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Modelling robotic palletising process with two robots using queuing theory

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Analysis and modelling

ABSTRACT

Purpose: This paper presents the modeling of a typical high-speed robotic palletizing process involving two robots using queuing theory with the aim of providing design guidelines of such a system for dynamic material flows.
Design/methodology/approach: In this study, our calculation is carried out using the queue models and the production parameters as given in Section 2. We select three types of performance indexes, i.e. average length of queue (L_q), utilization of server (US) and mean waiting time (MWT) in the queue to interpret the prediction of performance of the high-speed palletizing systems to handle the prescribed material flow.
Findings: The time average properties of the system, such as the average number of cartons in the system, mean waiting time, the congestion probability, utilization of machine have been calculated. The calculation results have shown that the work performance of the system is related to the characteristics of the material flow, and the product design should consider both the time-average parameters and the dynamic features of the material flow.
Practical implications: The paper has described a methodology of modeling a high-speed palletizing process with two robotic servers to handle high-speed dynamic materials flow using the queuing theory technique.
Originality/value: Based on the established model, a general design scheme can be derived to show how to consider the dynamic properties through the system design while meeting the prescribed utilization and congestion requirements of such a process.
Keywords: Dynamic material flow; Queuing theory; High-speed material handling; Palletizing

1. Introduction

Recently, the high-speed robotic palletizing system in material handling systems has drawn substantial attention in processing high-speed dynamic material flows, which are normally experienced in high-volume manufacturing lines. This type of machine, different from the traditional gantry robotic palletizers, is used to form complex material patterns [1] at the cost of some efficiency. It employs an innovative in-line pattern forming strategy by robot(s) and a stacking machine (palletizer) constructs the material stacks on the pallet. Typical applications can be found in food, dairy and beverage industries, where high-speed palletizing and storage are critical processes to keep the product fresh. As the technical core of this in-line pattern forming process, pattern forming algorithms have been extensively studied since 1960s. Both three-dimensional and two-dimensional pattern forming algorithms were designed by Tsai et al with dynamic scheduling techniques [2], and also by physical simulation method [3], and by linear programming approach [4]. Fleming et al [5] developed the global programming model for two-dimensional pallet loading problems. The palletizing process, too, was modeled by the advanced scheduling techniques. A real-time periodic scheduling strategy was also designed [6] , and John et al [7] based the work on job-machine model of sorter-palletizer subsystem. More recently, new heuristic algorithms were developed and many discussions were addressed regarding the performance, stability and control issues in pattern–forming process [8, 9]. However such a high-speed palletizing system has hardly been studied, particularly on the design process by considering the dynamic features of the product flow. Operation requirements, such as geometric constraints, productivity and operation safety are normally considered during the design and manufacturing of such a system. To handle the dynamic material flow, unrealistically high safety factor was usually applied to the design, which implies over-sized machine, high manufacturing cost and low equipment utilization rate. And in such cases, post-manufacture modification was usually needed, because the material flow properties were often ill-interpreted in the design. Either material congestion or deficiency was normally experienced during the operation.

This paper aims to optimize the design of a high-speed palletizing system involving two robot stations by considering the dynamic properties of the material flows to be handled. Using the queuing theory, the entire palletizing process is modeled as a One Job-Two Machine system using the product output rate and palletizing capacity. The time average features of the model are calculated as prediction of performance of high-speed palletizing system. With the established model, important machine features, such as the length of metering conveyor, velocity of pattern-forming conveyor, palletizing capacity and spacing are established. These can then serve as the quantitative guidelines to design such systems. The general design process considering the material flow properties is given and several different types of material flows are discussed to show how the modeling technique is integrated into the design process to extract the important working parameters for detailed design and manufacturing. The design of high-speed palletizing system can then be significantly improved, while maintaining high utilization and lower probability of material congestion.

2. Mathematical modelling

2.1. High-speed palletizing process

Product flow is basically characterized as a stochastic process inherent from the production processes, such as manufacturing, conveying, and packaging. However it used to be ignored or intentionally interfered with additional buffering, spacing and monitoring devices just for the sake of handling. Recently a high-speed palletizing system was designed by a local palletizing company. to handle the dynamic high-speed material flows with the especially developed pattern-forming algorithms and control strategies. In high-speed palletizing process, robots work to form the material sequence in-line rather than the patterns directly on the pallet, and the moving materials will end up at the palletizing plate to gradually form the final material pattern. Patterns-forming algorithms have been well established with a general aim to maximum space utilization with different size of cartons [10].

This reflected the application contexts in multi-product palletizing, or mixed order palletizing processes, however in high volume production process, such as in beverage industries, the real contexts are different from the previously prescribed. It usually has only one product type on one palletizing line, and the size of packages is homogeneous. Those patterning-forming algorithms are not as important as in the laboratory environment, because the pattern can be selected to form the repertoire based on the carton size and pallet size. A smooth and continuous product flow, and in-time handling are critically important in real production. This requires that the properties of materials flow should be studied and reflected in the system design.

Different from the medium-speed palletizing systems, a typical high-speed robotic palletizing system consists of a metering conveyor, where products accumulate from product conveyor for processing, the service robot(s), which position or rotate when necessary, the products on the pattern forming conveyor, and a palletizer at the end of production line. The metering conveyor functionally works as a buffer to accommodate the cartons queued to be pre-positioned. It is usually driven by a separate drive system, which is operated intermittently to form appropriate spacing for the material flow on the pattern-forming conveyor. The robot doesn't necessarily change the spacing when pre-positioning the cartons, but helps to form the carton lines and re-orientate them when necessary. Figure 1 shows the schematic description of such a high-speed palletizing system. At the left end is the product conveyor transferring the products in from the production line. We presume that the products are packaged in cartons. The palletizer transfers the patterned product layer onto the pallet. There are two robots in our study for the convenience of calculation.

A complete carton sequence is depicted for a better understanding of the pattern forming process, but in real production process, the components in dashed line do not exist. The distance between the robot and the palletizer could be as short as possible to save travel time and material cost. This suggests that the length between robot and product conveyor is the most sensitive part in the system to the high-speed dynamic carton flow, because the working capacity of the robot and the palletizer is usually determined and limited by the mechanical movements.

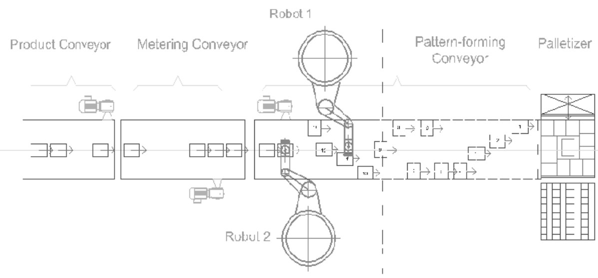

Fig. 1. Schematic description of high-speed palletizing system

In order to process all the cartons flowing in, the metering conveyor, where the cartons accumulate and spaced, is the buffer to temporarily hold the cartons to-be processed. For continuous operation of the production line, the metering conveyor should be able to offer spaces for all the waiting cartons. And the robot must be quick enough to pre-position them on the pattern-forming conveyor.

2.2. A one job-two machine model of high-speed palletizing process

Figure 2 shows a layout of One Job – Two Machine model for high speed palletizing. The system is modeled as a job-machine system, which allows the study of the time evolution of high-speed palletizing system with particular material flows. The machines, also called the server, are the robots in the system, and the jobs are the cartons flowing in from the packaging line. The total number of cartons in the system forms a queue, and those in the waiting list, either moving or not, is called the length of queue. In this case, the length of queue includes both the cartons on metering conveyor and the cartons to be processed on pattern-forming conveyor. In our calculation, we assumed that pattern-forming pattern can hold at most one carton during palletizing. So the entire queue capacity (places) is one more than the places available on the metering conveyor. As the instantaneous number of cartons in the system is directly related to the stochastic character of arrival and servicing, queuing theory is used to evaluate the time evolution of the system.

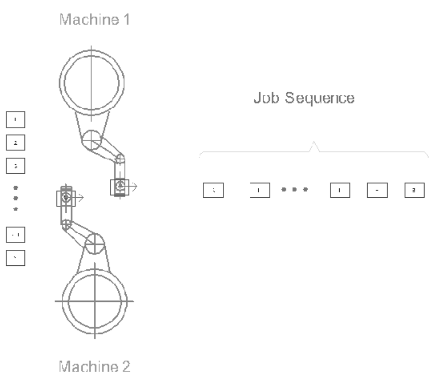

Fig. 2. One job-two machine model for high-speed pattern forming process

Not to lose the generality, no bulk arrival is assumed as the condition of arrival of cartons. The cartons flow into the metering conveyor at random interval, *A* minutes, approaching the robot. The density function of *A* can be expressed as

$$a(t) = \lambda \cdot e^{-\lambda \cdot t} \quad (1)$$

where λ is the mean inter-arrival time, and its variance is 1/λ2.

Because the pattern to be built has been standardized [13], and the cartons are of homogenous size, the pattern can be selected aiming at the maximum utilization of pallet. The above assumptions are more accurately reflecting the operation characteristic of high-speed palletizing process in real production process than those found in literature [7].

The servicing time of each carton is different from one another depending on the number of movement and operation of the end-effecter. It is very difficult to use one distribution to present the servicing time character, and we assumed that exponential distribution is to be followed.

In case of exponential distribution, the density function can be expressed as

$$s(t) = \mu \cdot e^{-\mu \cdot t} \quad (2)$$

where μ is the service rate in unit of cartons per hour, which can be expressed as

$$\mu = 1 \Big/ \int_0^{\infty} t \cdot s(t) dt \quad (3)$$

The entire model is donated as *M/M/2/GD/∞/∞* queue. At this stage, we assume that the buffer size is infinite for cartons, and unlimited number of cartons flow in the system. The average queue length of such a system is obtained by the equation

$$Lq = \sum_{j=1}^{\infty} (j-1) \cdot \pi_j = \sum_{j=1}^{\infty} j \cdot \pi_j - \sum_{j=1}^{\infty} \pi_j = \lambda^2 \Big/ \mu \cdot (\mu - \lambda) \quad (4)$$

where π_j is the probability of the state, in which *j* cartons are in the waiting queue.

The production parameters are obtained from one dairy production process in a local cream manufacturing company. There are six product lines operating in parallel, which produce up to 300 bottles of cream per minute. All of the bottles then have to be stacked, palletized, wrapped and refrigerated within 10 minutes. Six robots serve the pattern forming processes, each for one product line and works independently. Each product line has one stacker (palletizer) to carry out the stacking patterned cartons at the end. Two laser-guided forklifts transfer the product stacks to the storage for refrigeration. As for the arrival rate, the maximum production rate is 50 bottles/min on one line with a variance of 10% according to previous observations. The robot servers are two ABB IRB 640 robots, whose economical operating speed is 30 picking-and-placing per minute. Due to the different travel lengths and movement required by each package on the pallet, the servicing times for each product vary. All the operation parameters are summarized in Table 1.

Table 1.
Model Parameters

M/M/2/GD/∞/∞	
Arrival Distribution	$\lambda = 50$, $v(a) = 5$
Service Distribution	$\mu = 30$, $v(\mu) = 10$
Server Number	2
Queue Capacity	∞

3. System design based on modelling results

In this study, our calculation is carried out using the queue models and the production parameters as given in Section 2. We select three types of performance indexes, i.e. average length of queue (L_q), utilization of server (US) and mean waiting time (MWT) in the queue to interpret the prediction of performance of the high-speed palletizing systems to handle the prescribed material flow. Utilization represents the average working time of the robot machine, which reflects capacity matching robot to the current material flow. It also guides the robot capacity selection through the design process. The mean waiting time is used to be an index of the service satisfaction, but in this case it can serve as the guideline for metering conveyor design as it is directly related to the average travel time of a carton between the product conveyor and the robot. Table 2 shows all of the results of the calculation.

Table 2.
Calculation Results

	M/M/2/GD/∞/∞
Lq	1.586
US	79%
MWT	0.034 min

For the *M/M/2/GD/∞/∞* model result shown in Table 2, an average of 1.586 cartons can be found in the system with the given production parameters. In a previous study of One Job–Two Machine problem, it was found that the dynamic flow properties might influence the time average properties [11]. This result suggests that the stochastic feature of material flow could have a significant influence on the system performance; and a systematic investigation of servicing time observations is needed for more accurate calculation in the future. The observed utilization of each server is 79%. The working status of robot machine is not influenced by the servicing distribution type, but determined by the time average features of the material flow.

The rest of this section will discuss the design of high-speed palletizing system for high-speed dynamic material flow handling by interpreting and implementing of modelling results as given above. Operation specifications to be met by the system, such as the robot capacity, metering conveyor length, carton spacing definition and pattern-forming conveyor speed are extracted from the calculated results, will guide the designing.

When designing the robot, the working capacity has to satisfy the material flow with as-high-as-possible utilization and reliable operation speed. This can be evaluated by the calculated utilization and the maximum capacity in unit of carton per minute (cpm). We suggest that the robot capacity should be higher than the average arrival rate, in this case, 50 cpm. The preferable utilization rate is around 80% as suggested by ABB. But utilization should not be lower than 75 % to reduce the capital occupation.

According to the above calculation, the queue capacity defined in the system should be able to hold at least L_q cartons obtained in *M/M/2/GD/∞/∞* model. In our designing example, 5 is defined as the queue capacity with 1 on the pattern-forming conveyor, the other 4 cartons are on the metering conveyor. And the total length should be longer than $L_q*L_b + (L_q-1)*L_s$, where L_b is the box length, L_s is the spacing, because spacing is necessary between cartons for numbering and enough clearance. The new model can be expressed as *M/M/2/GD/7/∞*. The length of metering conveyor is determined as L_q*L_b. The carton size is 400 x 400 x 400 (L x W x H in mm), therefore the total length of conveyor between the product line and the robot is at least $6*L_b$, but one spacing is defined at the pattern-forming conveyor before robot.

The average speed of pattern-forming conveyor is designed to meet the mean waiting time of a carton in the queue. The average speed of the pattern-forming conveyor can be calculated by dividing the material queue length by mean waiting time, which finally is $L_q*L_b + (L_q-1)*L_s /MWT$.

Results show that the proposed modeling technique with queuing theory can guide the high-speed palletizing system design process to extract the technical interpretation from the high-speed dynamic material flow. In the following design case, the specific operation parameters for high-speed palletizing system of *M/M/2/GD/7/∞* model are extracted by queuing analysis. We calculated several material flows whose queue models are identical but with different cartons size. A corresponding production context can be that the cartons flow into the palletizing system directly from the packaging process, where processing times are independent to the carton size. Therefore the dynamic properties remain identical for different cartons. In the following calculation, we assumed box sizes from 400mm down to 100 mm at an interval of 50 mm. The spacing is defined as 1.6 second from ABB operation manual. We assume that the robot can preposition different types of boxes without extra operation time in order to keep the identical queue model parameters. The least utilization of robot is set at 75 %, congestion probability is introduced to monitor the congestion risk of the system. Its threshold is set at 10%. Table 3 shows the list of the extracted design parameters. With the modeling technique, the metering conveyor length, the velocity of pattern-forming conveyor, and carton spacing can be determined to meet the utilization and congestion objectives.

Table 3.
Operation Parameters of High-Speed Palletizing Systems Obtained from Modelling Process Using Queuing Theory
($\lambda = 50$, $\mu = 30$, *M/M/2/GD/5/∞*)

C_s (mm)	L_{MC}	V_{PC}	C_{SP}	US	Cp
400	1600.0	94.0	226.4	76%	8.75%
350	1400.0	82.2	198.1	76%	8.75%
300	1200.0	70.6	169.8	76%	8.75%
250	1000.0	58.7	141.5	76%	8.75%
200	800.0	47.0	113.2	76%	8.75%
150	600.0	35.2	84.9	76%	8.75%
100	400.0	23.5	56.6	76%	8.75%

Cs Carton size (mm)
L_{MC} Length of metering conveyor (mm)
V_{PC} Velocity of pattern-forming conveyor (m/s)
C_{SP} Carton spacing (mm)
C_p Congestion probability

4. Summary

The paper has described a methodology of modeling a high-speed palletizing process with two robotic servers to handle high-speed dynamic materials flow using the queuing theory technique. The time average properties of the system, such as the average number of cartons in the system, mean waiting time, the congestion probability, utilization of machine have been calculated. The calculation results have shown that the work performance of the system is related to the characteristics of the material flow, and the product design should consider both the time-average parameters and the dynamic features of the material flow. Based on the established model, a general design scheme can be derived to show how to consider the dynamic properties through the system design while meeting the prescribed utilization and congestion requirements of such a process.

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