge of the average coasts, which in fact impose a statistical calculation of the dimensioning (Appendix A1). The law governing this calculation is based on the law of the big numbers [2].

b. Mechanical resistance:

The tooth is considered an impacted beam, where we suppose that an effort is applied to the extremity of one single tooth (most unfavourable case). This effort is decomposed into a normal effort (N) and a tangential effort (T). The tangential effort produces the flexion of the tooth [8, 11, 12]. Thus the conditions of mechanical resistance are:

- the normal constraint (σ_n) must be less than that the practice constraint in the flexion (σ_f)
- the second constraint consist to the normal constraint should not exceed the Hertzian practice constraint (σ_p,) in the superficial pressure. The Appendix A2 gives the calculation of the normal constraint [8, 11, 12].

4. Formulation of problem

Doing the change of variable following m= X, Thus, the objective function is written as:

V(X) = Max Σ R$_i$(X) b$_i$(X) e$_i$(X)

Subjected to the constraints defined as follows:

V (X) > Vr

da (X) < De

σ_n (X) < σ_f

σ_n (X) < σ_p

Our application concerns the pairs of conical spur gear of a glass machine moulding.

5. Results and discussion

The Table 1, indicates the characteristics of the machine and the results of the module calculation established for different geometry of teeth.

Table 1.
Machine and pinion characteristics

Machine characteristics			Pinion characteristics		
N (tr/mn)	Vg (m/s)	Ng (tr/mn)	material	tooth	Calculated module
350	1.03	217.217	25 CD4	spherical	4
80	0.24	49.64	25 CD4	spherical	4
350	1.09036	216.921	25 CD4	oct/1st type	4
80	0.249	45.68	25 CD4	oct/1st type	4
350	1.09334	217.513	25 CD4	oct/1st type	4
80	0.24990	49.71	25 CD4	oct/1st type	4

The calculation results on the pairs of conical spur gear indicate that it is possible to adapt geometry of original tooth by another adaptable spherical or octoil shape, with the available machine tools. It is important to remark that the optimised module does not alter the variation of the strength or that of profile. Among others, one recovers the same material, which resists to the imposed constraints. This not variation of the module, allows us to choose for the same material different shape profiles according to the available means of manufacture The working of drawing is established with the aid of a 3D volume model, shown in Fig. 3.
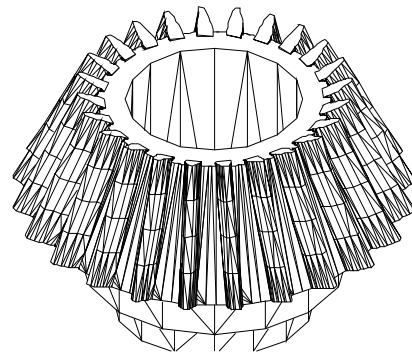

Fig. 3. Volume drawing in 3D of the pinion for m = 4 and first kind octoid

6. Conclusions

The suggested method allows determining the geometric features of a pair of conical spur gear after worn. This approach allows giving an approximation of the conditions of the machine function. However, the optimisation of the module is only a link one chain, for other parameter influences on the good working of pairs; we could mention the condition of assembly and the tribological aspects.

A simulation package, R2000, was used and a special "AutoCAD" software has been developed to accomplish the drawing of 2D wheel conical spur gear, the verification of the system assembly and the drawing of a 3D volume pattern.

Appendix

A1. Statistical dimensioning

Stipulates that for the validation of a sampling it is enough that the number of results (measures) answers in:

$$n \geq \frac{\sigma^2}{\left[\left(1 - p\right).E^2\right]}$$

Remark:

For our case, we have imposed a threshold of probability p = 0.95 is 95 %.

Among others, we have taken E = 0.02 (the acceptable threshold for our measuring instruments).

Determination of standard deviation that is given by the following equations:

$$\sigma = \sqrt{\frac{(m_1 - \bar{m})^2 + (m_2 - \bar{m})^2 + ... + (m_n - \bar{m})^2}{n-1}} = \left[\frac{1}{n-1}\sum_{i=1}^{n}(m_i - \bar{m})^2\right]^{\frac{1}{2}}$$

$$\bar{m} = \frac{(m_1 + m_2 + ... + m_n)}{n} = \frac{1}{n}\sum_{i=1}^{n}m_i$$

A2. Determination of mechanical resistance

The normal constraint and tangential effort are gives by following equations

$$\sigma_n = \frac{5.48.T}{10.bn.m}$$

where $b_n = K . m$
and $6 \leq K \leq 12$ [8, 11, 12]

$$T = \frac{60P}{\pi.m.Zp.N}$$

Nomenclature

b:	The width of the tooth
da:	Diameter of head
de:	Diameter area of site
df:	Diameter of tooth
E:	The level of the reserved absolute error
e_b:	Thickness at the base
e_m:	Thickness of the tooth
m:	Module of cutting
$\bar{m}$:	Indicate the average of the measures
m_i:	Indicate the number of measure
n:	Indicate the number of measure of the same dimension
N:	Rotation velocity
Ng:	Slip rotation velocity
p:	The compulsory threshold of probability (vary from 0 to 1)
P:	Power the machine
Ra:	Extern head ray
Rai:	Intern head ray
Rb:	Base ray
Rfi:	Inter foot ray
R_f:	Foot ray
Rp:	Addendum ray
Vg:	Slip velocity
Z_1:	Pinion tooth number
Z_2:	Wheel tooth number
α:	Pressure angle
δ:	Angle of addendum cone
σ:	Standard deviation
σ_n:	Normal constraint
σ_f:	Practice constraint in the flexion
σ_p:	Hertzian practice constraint

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