

# Process control using reliability based control charts

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## Analysis and modelling

### ABSTRACT

**Purpose:** The paper presents the method to monitor the mean time between failures (MTBF) and detect any change in intensity parameter. Here, a control chart procedure is presented for process reliability monitoring. Control chart based on different distributions are also considered and were used in decision making. Results and discussions are presented based on the case study at different industries.

**Design/methodology/approach:** The failure occurrence process can be modeled by different distributions like homogeneous Poisson process, Weibull model etc. In each case the aim is to monitor the mean time between failure (MTBF) and detect any change in intensity parameter. When the process can be described by a Poisson process the time between failures will be exponential and can be used for reliability monitoring.

**Findings:** In this paper, a new procedure based on the monitoring of time to observe  $r$  failures is also proposed and it can be more appropriate for reliability monitoring.

**Practical implications:** This procedure is useful and more sensitive when compared with the  $\lambda$ -chart although it will wait until  $r$  failures for a decision. These charts can be regarded as powerful tools for reliability monitoring.  $\lambda r$  gives more accurate results than  $\lambda$ -chart.

**Originality/value:** Adopting these measures to system of equipments can increase the reliability and availability of the system results in economic gain. A homogeneous Poisson process is usually used to model the failure occurrence process with certain intensity.

**Keywords:** Availability; Process reliability; Control charts; Failure rate; Mean time between failures

## 1. Introduction

Reliability is the probability of the equipment or process functioning without failure, when operated as prescribed for a given interval of time, under stated conditions. High costs motivate seeking engineering solutions to reliability problems for reducing financial expenditures, enhancing reliability, satisfying customers with on-time deliveries through increased equipment availability, and by reducing the cost and problems arising from products that fails easily. Long failure free periods result in increased productivity, fewer spare parts need to be stocked and less manpower employed in maintenance activities, and hence lower costs. Increased availability, decreased down time, smaller

maintenance costs and lower secondary expenditure result in a bigger profit.

Setting reliability requirements is a corner stone of any reliability strategy. The reliability requirement have a quantitative part and that is mean time between failure (MTBF). MTBF is specified for a component and systems whose failures are characterized by a constant hazard rate. A method for improving the reliability requirements is minimum mean time between failures (MMTBF). The requirements include two key components: a specified MMTBF and a maximum acceptable probability of premature failure. Probability of premature failure  $p_r$  is the probability that the time to failure will be smaller than minimum mean time between failure (MMTBF).

Failure process monitoring is an important issue for complex or fleet systems. Statistical control charts are widely used process monitoring tool in manufacturing industry, and can be used in this type of failure monitoring process. This is usually done by plotting the number of failures or breakdowns per unit time such as week or month. Standard c- chart or u- chart, which is for monitoring number of defects can be used for this purpose. However, this procedure requires large number of failures and it is not appropriate for application to a high reliable system such as process industries.

When there is excessive number of failure, the chart will signal in such an out of control situations. Although the anticipated false alarm probability, the probability that the process is not changed when plot shows an alarm, is 0.27% by a traditional chart, it could be much higher because when the number of failures is Poisson distributed. A new procedure proposed by Chan [5] is based on the monitoring of cumulative production quality between the observations of two defects in a manufacturing process. This approach can be extending to cumulative time to  $r$  failures. It also possible to find the value of  $r$  which will give maximum accurate result.

The value of reliability to be improved can be decided based on the control chart. Process industries can apply control chart method to monitor improvement in reliability so that the necessary steps can take, if the system is not getting the required value. Every process industry follows a probability distribution based on the failure data.

## 2. Methodology

### 2.1. Setting reliability requirements to minimize the total cost

Decreasing the probability of premature failure  $p_f$  can only be achieved by increasing the reliability of the system. Since increasing reliability of the system requires resources, an optimization procedure can be constructed to minimize the sum of potential losses due to premature failure and cost of resources invested in reliability improvement.

Assume a specified MMTBF  $s$  and a current hazard rate  $\lambda$  of the system. Assume that  $C$  gives the mean value of consequences due to premature failure. The consequences of premature failure at the beginning of MMTBF is different from that occurring at the end. Let  $Q$  be the cost of decreasing the current system hazard rate  $\lambda$  by an amount  $x$ ; when  $x=0$ ,  $Q=0$ .

The total losses  $L(x)$  from decreasing the current system hazard rate by an amount  $\lambda$  by an amount  $x$  are then,

$$L(x) = Q + \int_0^s (\lambda - x) \exp[-(\lambda - x)t] C dt \quad (1)$$

The potential losses without reducing the hazard rate are,

$$L(0) = \int_0^s \lambda \exp(-\lambda t) C dt \quad (2)$$

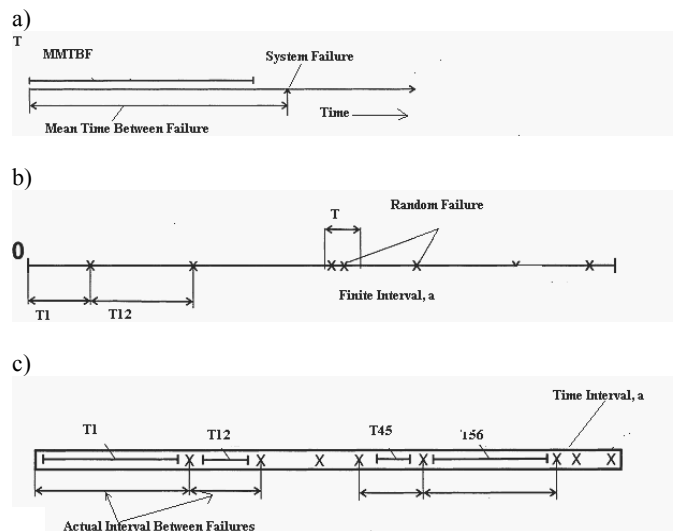


Fig. 1. (a) The MMTBF reliability requirements of an MMTBF interval and a maximum acceptable probability of premature failure. (b) clustering of two or more failures within a small interval of time. (c) for a given number of failures on a finite interval, calculating the probability of a set of specified MMTBF

The relative change of losses due to  $x$  of the hazard rate is,

$$\Delta L = L(x) - L(0) \quad (3)$$

The optimal value of  $\lambda$  giving the smallest amount of total losses  $L(x)$  can be regarding  $x$  ( $0 \leq x \leq \lambda$ ) where  $\lambda$  is the current hazard rate.

### 2.2. $\lambda$ chart

The failure occurrence process can be modeled by different distributions like homogeneous Poisson process, Weibull model etc. In each case the aim is to monitor the mean time between failure (MTBF) and detect any change in intensity parameter. When the process can be described by a Poisson process the time between failures will be exponential and can be used for reliability monitoring.

The distribution function of exponential distribution with parameter  $\frac{1}{MTBF}$  is

$$F\left(t, \frac{1}{MTBF}\right) = 1 - e^{-\frac{t}{MTBF}} \quad (4)$$

The distribution function of Weibull distribution is

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right] \quad (5)$$

The control limits of  $\lambda$  chart are defined in such a manner that the process is said to be out of control when the time to observe exactly one failure is less than LCL or above UCL. When the process is normally distributed there is chance for this to happen and it is commonly known as false alarm. The traditional false alarm probability is 0.27%, but any other value can be used. The actual false alarm probability depends on the product or process. It will vary from company to company.

Assuming an acceptable false alarm probability  $\alpha$ , the control limits can be obtained from the following equations:

$$\begin{aligned} 1 - e^{-\frac{LCL}{MTBF}} &= 1 - \alpha / 2 \\ 1 - e^{-\frac{CL}{MTBF}} &= 0.5 \\ 1 - e^{-\frac{UCL}{MTBF}} &= \alpha / 2 \end{aligned} \quad (6)$$

The control limits are:

$$\begin{aligned} LCL &= \left( \frac{1}{MTBF} \right)^{-1} \ln \left( \frac{1}{1 - \alpha / 2} \right) \\ CL &= \left( \frac{1}{MTBF} \right)^{-1} \ln 2 \\ UCL &= \left( \frac{1}{MTBF} \right)^{-1} \ln \left( \frac{2}{\alpha} \right) \end{aligned} \quad (7)$$

These control limits can be utilized to monitor the reliability. After each failure, the time can be plotted on the chart. If the plotted points falls between the calculated control limits, it indicates that the process in the state of statistical control and no action is warranted. If the point falls above the UCL, it indicates that the process average, or the failure occurrence rate, may have decreased which results in an increased time between failures. This is an important indication of possible process improvement. If it happens, the management should take action to find out the causes and maintain it. If the plotted point falls below the LCL, it indicates that the process average, or the failure occurrence rate, may have decreased which resulted in the decrease in failure time. This means that the process may have deteriorated and thus actions should be taken to identify and remove them.

In either case, the people involved can know when the reliability of the system is changed and a proper follow up can be made to improve the reliability. The control chart will also help the maintenance personals.

### 2.3. $\lambda_r$ chart

Monitoring the failure occurrence process using  $\lambda$  chart is straight forward. However, since the decision is based on one observation, it may cause many false alarms or it is insensitive to process shift if the control limits are wide. To deal with this problem the time between  $r$  failures can be monitored. The time between  $r$  successive failures can be denoted by  $\lambda_r$ .

To monitor the system based on the time between the occurrences of  $r$  failures, we need a distribution to model the cumulative time till the  $r$ th failure,  $t_r$ : It is well known that the sum of  $r$  exponentially distributed random variables is the Erlang distribution. An Erlang random variable is defined as the length until the occurrence of  $r$  defects (failures) in a Poisson process.

Then the probability density function of  $\lambda_r$  is given as:

$$f\left(t_r, r, \frac{1}{MTBF}\right) = \frac{\left[\frac{1}{MTBF}\right]^r t_r^{r-1}}{(r-1)!} \exp\left\{-\left[\frac{1}{MTBF}\right] t_r\right\} \quad (8)$$

The cumulative Erlang distribution is

$$F\left(t_r, r, \frac{1}{MTBF}\right) = 1 - \sum_{k=0}^{r-1} \frac{\left(\left[\frac{1}{MTBF}\right] t_r\right)^k}{k!} \exp\left\{-\left[\frac{1}{MTBF}\right] t_r\right\} \quad (9)$$

To calculate the control limits of the  $\lambda_r$ -chart, the exact probability limits will be used. If  $\alpha$  is the accepted false alarm risk then the upper control limit,  $UCL_r$ , the center line,  $CL_r$ , and the lower control limit,  $LCL_r$ , can be easily calculated by using Eq. (10), (11) & (12) in the following manner:

$$F\left(UCL_r, \frac{1}{MTBF}\right) = \sum_{k=0}^{r-1} \frac{\left(\left[\frac{1}{MTBF}\right] UCL_r\right)^k}{k!} \exp\left[-\frac{1}{MTBF} UCL_r\right] = 1 - \alpha / 2 \quad (10)$$

$$F\left(CL_r, \frac{1}{MTBF}\right) = \sum_{k=0}^{r-1} \frac{\left(\left[\frac{1}{MTBF}\right] CL_r\right)^k}{k!} \exp\left[-\frac{1}{MTBF} CL_r\right] = 0.5 \quad (11)$$

$$F\left(LCL_r, \frac{1}{MTBF}\right) = \sum_{k=0}^{r-1} \frac{\left(\left[\frac{1}{MTBF}\right] LCL_r\right)^k}{k!} \exp\left[-\frac{1}{MTBF} LCL_r\right] = \alpha / 2 \quad (12)$$

The decision-making procedure for the  $\lambda_r$ -chart remains same as the  $\lambda$ -chart. A point plotted below the LCL signifies deterioration of process and warrants corrective action, while a point plotted above the UCL denoted process improvement and action should be taken to identify the cause of improvement and to maintain it. From reliability point of view, a point plotted below the control limit points out that the time between failures may have decreased while a point plotted above the control limit indicates that the time between failures may have increased.

## 3. Case studies

### 3.1. Chocolate manufacturing company

The control chart procedure described above is applied in a carton manufacturing company. The block diagram for the process is shown in Fig.2. The system consists of draft fan, feed pump, boiler, condensate recirculation pump, cookers, cooling tables, mixer, forming machines, conveyer, wrapping machine and scrap remover. Here all components are connected in series and the

failure of one component results in the failure of system. In order to identify the critical element, the component which contributes more unavailability is identified which is illustrated in Fig.3 .Condensate recirculation pump contributes 51.6295579 % of the unavailability and is the critical element. The value of failure rate ( $\lambda$ ) is 0.0037 for condensate recirculation pump and is modified.

Table 1.  
Failure data

Component	MTBF	MTT	R(t)	Ai
Time(t) = 700 Hrs				
Draft Fan	3967	3	0.838235862	0.99924
FeedFump	3426	2	0.815201541	0.99941
Boiler	4675	12	0.860938143	0.99743
CEP	267.5	4	0.073034567	0.98526
Cooker 1	3693	10	0.827333175	0.99729
Cooker2	3403	10	0.814076569	0.99707
Cooker3	3487	10	0.818120513	0.99714
Cooker4	3298	10	0.808762604	0.99697
CT1	5689	6	0.884224339	0.99894
CT2	6345	6	0.89554474S	0.99905
CT3	60S9	6	0.891400546	0.99901
CT4	6234	6	0.893787296	0.99903
Mixer	2476	1	0.753735284	0.99959
Forming M/c	6476	4	0.897545544	0.99938
Conveyer	4276	4	0.84899295	0.99906
Rope sizer	3509	2	0.819150842	0.99943
Wrappin M/cl	4008	7	0.839750295	0.99825
Wrappin M/c3	4760	7	0.863243197	0.99853
Wrappin M/c3	4230	7	0.847482886	0.99834
Scrap remover	6057	3	0.890859312	0.99950

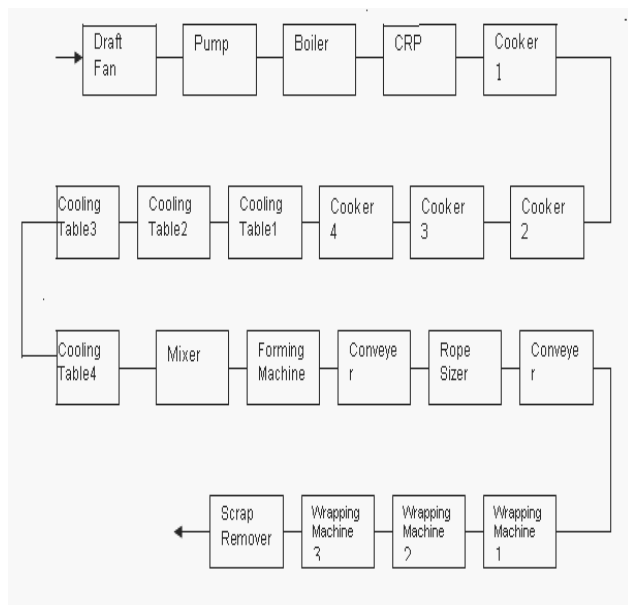


Fig. 2. Process description

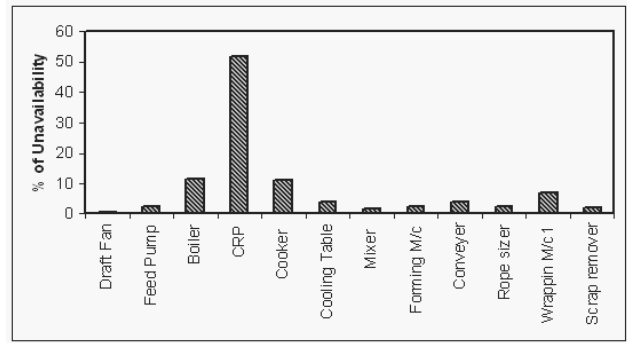


Fig. 3. Percentage of unavailability due to each component

Table 2.  
Some control limits of  $\lambda_r$  chart

12 Chart				13 Chart		
A	UCL	CL	LCL	UCL	CL	LCL
0.0010	8900	1678	52	10869	2633	211
0.0020	4450	839	26	5434	1316	105
0.0030	2966	559	17	3748	908	72
0.0040	2250	419	13	2717	658	53
0.0050	1780	335	10	1976	486	38
0.0060	1483	279	9	1672	417	32
0.0070	1271	239	7	1509	371	29
0.0080	1112	209	6	1358	334	26
0.0090	988	186	5	1207	297	23
0.0100	890	167	4	1036	267	21

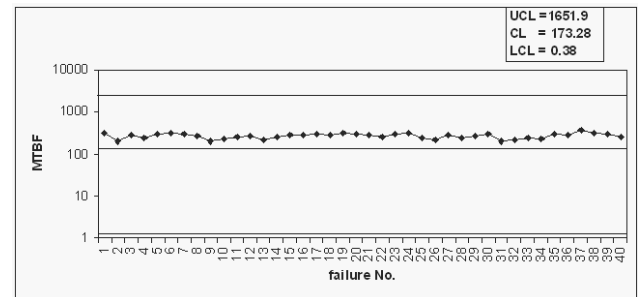


Fig. 4.  $\lambda$  Chart before modification

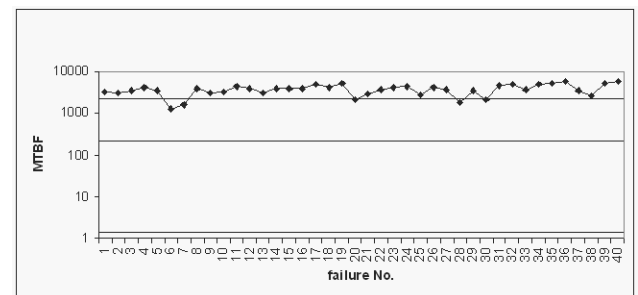
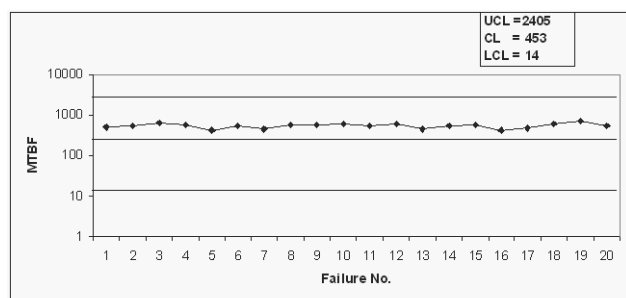
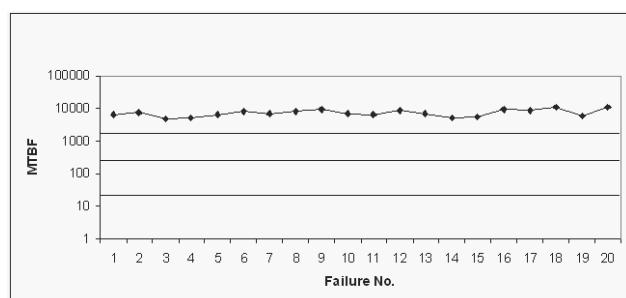
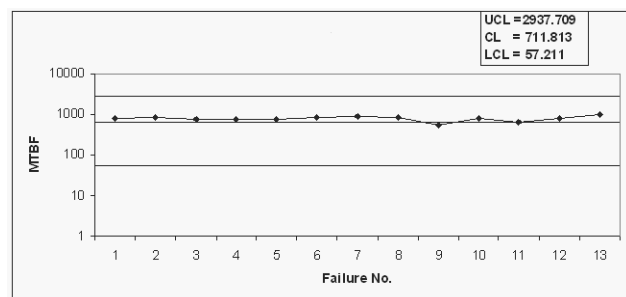
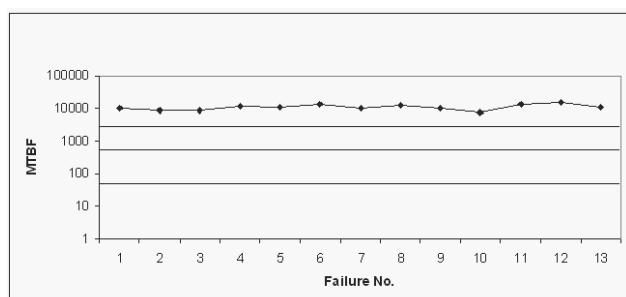


Fig. 5.  $\lambda$  Chart after modification

Fig. 6.  $\lambda_2$  Chart before modificationFig. 7.  $\lambda_2$  Chart after modificationFig. 8.  $\lambda_3$  Chart before modificationFig. 9.  $\lambda_3$  Chart after modification

### 3.2. Carton manufacturing company

The control chart procedure explained above is applied in a carton manufacturing company. The block diagram for the process is shown in Fig. The paper roll is fed in the rolling unit and passes to the corrugation unit. The other processes include cutting, printing, pressing, finishing, slotting and stitching.

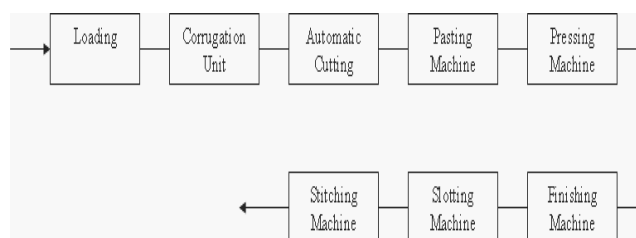


Fig. 10. Process description

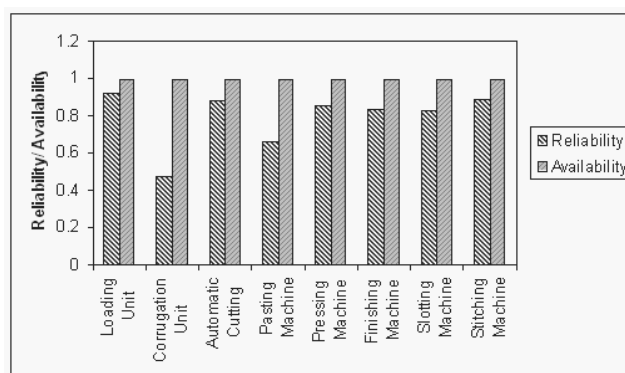


Fig. 11. Reliability and availability before modification

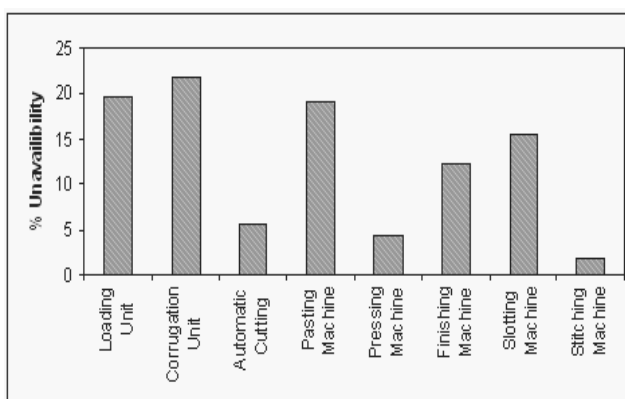


Fig. 12. Percentage of unavailability due to each component

The control chart construction starts with the identification of critical component. The procedure involves finding out the component whose variation in unavailability has a major bearing on the system availability or unavailability.

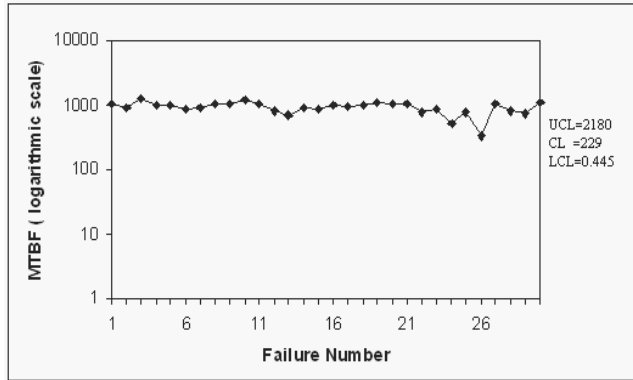


Fig. 13.  $\lambda$  Chart before modification

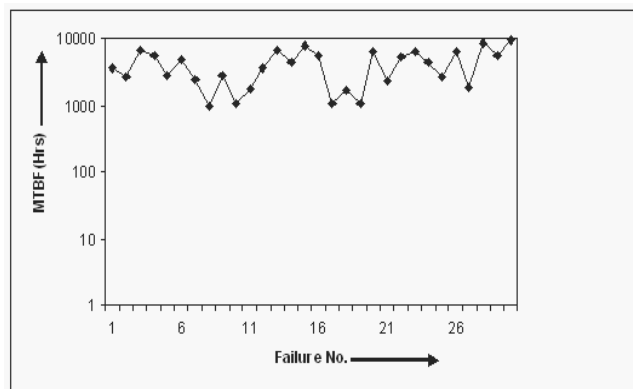


Fig. 14.  $\lambda$  Chart after modification

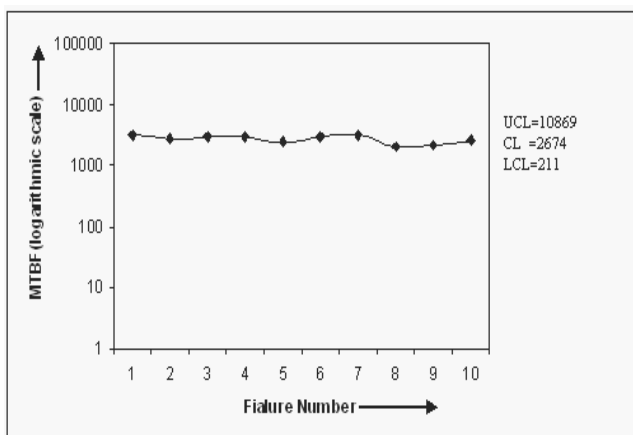


Fig. 15.  $\lambda_r$  Chart before modification

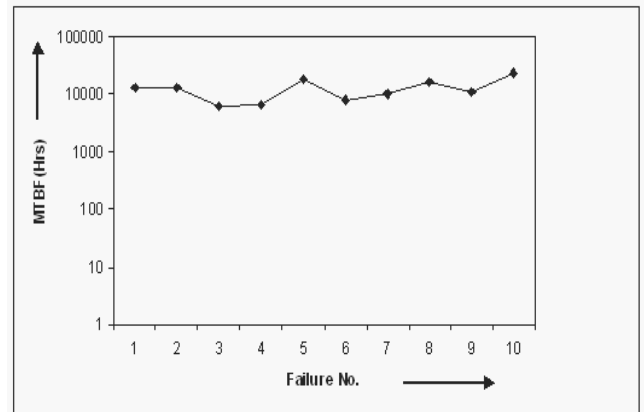


Fig. 16.  $\lambda_r$  Chart after modification

## 4. Conclusions

Until now, statistical control charts have been mostly used to monitor production processes. Although reliability monitoring, especially that for complex equipment or fleet of systems is an important subject. In this paper, it is studied the use of control charting technique to monitor the failure of components. To deal with the problems suffered by conventional quality control charts, the monitoring procedure specifically based on exponential distribution can be used. In this paper, a new procedure based on the monitoring of time to observe  $r$  failures is also proposed and it can be more appropriate for reliability monitoring. It differs from the  $t$  chart in the sense that it plots the cumulative time until observing  $r$  failures. This procedure is useful and more sensitive when compared with the  $\lambda$ -chart although it will wait until  $r$  failures for a decision. These charts can be regarded as powerful tools for reliability monitoring.  $\lambda_r$  gives more accurate results than  $\lambda$ -chart.

Before conducting the modifications the changes can be simulated to analyse the failure pattern so that appropriate decisions regarding plant modifications can be made.

## Nomenclature

- A Availability
- CL Central line
- $L(0)$  Potential losses without decreasing the hazard rate
- $L(x)$  Total losses from decreasing the current system hazard ' $\lambda$ ' by an amount ' $x$ '
- LCL Lower control limit
- MDT Mean down time
- MFFOP Minimum failure free operating period
- MTTF Mean time to failure
- MTTR Mean time to repair
- Q Cost of decreasing the current system hazard rate ' $\lambda$ ' by an amount ' $x$ '
- $R(t)$  Reliability at time any time  $t$
- S Minimum failure free operating interval

T Time  
 UCL Upper control limit  
 $\Delta L$  Relative changes of losses due to 'x' of the hazard rate  
 A Probability of false alarm  
 $\lambda$  Failure rate, i.e. Number of failures per unit time

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