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TOPSIS and fuzzy multi-objective model integration for supplier selection problem

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ABSTRACT

Purpose: The problem includes the three objective functions: minimizing the total cost, the net rejected items and the inverse total value of purchasing (TVP), while satisfying capacity and demand requirement constraints.

Design/methodology/approach: The model is established for supplier selection problem and later the proposed single objective model is used to calculate the optimum order quantities among the selected suppliers. A numerical example is given to illustrate how the model is applied.

Findings: In this article, we proposed a single objective function to solve the fuzzy multi-item multi-objective model in order to calculate the optimum order quantities to each supplier.

Practical implications: Single objective function, which is able to consider the relative importance of the goals, is proposed to solve the model. A numerical example is given to illustrate how the model is applied.

Originality/value: This approach is able to help the DM evaluate the suppliers in order to find out the appropriate order to each them, and allows purchasing manager(s) to manage supply chain performance on service, cost, quality and etc. the suppliers' price breaks, which depend on the sizes of order quantities, affects the selection process.

Keywords: Supplier selection; MCDM; TOPSIS method; Fuzzy multi-objective model; Relative importance of criteria

1. Introduction

Companies have to work with different suppliers to continue their activities. In manufacturing industries the component parts and raw materials can equal up to 70% of the product cost. In such circumstances the purchasing department has a key role in cost reduction, and supplier selection is one of the most important functions of purchasing management according to Ghodsypour and O'Brien [1]. Supplier selection is a multiple-criteria decision-making (MCDM) problem that is affected by quantitative and qualitative factors. As a result, a purchasing manager has to analyze the trade-off among the several conflicting criteria. MCDM techniques help the decision makers (DMs) evaluate a set of alternatives. In real situation, for this problem, the weights of criteria are different and depend on purchasing strategies in a supply chain [2].

In real case, suppliers usually offer quantity discounts to encourage the buyer towards larger order. In this situation, the buyer need to decide what order quantities to assign to each supplier. This is a complicated multi-objective decision making problem which is affected by several conflicting factors [17]. Although comprehensive research on economic order quantities with quantity discounts exists, only a few methods address the problem from the perspective of supplier selection and order quantity allocation [17] that are Gaballa [3], Bender et al. [4], Turner [5], Sharma et al. [6], Benton [7], Chauhdry et al. [8], Rosenthal et al. [9] and Ghodsypour [10] discussed on this type of model [17]. A linear mixed integer programming was developed by Chauhdry et al. [8] for supplier selection. In the developed model delivery, price, quality and quantity discount are included. The objective of the model is to minimize aggregate price by considering both incremental and cumulative discount, while delivery and quality were considered as constraints. In this article, goal programming was suggested as an appropriate technique for this multi-objective problem. In another research, a mixed integer programming model was developed by Rosenthal et al. [9] for supplier selection with bundling, in which a buyer has to buy various items from several suppliers whose quality, capacity and deliveries are limited and who offer bundled products at discounted prices. Single objective programming was used in their model. An integrated AHP and linear programming model was proposed by Ghodsypour and O' Brien [11] in order to help managers consider both quantitative and qualitative factors in their purchasing activity in a systematic approach. The author also considered quality, and service, and buyer's limitations on budget, and price discount.

In real cases, for supplier selection problem, majority of the input information is not known precisely, so that the values of many criteria are expressed in uncertain terms such as "good in price" or "very high in quality". This vagueness cannot be easily considered by deterministic models. In these cases, the fuzzy theory, which is one of the best tools to handle uncertainty, can help solve the supplier selection problem. In fuzzy programming, the problem is no longer forced to be formulated in precise and rigid form. Based on fuzzy logic approaches, a model, which combines the use of fuzzy set theory (FST) with AHP and implements it to evaluate small suppliers in the engineering and machine sectors, was developed by Morlacchi [12]. Moreover, the application of FST was discussed by Holt [13] and Erol et al. [14] in order to find the best supplier among suppliers. These papers deal with a single-sourcing supplier selection problem where all buyers' demands can be met by only one supplier. Fuzzy goal programming was proposed by Kumar et al. [15] for supplier selection problems with multiple sourcing that include three objectives: minimizing the net cost, the net rejections and the net late deliveries subject to realistic constraints regarding suppliers' capacity and buyer's demand. The authors used Zimmermann's [16] weightless technique where there is no difference between objective functions.

In the literature, relatively scarce effective models have been developed by the papers for supplier selection problems simultaneously trying to deal with unstructured relevant information and imprecise input data and different weights of evaluative criteria, under conditions of price breaks and multiple sourcing. Usually, these aspects have been analyzed one at a time by each model [17]. To overcome the above problem, Amid et al. [17] developed a fuzzy multi-objective and mixed integer linear programming for the supplier selection problem to consider different weights of evaluative criteria, under conditions of price breaks and multiple sourcing but only for one product.

Motivated by above discussion, an integration of TOPSIS method and Fuzzy multi-objective mixed integer linear programming (Fuzzy MOMILP) is proposed to consider both quantitative and qualitative factors for choosing the best suppliers and define the optimum quantities among the selected suppliers under conditions of price breaks and multiple sourcing where buyer wants to buy multiple products. By this fuzzy multi-objective model, purchasing managers are able not only to take into account the imprecision of information but also to consider the limitations of buyers and suppliers to calculate the order quantity assigned to each supplier. The suppliers' price breaks, which depend on the sizes of order quantities for each product, affect the selection process. First, the suppliers are evaluated by TOPSIS technique based on tangible and intangible factors, then the fuzzy MOMILP model is established, and finally, a single objective function, which considers the relative importance of the fuzzy objectives, is proposed to solve the model to calculate the optimum quantities among the selected suppliers. The problem also includes the three objective functions: to minimize the inverse total value of purchasing (TVP), the total cost and total defect rate, while satisfying capacity and demand requirement constraints. The methodology is illustrated in Fig.1

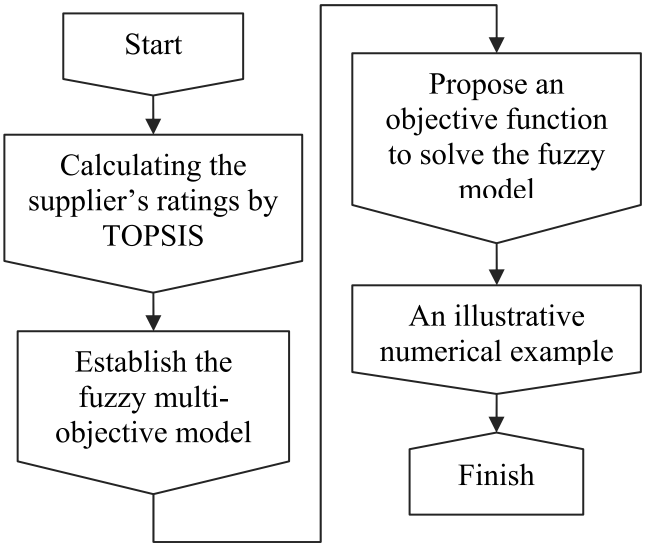

Fig.1. Flowchart of the methodology

2. Integration of topsis and fuzzy multi-objective model in supplier selection and order allocation

The model presented in this article allocates order quantities between the suppliers by using TOPSIS to make the trade off between tangible and intangible factors and calculate a rating of suppliers, and then by applying these ratings as coefficients in the inverse TVP objective functions of the fuzzy multi-objective model, and finally by solving the model by the proposed single objective function. Therefore the main steps of the algorithm include supplier evaluation and shipment allocation.

2.1. Supplier evaluation

TOPSIS Concepts

TOPSIS was developed by Hwang and Yoon [18], based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS) for solving a multiple criteria decision making problem. Briefly, the PIS is made up of all best values attainable of criteria, whereas the NIS is composed of all worst values attainable of criteria. The calculation processes of this method are as follows.

Compute the Overall Score of Each Supplier

TOPSIS is proposed for prioritizing the preference of supplier that is very suitable for solving the group decision making problem in an uncertain environment. In this article, $S=\{S_1,S_2,\dots,S_n\}$ is a discrete set of n possible suppliers and $Q=\{Q_1,Q_2,\dots,Q_\theta\}$ is a set of θ attributes of suppliers. $w=\{w_1,w_2,\dots,w_\theta\}$ is the vector of attribute weights so that they must sum to 1 otherwise it is normalized. In this paper, the attribute ratings of suppliers for the subjective attributes and the attribute weights are considered as linguistic variables. Here, the attribute weights can be expressed by the 1–9 scale shown in Table 1. The attribute ratings G can also be expressed by the 1–5 scale shown in Table 2. And the quantitative attributes are scaled using their own real number.

Table 1.
The scale of attribute weights w

Scale	w
Very very low (VVL)	*0.050*
Very low (VL)	*0.125*
Low (L)	*0.175*
Medium low (ML)	*0.225*
Medium (M)	*0.275*
Medium high (MH)	*0.325*
High (H)	*0.375*
Very high (VH)	*0.425*
Very very high (VVH)	*0.475*

Table 2.
The scale of attribute ratings G

Scale	G
Poor (P)	1
Medium poor (MP)	3
Fair (F)	5
Medium good (MG)	7
Good (G)	9
Intermediate values between the two adjacent judgments	2,4,6,8

The procedures are summarized as follows:

Step 1
Arrange a committee of DMs to express their preferences on attribute weights and ratings of suppliers:
(1) Use linguistic variables (Table 1) to identify the attribute weights of suppliers. The attribute weight of attribute Q_γ can be calculated in Eq. (1).

$$W_\gamma = \frac{1}{K}\left[W_\gamma^1 + W_\gamma^2 + \dots + W_\gamma^k\right]. \tag{1}$$

where $W_\gamma^K (\gamma = 1,2,\dots,\theta)$ is the attribute weight of Kth DMs and can be described by linguistic variable.

(2) Use linguistic variables (Table 2) to identify the attribute ratings of suppliers for the subjective attributes. Then, the rating value can be calculated in Eq. (2).

$$G_{i\gamma} = \frac{1}{K}\left[G_{i\gamma}^1 + G_{i\gamma}^2 + \dots + G_{i\gamma}^K\right]. \tag{2}$$

where $G_{i\gamma}^K (i = 1,2,\dots,n; \gamma = 1,2,\dots,\theta)$ is the attribute rating value of Kth DMs.

Step 2
Construct the decision matrix D that the structure of the matrix can be expressed in Eq. (3).

$$D = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1\theta} \\ G_{21} & G_{22} & \cdots & G_{2\theta} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{n\theta} \end{bmatrix}. \tag{3}$$

Step 3
Standardize the evaluation matrix in Eq. (4): the process is to transform different scales and units among various criteria into common measurable units to allow comparisons across the criteria.

$$D^* = \begin{bmatrix} G_{11}^* & G_{12}^* & \cdots & G_{1\theta}^* \\ G_{21}^* & G_{22}^* & \cdots & G_{2\theta}^* \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1}^* & G_{n2}^* & \cdots & G_{n\theta}^* \end{bmatrix}. \tag{4}$$

Assume $G_{i\gamma}$ to be of the evaluation matrix D of alternative i under evaluation criterion γ then an element $G_{i\gamma}^*$ of the normalized evaluation matrix D^* can calculated by Eq(5).

$$G_{i\gamma}^* = \frac{G_{i\gamma}}{\sqrt{\sum_{i=1}^{n}\left(G_{i\gamma}\right)^2}}. \tag{5}$$

Step 4
Construct the weighted normalized decision matrix in Eq. (6). Considering the relative importance of each attribute, the weighted normalized evaluation matrix can be calculated by multiplying the normalized evaluation matrix $G_{i\gamma}^*$ with its associated weight W_γ to obtain the result $V_{i\gamma} = G_{i\gamma}^* \times W_\gamma$. The weighted normalized decision matrix $D^{*'}$ is:

$$D^{*'} = \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{1\theta} \\ V_{21} & V_{22} & \cdots & V_{2\theta} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & V_{n\theta} \end{bmatrix}. \tag{6}$$

Step 5
Construct the ideal solutions $S^{\max} = \left\{G_1^{\max}, G_2^{\max}, \dots, G_\theta^{\max}\right\}$ and negative ideal solutions $S^{\min} = \left\{G_1^{\min}, G_2^{\min}, \dots, G_\theta^{\min}\right\}$ in Eqs. (7)-(8) respectively.

$$S^{\max} = \left\{ \left[\max_{1 \le i \le n} V_{i\gamma} \middle| \gamma \in J_1 \right], \left[\min_{1 \le i \le n} V_{i\gamma} \middle| \gamma \in J_2 \right] \right\}. \quad (7)$$

$$S^{\min} = \left\{ \left[\min_{1 \le i \le n} V_{i\gamma} \middle| \gamma \in J_1 \right], \left[\max_{1 \le i \le n} V_{i\gamma} \middle| \gamma \in J_2 \right] \right\}. \quad (8)$$

where J_1 is associated with the benefit criteria and J_2 is associated with the cost criteria; $\gamma=1,\ldots,\theta$.

Step 6

Calculate the separation of each alternative from the ideal solution and negative ideal solutions in Eqs. (9)-(10) respectively. That means S_i^+ is the distance (in Euclidean sense) of each alternative from the ideal solution and S_i^- is the distance from the negative ideal solution and are defined as:

$$S_i^+ = \sqrt{\sum_{\gamma=1}^{\theta} \left(V_{i\gamma} - G_i^{\max}\right)^2}. \quad (9)$$

$$S_i^- = \sqrt{\sum_{\gamma=1}^{\theta} \left(V_{i\gamma} - G_i^{\min}\right)^2}. \quad (10)$$

where $i=1,\ldots,n$.

Step 7

The relative closeness to the ideal solution is calculated in Eq. (11).

$$C_i^* = \frac{S_i^-}{S_i^+ + S_i^-}; i = 1,\ldots,n. \quad (11)$$

where $0 \le C_i^* \le 1$.

Step 8

Rank the supplier alternatives. When C_i^* is bigger, the ranking order of S_i is better. If there are no constraints, choose the maximum score supplier and buy all demand from this supplier, otherwise go to next step.

Establish the Fuzzy MOMILP Model for Order Allocation

The model is established for supplier selection problem and later the proposed single objective model is used to calculate the optimum order quantities among the selected suppliers. The objective functions and the constraints of this model are described as follows:

The following notations are defined to formulate the model:

Notations

Indices:

$i=1,\ldots,n$ index of suppliers;

$t=1,\ldots,T_i$ index of price level

$j=1,\ldots,m_i$ index of products (items)

Parameters:

D_j demand of product j

O_i order cost for supplier i

q_{ij} expected defect rate of product j offered by supplier i

V_{ij} capacity of supplier i for product j

R_{ijt} maximum purchased volume of product j from supplier i at price level t

R^*_{ijt} slightly less than R_{ijt}

C_{ijt} purchasing price of the product j from supplier i at price level t

W_i the overall score of the supplier i obtained from TOPSIS Model that is equal to C_i^*

Decision variables:

X_{ijt} number of product j ordered from supplier i at price level t

Y_{ijt} 1 if an order is placed on supplier i at price level t for product j, 0 otherwise

Y_i 1 if at least an order is placed on supplier i, 0 otherwise

Objective functions

Total cost: The sum of the raw materials or component parts cost and order cost should be minimized; therefore, the total cost function is defined in Eq. (12).

$$\min Z_1 = \sum_{i=1}^{n} \left(\sum_{j=1}^{mi} \sum_{t=1}^{T_i} C_{ijt} X_{ijt} + O_i Y_i \right). \quad (12)$$

Total defect rate: q_{ij} is the expected defect rate of jth product for supplier i, which should be minimized; therefore, it can be stated in Eq. (13).

$$\min Z_2 = \sum_{i=1}^{n} \sum_{j=1}^{mi} \sum_{t=1}^{T_i} q_{ij} X_{ijt}. \quad (13)$$

Inverse TVP: As $(1-W_i)$ and X_{ijt} denote the inverse normal weights of the suppliers and the numbers of purchased units of product j from ith supplier at price level t, respectively, and Eq. (14) is designed to minimize the inverse TVP.

$$\min Z_3 = \sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{t=1}^{T_i} (1-W_i) X_{ijt} \quad (14)$$

Proposed Single Objective Function for Fuzzy Situation

In a real case, for this problem, all objectives may not be achieved at the same time under the system constraints; the DM may define a tolerance limit and membership function $\mu_{Z_k}(x)$ for the kth fuzzy goal [17]. The objective functions Z_k, $k=1,\ldots,p$, are expressed by fuzzy sets whose membership functions increase linearly from 0 to 1. In this approach, the membership function of objectives is formulated by separating every objective function into its maximum and minimum values [17, 16]. The linear membership function for minimization goals (Z_k) is given as follows (Fig. 2) [17]:

$$\mu_{Z_k}(x) = \begin{cases} 1 & Z_k(x) \le Z_k^- \\ \dfrac{Z_k^+ - Z_k(x)}{Z_k^+ - Z_k^-}, & Z_k^- \le Z_k(x) \le Z_k^+, k = 1,2,\ldots,p \\ 0 & Z_k(x) \ge Z_k^+ \end{cases} \quad (15)$$

Z_k^- is minimum value (best solution), and Z_k^+ is the maximum value (worst solution) of negative objective Z_k [17].

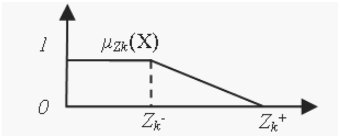

Fig.2. Objective function as fuzzy number for minimization objective [17].

To satisfy the membership function, we can define $Z_k(x) - \lambda_k Z_k^+ \le 0$ where $\lambda_k \in [Z_k^- / Z_k^+, 1]$. It is clear when λ_k is equal to Z_k^- / Z_k^+, $Z_k(x)$ decreases until Z_k^-, this is why, to minimize $Z_k(x)$, λ_k should be minimized. Therefore, the proposed single objective function and its constraints to consider relative importance of each fuzzy goal are as follows:

$$\min Z = \sum_{k=1}^{p} w_k \lambda_k \tag{16}$$

s.t

$$Z_k(x) - \lambda_k Z_k^+ \le 0 \qquad k = 1,2,\ldots,p \tag{17}$$

$$\frac{Z_k^-}{Z_k^+} \le \lambda_k \le 1 \qquad k = 1,2,\ldots,p \tag{18}$$

$$\sum_{k=1}^{p} w_k = 1 \qquad 0 \le w_k \le 1 \tag{19}$$

where w_k is the weighting coefficient that present the relative importance of the fuzzy goals.

Constraints

The constraints of the problem are formulated as follows:

Capacity constraints: As supplier i can supply up to V_{ij} units of product j and its order quantity for product j (X_{tij}) should be equal or less than its capacity, these constraints are given in Eq. (20).

$$\sum_{t=1}^{T_i} X_{ijt} \le V_{ij}, \quad i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i\ . \tag{20}$$

Demand constraints: As sum of the assigned order quantities of products to n suppliers should meet the buyer's demand, demand constraint is designed in Eq. (21).

$$\sum_{i=1}^{n}\sum_{t=1}^{T_i} X_{ijt} = D_j, \quad j = 1,2,\ldots,m_i\ . \tag{21}$$

Combining the ordering cost: In order to combine the ordering costs of several products into one single order for each supplier, Y_i variables are employed so that only if the buyer buys at least one product from supplier i ($\sum_{j=1}^{m_i}\sum_{t=1}^{T_i} X_{ijt} > 0$, $i = 1,2,\ldots,n$.), the integer variable is 1, 0 otherwise. These integer variables are taken into account by using the Eq. (22).

$$\left(\sum_{j=1}^{m_i}\sum_{t=1}^{T_i} X_{ijt}\right) \le \left(\sum_{j=1}^{m_i} V_{ij}\right) Y_i, \qquad i = 1,2,\ldots,n\ . \tag{22}$$

Quantity discount constraints: To consider quantity discount that is given by each supplier, the constraints are designed in Eqs. (23)-(24)-(25).

$$R_{ij(t-1)} Y_{ijt} \le X_{ijt},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i;\ t = 1,2,\ldots,T_i\ . \tag{23}$$

$$R_{ijt}^* Y_{ijt} \ge X_{ijt},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i;\ t = 1,2,\ldots,T_i\ . \tag{24}$$

$$\sum_{t=1}^{T} Y_{ijt} \le 1, \qquad i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i\ . \tag{25}$$

(At most one price level per supplier for each product can be chosen)

Non-negativity and binary constraints: The constraints can be shown in Eqs. (26)-(27)-(28).

$$X_{ijt} \ge 0,\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i;\ t = 1,2,\ldots,T_i\ . \tag{26}$$

$$Y_{ijt} = 0 \text{ or } 1,\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i;\ t = 1,2,\ldots,T_i \tag{27}$$

$$Y_i = 0 \text{ or } 1,\ i = 1,2,\ldots,n\ . \tag{28}$$

Table 3.
Suppliers' quantitative information

S_i	Q_1	Q_2
S_1	0.03	0.95
S_2	0.05	0.98
S_3	0.01	0.85

Table 4.
Attribute weights for five suppliers

Q_y	D_1	D_2	D_3	D_4	w_y	Normalized w_y
Q_1	H	M	ML	VL	0.25	0.278
Q_2	VH	VL	VVL	ML	0.21	0.229
Q_3	L	VL	VVL	MH	0.17	0.187
Q_4	VH	L	M	ML	0.28	0.306
TOTAL					0.91	1.000

Table 5.
Attribute rating values for supplier

Q_y	S_i	D_1	D_2	D_3	D_4	G_{iy}
Q_1						
	S_1	0.03	0.03	0.03	0.03	0.030
	S_2	0.05	0.05	0.05	0.05	0.050
	S_3	0.01	0.01	0.01	0.01	0.010
Q_2						
	S_1	0.95	0.95	0.95	0.95	0.950
	S_2	0.98	0.98	0.98	0.98	0.980
	S_3	0.85	0.85	0.85	0.85	0.850
Q_3						
	S_1	G	P	MP	MP	4.000
	S_2	MP	MP&F	MP&F	MP&F	3.750
	S_3	F	F	MP&F	F	4.750
Q_4						
	S_1	G	MP	P	MP	4.000
	S_2	MP	MP&F	F	F	4.250
	S_3	G	G	MP&G	MG&G	8.500

Table 6.
Normalized decision table

S_i	Q_1	Q_2	Q_3	Q_4
S_1	0.507	0.591	0.551	0.388
S_2	0.845	0.609	0.517	0.412
S_3	0.169	0.529	0.655	0.824

Table 7.
Weighted normalized decision table

S_i	Q_1	Q_2	Q_3	Q_4
S_1	0.141	0.135	0.103	0.119
S_2	0.235	0.140	0.097	0.126
S_3	0.047	0.121	0.122	0.252

3. A numerical example

This numerical example proposes a two-stage mathematical model to evaluate suppliers and their shipment allocations, given a number of quantitative and qualitative criteria, some of which may conflict. In the first stage, three different suppliers are evaluated by using TOPSIS. In the second stage of the model, the weights computed by TOPSIS serve as coefficients in the inverse TVP objective functions of the fuzzy multi-objective model. The objective is to find the optimum order quantities subject to satisfy capacity and demand requirement constraints. The main steps are explained by the following order.

Applying TOPSIS Method to Calculate the Overall Score of Each Supplier

Here, there are three suppliers $S_i=\{S_1,S_2,S_3\}$ selected as alternatives against four attributes $Q_y=\{Q_1,Q_2,Q_3,Q_4\}$. The four attributes are a special factor, on-time delivery, performance history and technical capability, respectively. Q_2, Q_3 and Q_4 are benefit attributes, the greater values being better, and Q_1 are cost attributes, the smaller values are better. Performance history (Q_3) and technical capability (Q_4) are subjective criteria that are considered as linguistic variables, and other attributes are also scaled using their own real numbers, respectively that is shown in Table 3.

Step 1
Make the weights of attributes Q_1, Q_2, Q_3 and Q_4. A committee of four DMs, D_1, D_2, D_3 and D_4 has been formed to express their preferences and to select the best suppliers. Based on Eq. (1), the evaluation values of attribute weights from four MDs can be obtained and the results are shown in Table 4.

Step 2
Make attribute rating values for three supplier alternatives. Based on Eq. (2), the results of attribute rating values are shown in Table 5.

Step 3
Construct the decision matrix. Based on Eq. (3), the decision matrix of suppliers is obtained.

Step 4
Construct the normalized decision table. Based on the normalized decision matrix shown in Eq. (4), the normalized decision matrix of suppliers is shown in Table 6.

Step 5
Construct the weighted normalized decision table. Based on the weighted normalized decision matrix shown in Eq. (6), the weighted normalized decision matrix of suppliers is shown in Table 7.

Step 6
Construct the ideal supplier $S^{\max}$ and negative ideal supplier $S^{\min}$ as referential suppliers. Based on Eqs. (7)-(8), the ideal and negative ideal suppliers are shown as follows, respectively:

$$S^{\max} = [0.047,..,0.121,..,0.097,..,0.119]$$
$$S^{\min} = [0.235,..,0.140,..,0.122,..,0.252]$$

Step 7
Calculate the separation of each alternative from the ideal and negative ideal suppliers. Based on Eqs. (9)-(10), the results of the separation are shown as follows:

$$S_1^+ = 0.095 \quad S_2^+ = 0.189 \quad S_3^+ = 0.136$$
$$S_1^- = 0.164 \quad S_2^- = 0.129 \quad S_3^- = 0.189$$

Step 8
Calculate the relative closeness of each alternative to the ideal supplier. Based on Eq. (11), the results of the relative closeness are shown as follows:

$$C_1^* = 0.391 \quad C_2^* = 0.250 \quad C_3^* = 0.359$$

Step 9
Rank the order of three suppliers. Based on step 8, the result of ranking order is shown as follows:

$$S_1 > S_3 > S_2$$

Note: The ratings computed in step 8 will serve as coefficients in the inverse TVP objective functions of the fuzzy multi-objective model.

Fuzzy Multi-Objective Model and Solution Process

The three suppliers should be managed for two products in two price levels so that the prices (C_{ijt} in \$) of each product quoted by each supplier, and the defect rate (q_{ij}), the capacity of suppliers (V_{ij}) and the order cost (O_i) are provided in Table 8. The demand vector D_j is (15000, 7000): i.e. the demand of the product 1 is 15,000. The overall score vector W_i is (0.391, 0.25, 0.359). The linear membership function is used for fuzzifying the objective functions of the above problem according to Eq. (15). The data set for the values of the lower bounds and upper bounds of the objective functions are shown in Table 9 and Fig. 3. The model can be formulated based on Eqs. (16), (17), (18), (19), (20), (21), (22), (23), (24), (25), (26), (27) and (28) as follows:

$$\min Z = \sum_{k=1}^{3} w_k \lambda_k \qquad \text{s.t}$$
$$Z_k(x) - \lambda_k Z_k^+ \le 0 \qquad k = 1,2,3\,.$$
$$0.75 \le \lambda_1 \le 1 \qquad Z_1^+ = 400{,}000\,, \qquad Z_1^- = 300{,}000\,.$$
$$0.28 \le \lambda_2 \le 1 \qquad Z_2^+ = 70\,, \qquad Z_2^- = 20\,.$$
$$0.81 \le \lambda_3 \le 1 \qquad Z_3^+ = 16{,}000\,, \qquad Z_3^- = 13{,}000\,.$$
$$\sum_{t=1}^{2} X_{ijt} \le V_{ij}\,, \qquad i = 1,2,3;\ j = 1,2\,.$$
$$\sum_{i=1}^{3}\sum_{t=1}^{2} X_{ijt} = D_j\,, \qquad j = 1,2\,.$$

$$\left(\sum_{j=1}^{2}\sum_{t=1}^{2} X_{ijt}\right) \leq \left(\sum_{j=1}^{2} V_{ij}\right) Y_i , \qquad i = 1,2,3 .$$

$$R_{ij(t-1)} Y_{ijt} \leq X_{ijt} , \qquad i = 1,2,3; j = 1,2; t = 1,2 .$$

$$R^{*}_{ijt} Y_{ijt} \geq X_{ijt} , \qquad i = 1,2,3; j = 1,2; t = 1,2 .$$

$$\sum_{t=1}^{T} Y_{ijt} \leq 1 , \qquad i = 1,2,3; j = 1,2 .$$

$$X_{ijt} \geq 0 , \qquad i = 1,2,3; j = 1,2; t = 1,2 .$$

$$Y_{ijt} = 0 \text{ or } 1, \qquad i = 1,2,3; j = 1,2; t = 1,2$$

$$Y_i = 0 \text{ or } 1, \qquad i = 1,2,3$$

The three objective functions Z_1, Z_2 and Z_3 are total cost, net rejections and inverse TVP, respectively, and X_{ijt} is the number of purchased units of product *j* from the *i*th supplier at price level *t*. Moreover, w_k (k=1,2,3) is the weights that is associated with the *k*th objective.

Case 1: the weights of the objectives are given as w_1=1, w_2=0 and w_3=0 are weights of total cost, net rejections and inverse TVP objective functions, respectively. The linear programming software LINDO or Solver from Microsoft Excel is used to solve this problem. The optimal solution for the above formulation is calculated as follows:

X_{112} =4,000 Y_{212} =11,000 X_{121} =1,001 X_{222} =5,999
Z_1=325,656 Z_2=69 Z_3=15,745

Table 8.
Collected data for numerical example

S_i	Quantity level for product 1	Price ($)	Quantity level For product 2	Price ($)	Order cost
S_1					
	Q<4,000	14	Q<3,000	20	85
	4,000 ≤ Q<10,000	13.5	3000 ≤ Q<6,000	19.5	
	V_{11}= 10,000; q_{11}= 0.002		V_{12}= 10,000; q_{12}= 0.002		
S_2					
	Q<4,000	13	Q<4,000	19.5	70
	4,000 ≤ Q<12,000	12.5	4,000 ≤ Q<6,000	19	
	V_{21}= 12,000; q_{21}= 0.0035		V_{22}= 12,000; q_{22}= 0.0035		
S_3					
	Q<5,000	15	Q<2,500	21	70
	5,000 ≤ Q<12,000	14.5	2,500 ≤ Q<4,500	21.5	
	V_{31}= 12,000; q_{31}= 0.001		V_{32}= 12,000; q_{32}= 0.001		

Case 2: in this case, inverse TVP is the most important factor for the DM in comparison with case 1; therefore, the relative importance of the objectives are assumed as w_1=0, w_2=0 and w_3=1 are weights of total cost, net rejections and inverse TVP objective functions, respectively. Thus, the value of objectives and the ordered quantities vary as follows:

X_{112} =9,999 Y_{312} =5,001 X_{122} =4,501 X_{321} =2,499
Z_1=347,904 Z_2=37 Z_3=13,638

Case 3: finally, quality is the most important factor for the DM in comparison with case 1 and 2; therefore, the weights of the goals are assumed as w_1=0, w_2=1 and w_3=0 are weights of total cost, net rejections and inverse TVP objective functions, respectively. Thus, the value of objectives and the ordered quantities vary as follows:

X_{111} =3,999 X_{312} =11,001 X_{121}=2,501 X_{322}=4,499
Z_1=362,404 Z_2=29 Z_3=13,894

The results of the three cases are shown in Table 10.

Table 9.
The data set for membership functions

	upper bounds, μ=0	lower bounds, μ=1
Min Z_1 (total cost)	Z_1^+ =\$400,000	Z_1^- =\$300,000
Min Z_2 (rejected items)	Z_2^+ =70	Z_2^- =20
Min Z_3 (inverse TVP)	Z_3^+ =16.000	Z_3^- =13.000

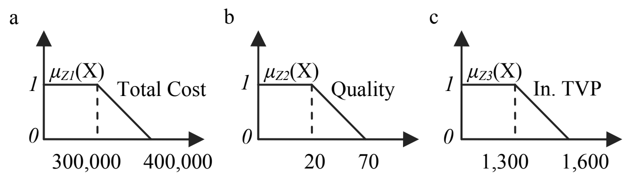

Fig.3. Membership functions: (a) total cost (Z_1), (b) quality (Z_2) and (c) inverse TVP (Z_3) objective functions.

Table 10.
Different cases solutions of the numerical example

	Case 1:		Case 2:		Case 3:	
	j = 1	j = 2	j = 1	j = 2	j = 1	j = 2
X_{1j}	4.000	1.001	9.999	4.501	3.999	2.501
X_{2j}	11.000	5,999	0	0	0	0
X_{3j}	0	0	5.001	2.499	11.001	4.499
Z_1	325.656		347.904		362.404	
Z_2	69		37		29	
Z_3	15.745		13.638		13.894	

In case 1, because of the DM's preference, cost is the most important factor and the performance of the factor is the best value of the three costs in comparison to other solutions. In case 2, since the inverse TVP is very important, its performance is improved from 15,745 to 13,638 in comparison with case 1. And finally, Because of the DM's preference in case 3, quality is the most important factor and its performance is the best value in comparison with other solutions. It is shown that the proposed model is able to improve the value of objectives function or performance on the objectives based on the DM's preference. Moreover, it is shown that variation in priority of factors will cause variation in optimum ordered quantities to each supplier. Thus, this model enables the purchasing managers to calculate optimum order quantities to each supplier based on the priority of criteria in a supply chain. In addition, our solutions are similar to the solutions obtained by Amid et al. [17] that used the weighted additive model.

The value of objectives for the three cases is shown in Fig. 4.

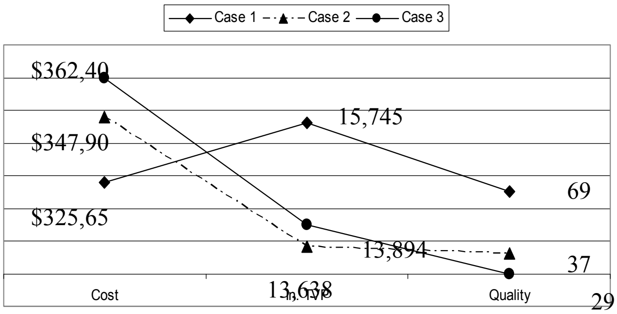

Fig. 4. Solutions of the different cases—for the three dimensions of cost, rejected items and inverse TVP

4. Conclusions

Supplier selection is a MCDM problem that is affected by several factors. For supplier selection problem the criteria are not equally important, and many input data or information is not known precisely as well. In this article, we proposed a single objective function to solve the fuzzy multi-item multi-objective model in order to calculate the optimum order quantities to each supplier. This proposed single objective considers vagueness of input data and relative importance of the criteria at the same time. This approach is able to help the DM evaluate the suppliers in order to find out the appropriate order to each them, and allows purchasing manager(s) to manage supply chain performance on service, cost, quality and etc. the suppliers' price breaks, which depend on the sizes of order quantities, affects the selection process. The problem includes the three objective functions: minimizing the total cost, the net rejected items and the inverse total value of purchasing (TVP), while satisfying capacity and demand requirement constraints.

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