Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 31
ISSUE 2
December
2008

Dynamical flexibilities of mechanical rotational systems

S. Żółkiewski *
Division of Mechatronics and Designing of Technical Systems,
Institute of Engineering Processes Automation and Integrated Manufacturing System,
Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: slawomir.zolkiewski@polsl.pl

Received 18.09.2008; published in revised form 01.12.2008

Analysis and modelling

ABSTRACT

Purpose: of this work is to present dynamical flexibilities of rotational beams and rods systems. The results of mathematical calculations were presented in the form of dynamical flexibility of analyzed systems. In final solution there were took into consideration the interactions between the major motions and local vibrations of subsystems.
Design/methodology/approach: The dynamical flexibilities were derived by the Galerkin's method. The dynamical flexibilities for example numerical cases were presented onto charts of attenuation-frequency characteristics. The mathematical models were derived on the basis of known equations of motion derived in previous thesis's.
Findings: After analysis of characteristics we can observe the transportation effect. We can notice additional poles on the characteristic of dynamical flexibility characteristics and after increasing angular velocity created modes symmetrically propagate from the origin mode and instead of the original mode there is created a zero's amplitude.
Research limitations/implications: Analyzed systems are beams and rods in rotational motion. Motion was limited to plane motion. Future works will be connected with consideration of complex systems.
Practical implications: of derived dynamical flexibilities of free-free and fixed beams and rods systems is a possibility of derivation of the stability zones of analyzed systems and derivation of eigenfrequencies and zeros in the function of angular velocity of work motion.
Originality/value: Models analyzed in this thesis apply to rotating rod and beam systems with taking into consideration the transportation effect. This new approach of analyzing rod and beam systems can be put to use in modelling, analyzing and designing machines and mechanisms with rotational elements.
Keywords: Applied mechanics; Numerical techniques; Vibrations; Transportation effect

1. Introduction

Many methods of analyzing vibrations are connected with problems of analyzing systems in motion. One of the most popular method of analyzing of dynamical state of systems is a method of dynamical flexibility. the dynamical flexibility can be put to use both to continuous and discrete systems. This method gives the possibility of observing of, in very easy way, assigning resonance zones and finding the amplitudes of vibrations of the analyzed subsystems, finding zeros of dynamical characteristics so therefore finding parameters of work where the vibrations are minimally.

In this publication there are presented problems applying to rotational beams and rods linear systems. The transporation movement was assumed as the rotation. In many publications [2-6, 12-13, 20] there are positions connected with the subject group of vibrating systems in transportation differ from the stationary systems [1, 7-11, 14-20]. The derived dynamical flexibilities are presented in mathematical form and in form of dynamical characteristics. The dynamical flexibilities were derived from Equations of motion derived in previous works.

Ways of minimizing amplitudes of vibrations are well-known and the example ways of changing forces acting into the systems, changing framework of system or changing the geometrical or physical parameters of the system can be used in the Modyfit.

2. Models and dynamical flexibilities of rods

This section is a presentation of the dynamical flexibilities of rod systems (Fig. 1) both in the mathematical form and in the form of dynamical characteristics on the chart (Fig. 2 and Fig. 3). Based on the Equations of motion [6]:

The projection into the X axis of the global reference system:

$$\frac{\partial^2 u_X}{\partial t^2} - a^2 \cdot \frac{\partial^2 u_X}{\partial x^2} = = \omega^2 \cdot \left(s \cdot \cos\varphi + u_X \right) + 2 \cdot \omega \cdot \frac{\partial u_Y}{\partial t}. \tag{1}$$

The projection into the Y axis of the global reference system:

$$\frac{\partial^2 u_Y}{\partial t^2} - a^2 \cdot \frac{\partial^2 u_Y}{\partial x^2} = = \omega^2 \cdot \left(s \cdot \sin\varphi + u_Y \right) - 2 \cdot \omega \cdot \frac{\partial u_X}{\partial t}, \tag{2}$$

there was derived the dynamical flexibility of stationary free-free system by the Galerkin's method as follow:

$$Y(\Omega) = \frac{2}{\rho \cdot A \cdot l} \cdot \sum_{n=0}^{\infty} \frac{\cos(n\pi) \cdot \cos\left(\frac{n\pi x}{l}\right)}{a^2 \cdot \left(\frac{n\pi}{l}\right)^2 - \Omega^2}, \tag{3}$$

where:

$Y(\Omega)$ – the dynamical flexibility in function of frequency of extorted force,
n – mode of vibrations of rod,
a – velocity of the wave propagation in the rod,

$$a = \sqrt{\frac{E}{\rho}}, \tag{4}$$

E – Young modulus,
Ω – frequency of vibrations,
x – the position of analyzed section.
A – the cross-section of rod,
l – length of the rod,
ρ – mass density of the rod,
ω – angular velocity of the rod,
φ – rotation angle,
t – time,
a vector of linear displacement of the rod's section along center line of the bar in the local reference system:

$$\overline{\mathbf{u}} = \left[u \quad 0 \quad 0 \right]^T, \tag{5}$$

a vector of linear displacement of the rod's in the global reference system:

$$\overline{\mathbf{u}} = \left[u_X \quad u_Y \quad 0 \right]^T, \tag{6}$$

a position vector:

$$\overline{\mathbf{S}} = \left[s \quad 0 \quad 0 \right]^T. \tag{7}$$

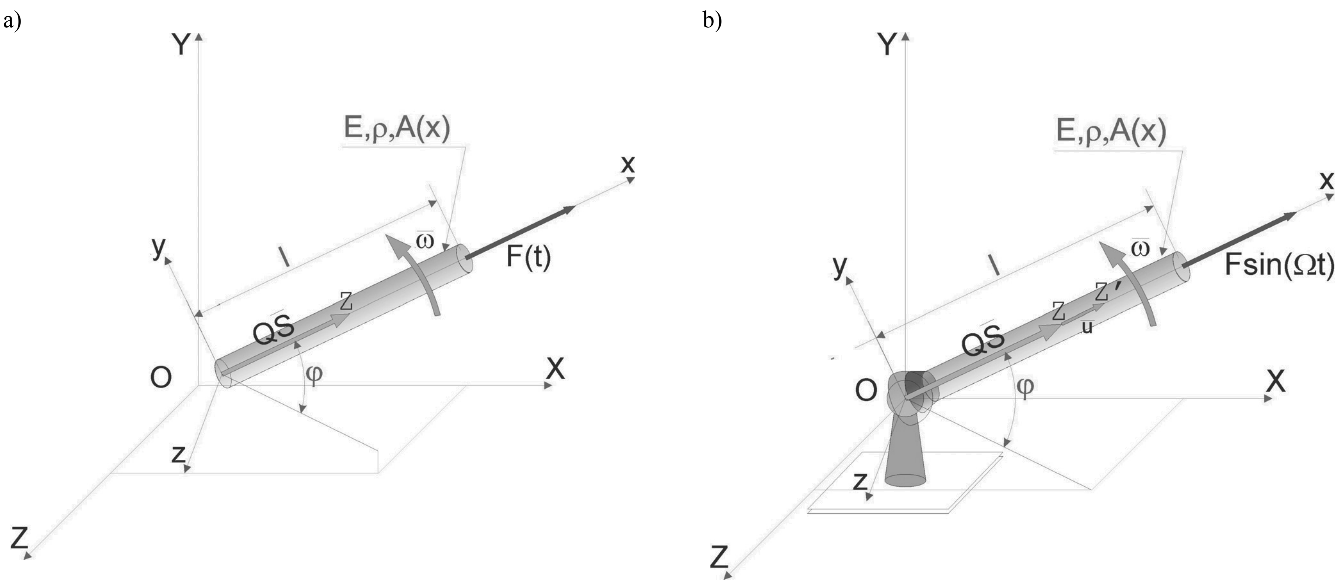

Fig. 1. The model of analyzed systems: a) free-free rod, b) fixed rod on the rotational table

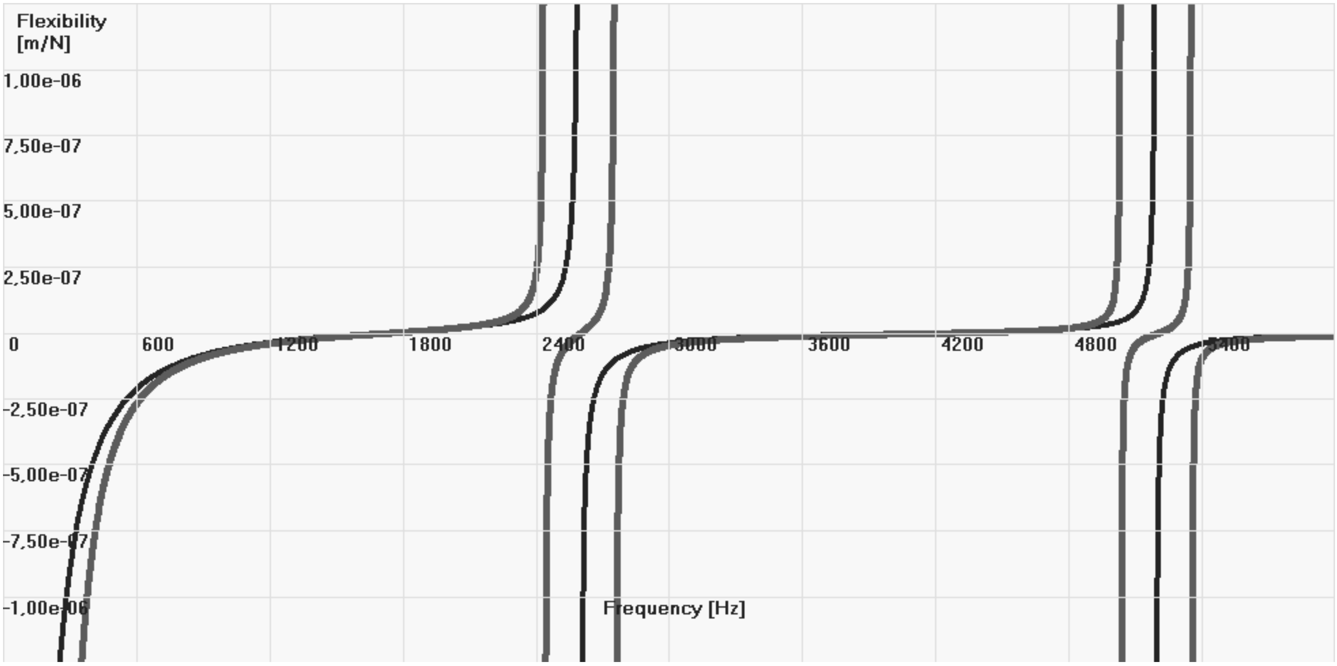

Fig. 2. The dynamical flexibility of free-free rod rotating with angular velocity equals 1000 rad/s (red) and stationary one (black)

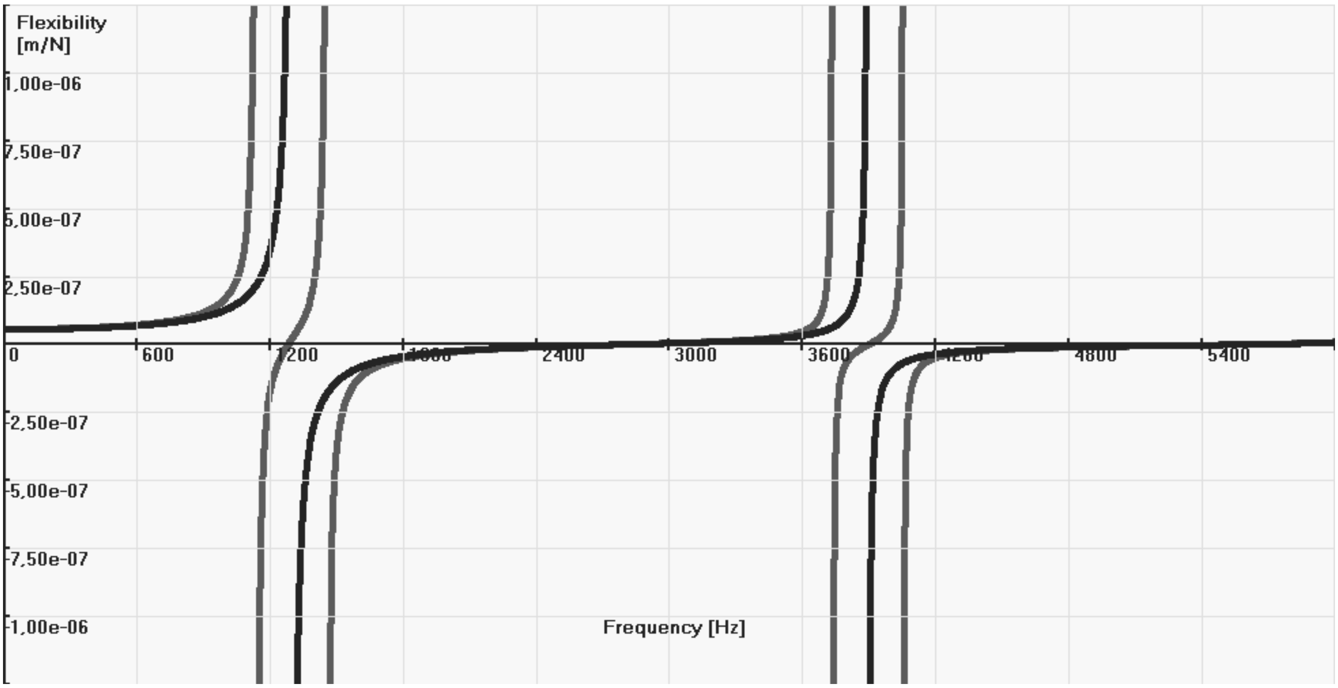

Fig. 3. The juxtaposition of dynamical flexibilities of fixed rod rotating with angular velocity equals 1000 rad/s (red line) and stationary one (black line)

The Equation (2) presents the dynamical flexibility of the stationary fixed rod.

$$Y(\Omega)=\frac{2}{\rho\cdot A\cdot l}\cdot\sum_{n=0}^{\infty}\frac{\sin\left[\left(n+\frac{1}{2}\right)\pi\right]\cdot\sin\left[\frac{(2n+1)\pi x}{2l}\right]}{a^2\cdot\left[\frac{(2\cdot n+1)\cdot\pi}{2\cdot l}\right]^2-\Omega^2}. \qquad (8)$$

The Equation (3) presents a dynamical flexibility of free-free rod system rotating with the angular velocity ω.

$$Y(\Omega)=\sum_{n=0}^{\infty}\frac{2\cdot\cos(n\pi)\cdot\left(a^2\cdot n^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right)\cdot\cos\left(n\pi\frac{x}{l}\right)}{\rho\cdot A\cdot l\cdot\left[\left(a^2\cdot n^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right)^2-4\cdot\omega^2\cdot\Omega^2\right]}. \qquad (9)$$

The Equation (4) presents a dynamical flexibility of fixed rod system rotating with the angular velocity ω.

$$Y(\Omega)=\sum_{n=0}^{\infty}\frac{2\cdot\sin\left(\frac{2n+1}{2}\pi\right)\cdot\left[a^2\cdot\left(\frac{2\cdot n+1}{2}\right)^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right]\cdot\sin\left(\frac{2n+1}{2}\pi\frac{x}{l}\right)}{\rho\cdot A\cdot l\cdot\left\{\left[\left(a^2\cdot\left(\frac{2\cdot n+1}{2}\right)^2\cdot\frac{\pi^2}{l^2}-\Omega^2-\omega^2\right)\right]^2-4\cdot\omega^2\cdot\Omega^2\right\}}. \qquad (10)$$

3. Models and dynamical flexibilities of beams

In this section there were presented the dynamical flexibilities of beam systems both the stationary ones and the dynamical flexibilities of beam systems in transportation. Generalized coordinates and generalized velocities were assumed as orthogonal projections of coordinates and velocities of the beam to axes of the global reference frame:

$$q_1=r_X,\quad q_2=r_Y, \qquad (11)$$

$$\dot{q}_1=\frac{dq_1}{dt}=\dot{r}_X=v_X,\quad \dot{q}_2=\frac{dq_2}{dt}=\dot{r}_Y=v_Y. \qquad (12)$$

Based on the Equations of motion [6]: The projection into the X axis of the global reference system:

$$\frac{\partial^2 w_X}{\partial t^2}+\frac{E\cdot I_Z}{\rho\cdot A}\cdot\frac{\partial^4 w_X}{\partial x^4}= =-\omega^2\cdot(s_X-w_X)+2\cdot\omega\cdot\frac{\partial w_Y}{\partial t}. \qquad (13)$$

The projection into the Y axis of the global reference system:

$$\frac{\partial^2 w_Y}{\partial t^2}+\frac{E\cdot I_Z}{\rho\cdot A}\cdot\frac{\partial^4 w_Y}{\partial x^4}= =-\omega^2\cdot(s_Y-w_Y)-2\cdot\omega\cdot\frac{\partial w_X}{\partial t}. \qquad (14)$$

On Figures 4 and 5 there are presented models of beam in transportation, the free-free one (Fig. 4) and the fixed one (Fig. 5).

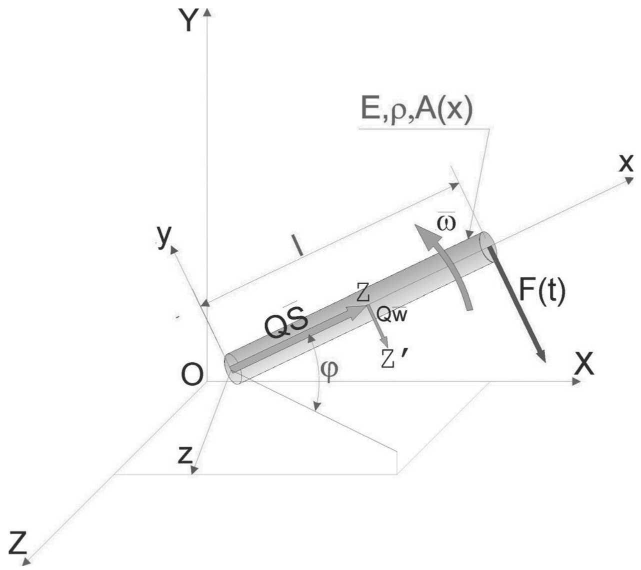

Fig. 4. The model of free-free beam in rotation

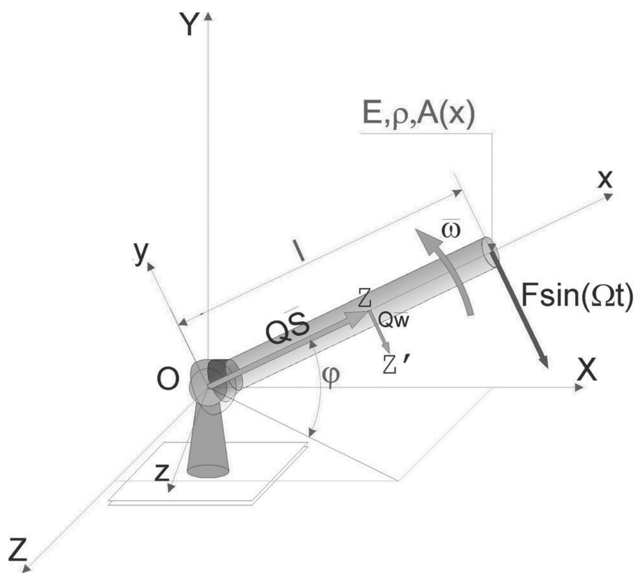

Fig. 5. The model of fixed beam in transporation

A vector of linear displacement of a cross-section in the beam is orthogonal to their center lines in the local reference system and can be written:

$$\overline{\mathbf{w}} = \begin{bmatrix} w & 0 & 0 \end{bmatrix}^T . \tag{15}$$

There was searched the solution in form of displacement function as the product of eigenfunctions series as follows:

$$w_X = \sum_{n=1}^{\infty} A_X \cdot X(x) \cdot \sin(\Omega t), \tag{16}$$

$$w_Y = \sum_{n=1}^{\infty} A_Y \cdot X(x) \cdot \cos(\Omega t), \tag{17}$$

The results were presented in form of mathematical models of analyzing systems. Dynamical flexibilities were derived by the Galerkin's method and presented as dynamical characteristics on charts (Figs. 6-9).

The Equation (18) presents dynamical flexibility of free-free stationary beam systems derived on base of known Equations of motion of this beam.

$$Y(\Omega) = \frac{1}{\rho \cdot A \cdot \gamma_n^2} \cdot \sum_{n=1}^{\infty} \frac{X(l) \cdot X(x)}{c^2 \cdot \left(\frac{2n-1}{2 \cdot l} \pi \right)^4 - \Omega^2}, \tag{18}$$

where:
I_Z – geometric momentum of inertia,
c – the formula (19):

$$c = \sqrt{\frac{E \cdot I_Z}{\rho \cdot A}}. \tag{19}$$

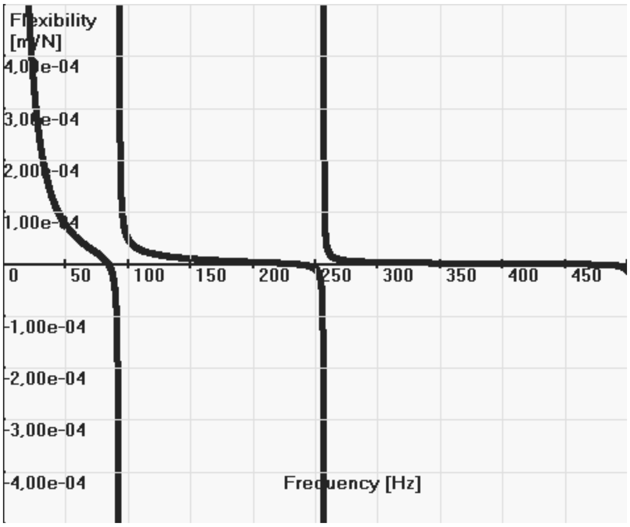

Fig. 6. The dynamical flexibility (18) of stationary free-free beam

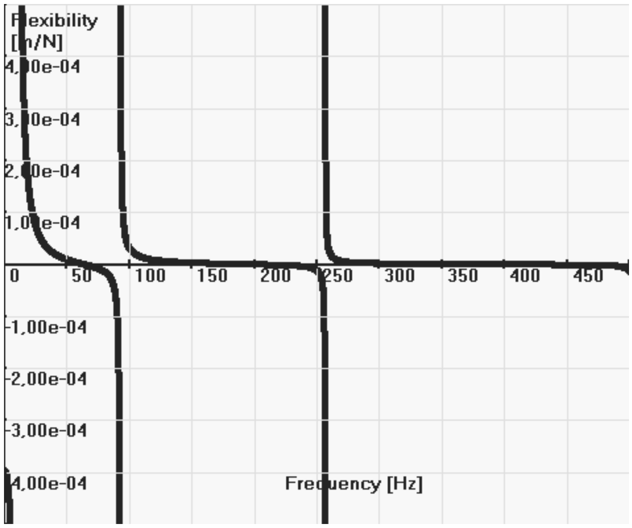

Fig. 7. The dynamical flexibility (18) of stationary free-free beam

An eigenfunction of displacement is as follow:

$$X(x) = \sin(kx) + \frac{\cos(kl) - \cosh(kl)}{\sin(kl) + \sinh(kl)} \cdot \cos(kx)$$
$$+ \sinh(kx) + \frac{\cos(kl) - \cosh(kl)}{\sin(kl) + \sinh(kl)} \cdot \cosh(kx) \tag{20}$$

and

$$k \approx \frac{2 \cdot n + 1}{2 \cdot l} \cdot \pi, \quad n = 0 \Rightarrow k = 0. \tag{21}$$

The Equation (22) presents dynamical flexibility of fixed stationary beam systems derived on base of known Equations of motion of this beam.

$$Y(\Omega) = \frac{1}{\rho \cdot A \cdot \gamma_n^2} \cdot \left[\frac{X(l) \cdot X(x_{k=0})}{\Omega^2} + \sum_{n=1}^{\infty} \frac{X(l) \cdot X(x)}{c^2 \cdot \left(\frac{2n+1}{2 \cdot l} \pi \right)^4 - \Omega^2} \right]. \tag{22}$$

Where an eigenfunction of displacement for fixed beam:

$$X(x) = \sin(kx) + \frac{\cos(kl) + \cosh(kl)}{\sin(kl) - \sinh(kl)} \cdot \cos(kx) +$$
$$- \sinh(kx) - \frac{\cos(kl) + \cosh(kl)}{\sin(kl) - \sinh(kl)} \cdot \cosh(kx), \tag{23}$$

$$k \approx \frac{2 \cdot n - 1}{2 \cdot l} \cdot \pi. \tag{24}$$

The dynamical flexibility of rotating free-free beam with angular velocity signed as ω with the same *X(x)* as in the Equation (20).

$$Y(\Omega)=\frac{-X(l)\cdot X(x_{k=0})\cdot(\Omega^2+\omega^2)}{\rho\cdot A\cdot\gamma_n^2\cdot(\Omega^2-\omega^2)^2}+$$
$$+\sum_{n=1}^{\infty}\frac{-\left[c^2\cdot\left(\frac{2\cdot n+1}{2\cdot l}\cdot\pi\right)^4-\Omega^2-\omega^2\right]\cdot X(l)\cdot X(x)}{\rho\cdot A\cdot\gamma_n^2\cdot\left[\left(c^2\cdot\left(\frac{2\cdot n+1}{2\cdot l}\cdot\pi\right)^4-\Omega^2-\omega^2\right)^2-4\cdot\omega^2\cdot\Omega^2\right]}. \tag{25}$$

The dynamical flexibility of rotating fixed beam with angular velocity signed as ω and *X(x)* the same as in (23).

$$Y(\Omega)=\sum_{n=1}^{\infty}\frac{\left[c^2\cdot\left(\frac{2\cdot n-1}{2}\cdot\frac{\pi}{l}\right)^4-\Omega^2-\omega^2\right]\cdot X(l)\cdot X(x)}{\rho\cdot A\cdot\gamma_n^2\cdot\left\{\left[c^2\cdot\left(\frac{2\cdot n-1}{2}\cdot\frac{\pi}{l}\right)^4-\Omega^2-\omega^2\right]^2-4\cdot\omega^2\cdot\Omega^2\right\}}, \tag{26}$$

where the norm is as follow:

$$\gamma_n^2=\int_0^l X^2(x)\,dx. \tag{27}$$

In the Figure 8 the dynamical flexibility of the free-free beam rotating with angular velocity equal 100 rad/s was presented.

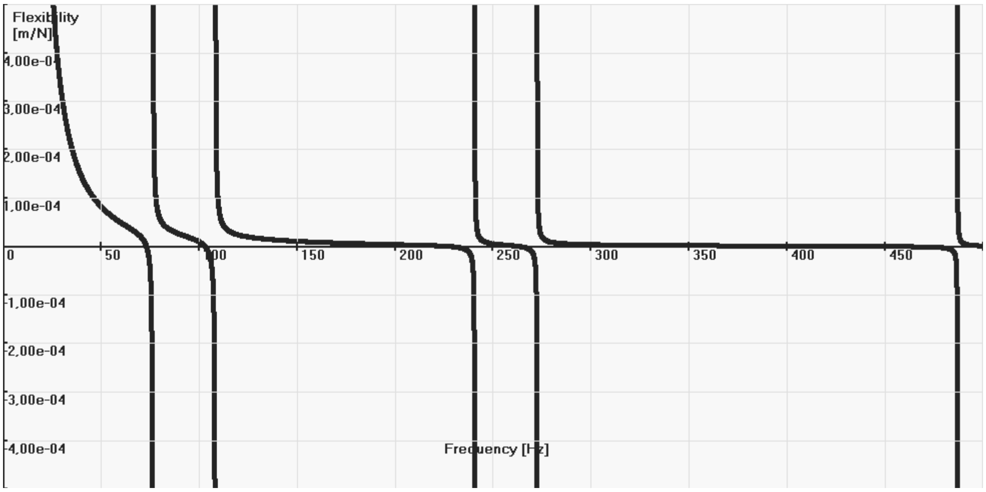

Fig. 8. Dynamical flexibility of the free-free beam system rotating with angular velocity equal 100 rad/s

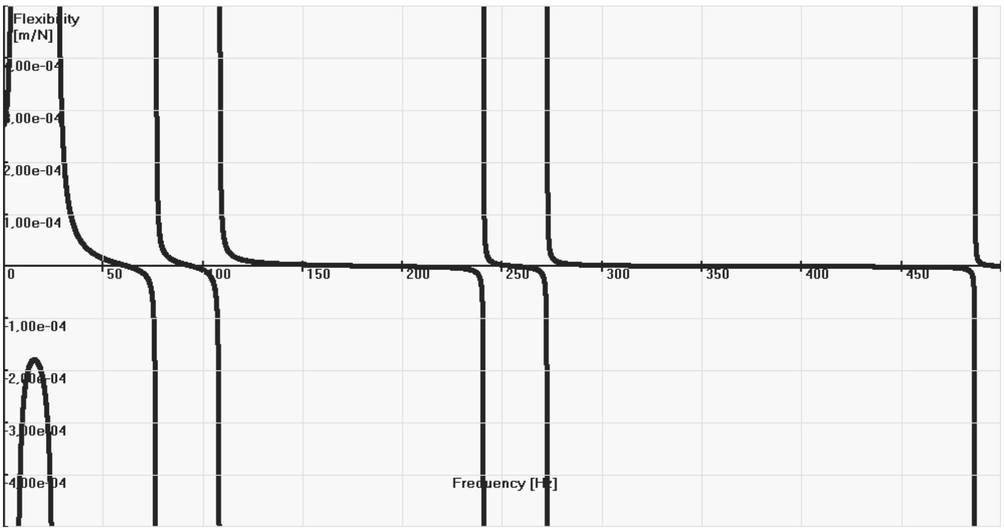

Fig. 9. Dynamical flexibility of the fixed beam system rotating with angular velocity equal 100 rad/s

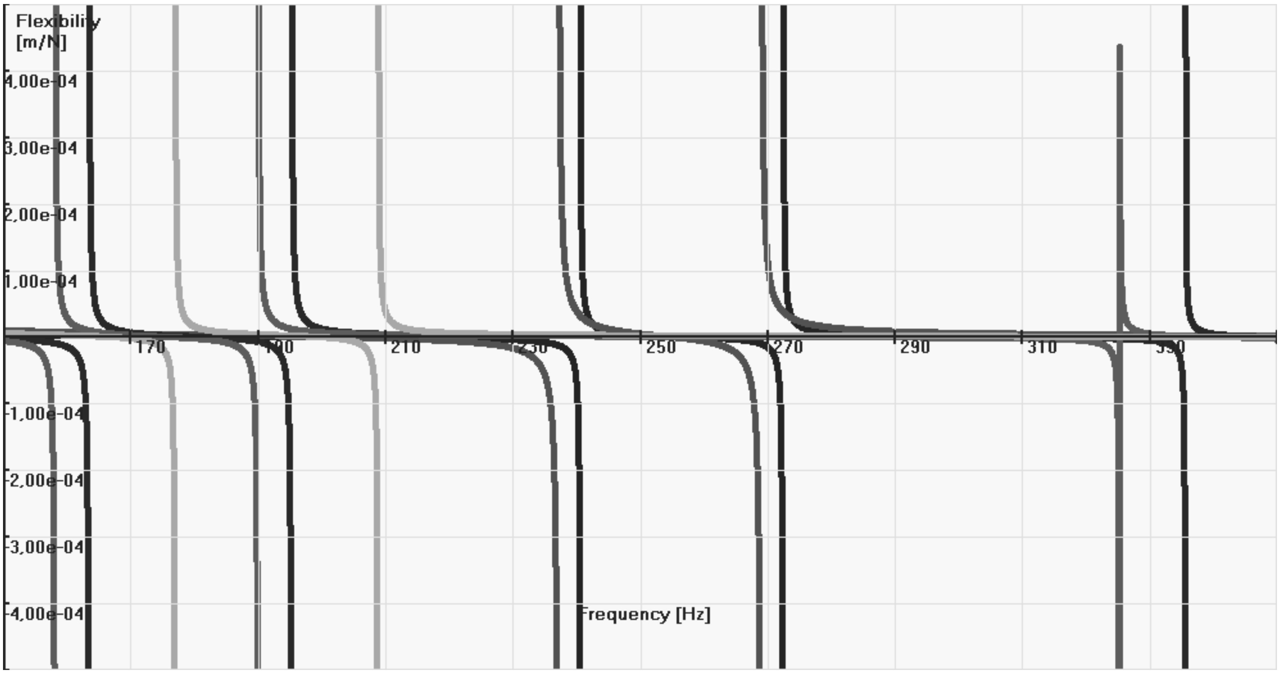

Fig. 10. The sample dynamical flexibilities for systems made up of different materials

In the Fig. 9 there was presented the dynamical flexibility of fixed beam rotating with angular velocity equal 100 rad/s. Figure 10 shows the sample dynamical characteristics for different materials. A red line for brass, the orange line for lead, the blue line for bronze, the green one for aluminium alloy and the black one for steel. This chart show that we can very easily control systems as early as on the designing level by changing their material parameters and predict their response for different loading terms.

4. Conclusions

Systems in rotational transportation are additionally loaded by forces arising from the so called transportation effect. Major additional forces that acting on systems in rotational motion and added to the mathematical model are the Coriolis force and centrifugal forces. One of the most popular method of of analyzing vibrations is a method of dynamical flexibility presented in this thesis in form of finish results as mathematical functions and dynamical characteristics. These dynamical characteristics were generated by numerical application Modyfit (Modelling of dynamical flexible systems in transportation). Taking into consideration into the considered models the main motion treated here as transportation effects onto the dynamical characteristics. This effect relies on moving the zeros and modes of dynamical flexibility together with increasing of angular velocity of rotational systems. The transportation effect has more affect onto beam systems than rod ones that was expressed onto characteristics. Presented characteristics are just only sample characteristics and therefore they should be analyzed separately in individual cases and can be regenerated for optional parameters in the Modyfit environment.

Acknowledgements

This work has been conducted as a part of research project N 502 071 31/3719 supported by the Ministry of Science and Higher Education in 2006-2009.

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