Journal of Achievements in Materials and Manufacturing Engineering

VOLUME 32
ISSUE 1
January
2009

# Longitudinal vibrations of mechanical systems with the transportation effect

**A. Buchacz, S. Żółkiewski** *
Division of Mechatronics and Designing of Technical Systems,
Institute of Engineering Processes Automation and Integrated Manufacturing System,
Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: slawomir.zolkiewski@polsl.pl

Received 11.09.2008; published in revised form 01.01.2009

**Analysis and modelling**

## ABSTRACT

**Purpose:** this thesis purpose is a new way of modelling systems working with high speeds of mechanisms. Systems are analyzed with taking into consideration the rotational movement and with criterions of using materials with high flexibility and high precision of work. The dynamical analysis was done with giving into consideration the interaction between working motion and local vibrations. During the motion a model is loaded by longitudinal forces.

**Design/methodology/approach:** equations of motion were derived by the Lagrange method, with generalized coordinates and generalized velocities assumed as orthogonal projections of individual quantities of the rod and manipulators to axes of the global reference frame.

**Findings:** the model of longitudinally vibrating systems in plane motion was derived, after that the model can be transformed to the dynamical flexibility of these systems. Derived equations are the beginning of analysis of complex systems, especially can be used in deducing of the substitute dynamical flexibility of multilinked systems in motion.

**Research limitations/implications:** mechanical systems vibrating longitudinally in terms of rotation were considered in this thesis. Successive problem of the dynamical analysis is the analysis of systems in spatial transportation and systems loaded by transversal forces.

**Practical implications:** effects of presented calculations can be applied into machines and mechanisms in transportation such as: high speed turbines, wind power plant, water-power plants, manipulators, aerodynamics issues, and in different rotors etc.

**Originality/value:** the contemporary analysis of beams and rods were made in a separate way, first working motion of the main system and next the local vibrations. A new way of modelling took into consideration the interaction between those two displacement. There was defined the transportation effect for models vibrating longitudinally in this paper.

**Keywords:** Applied mechanics; Longitudinal vibrations; Multibody systems; Transportation effect

## 1. Introduction

Applications of the mechanical and mechatronic systems make known a tendency to optimization of parameters of work of machines and mechanisms. High performance of modern technical systems, mainly concerning maximal precision of work of mechanical systems and positioning of manipulators and robots and also high quality assurance and reliability, is the cause of optimization of models. Nowadays solutions assumed the discrepancy between main global motion and local amplitudes of vibrations. In this thesis we search we search the more precisely ways of modelling. Up to now very often is used the superposition method that is not as accurate as presented method because of not taking into consideration interactions between main motion and local motion as well. Considered models are the free rod vibrating longitudinally, the rod fixed in the origin of the global reference system and manipulators in transportation. Equations of motion include centrifugal forces and Coriolis forces connected with the transportation effect. Considered systems are into plane rotational motion. Acting force is a harmonic axial one with the amplitude of the force equals one, that is consistent to the definition of the dynamical flexibility. Force generates longitudinal vibrations. In literature, in many publications the problem of analyzing of systems moving with low speed or while they vibrate only locally is a well-known problem [7-15,18]. In our model we took into consideration issues have not considered up to now, such as the relations between local vibrations and main motion [1-6,16-18]. Transportation in this thesis is considered as main motion. Nowadays problems of controlling mechanical systems need to be considered much more adequate to actual phenomena than so far solutions. Up to now, final results had done by consider main motion and local vibrations separately. That simplification has essential sense because vibrations from flexibility of elements of the mechanical composition are much smaller than main dislocation of this composition. More efficient drives caused increased scope of velocities and accelerations. Nowadays terms of operation of machines are connected with constraint power output of drives needful for motion and using materials with lower mass density such as aluminums alloys, lower than mass density of materials using up to now. Those and many other arguments are a cause of searching new models of designing systems, that take into consideration the flexibility of mechanism's elements.

## 2. Modelling of longitudinal vibrations of mechanical systems in transportation

There is considered models of the free rod and the rod that is fixed in the origin of the global reference system and systems with higher then one mobility such as two-linked manipulators. Elements of analyzing systems are the homogenous rods made from an elastic materials with a full cross-section on whole length.

### 2.1. The model of longitudinally vibrating and rotating rod fixed in the origin

The modeled rod is in rotational transportation. Motion of the rod is all round the origin of the global reference system. The rod is fixed in the center of rotation (Fig.1). Force acting on the system is the longitudinal one. Systems in motion were described by one component of the instantaneous angular velocity vector with respect to the axis of the global inertial frame XY. Considered systems vibrate with taking into consideration deformation in transportation. Rod is the homogenous elastic element with a full cross-section $A$ that is constant on the whole length of the rod $l$. Material of the rod has the longitudinal modulus of elasticity (the Young modulus) $E$ and mass density $\rho$. The initial conditions are known: preliminary deflection of the rod and initial velocity of vibrations. Transverse deformation of the rod caused by longitudinal vibrations is not considered based on the principle of plane cross-sections up to [12]. The solution is done in the global reference system, independent from the rod.

### 2.2. The vibrating longitudinally free rod in transportation – simplified model

The homogeneous flexible free rod is considered in this subchapter. The rod has a symmetrical section. Deriving of the dynamical model of the free rod vibrating with longitudinal vibrations and taking into consideration the transportation (Fig. 2) is the paper's objective.

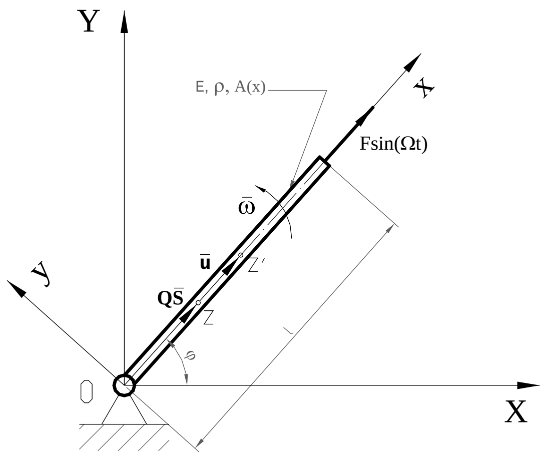


Fig. 1. The model of the rotating rod loaded by a longitudinal force

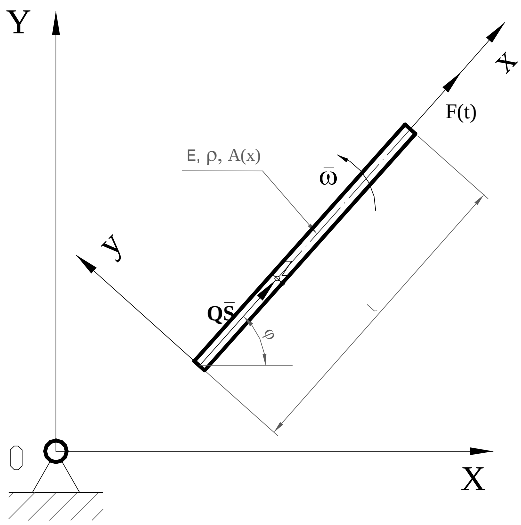


Fig. 2. The vibrating rod in terms of plane motion and loaded by a harmonic axial force

## 2.3. The longitudinally vibrating two-linked manipulator

In this subsection the two-link vibrating manipulator is considered. The manipulator is composted from rods with cross sections suitably $A_1$ as the section of first rod and $A_2$ as the section of second rod which are constant on the whole length of rods appropriately for first rod $l_{01}$ and in second rod $l_{12}$ (Fig. 3). Material of first rod has Young's modulus $E_1$ and material of second one $E_2$ and similarly mass densities $\rho_1$ for first one and $\rho_2$ and for second one. Rods were imposed by a harmonic longitudinal force. The solution was determined in global independent reference system in terms of plane motion.

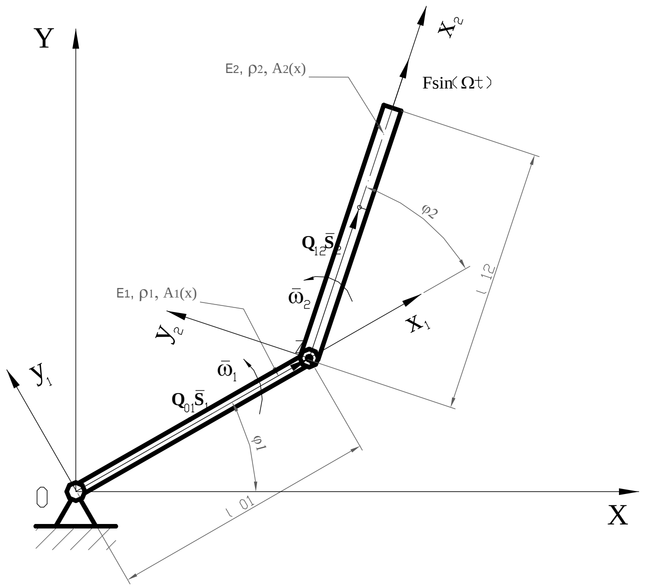


Fig. 3. The two-linked manipulator loaded by a longitudinal force in transportation

## 2.4. The model of the longitudinally vibrating three-linked manipulator

The tree-linked vibrating manipulator is considered. Rods from this system have cross sections constant on the whole length of rods (Fig. 4). Manipulator was loaded by a harmonic longitudinal force.

The mathematical model was determined in global independent reference system in terms of plane motion. Rods from this manipulator have cross sections suitably $A_1$ as the section of first link and $A_2$ as the section of second link and $A_3$ as the section of third link which are constant on the whole length of rods appropriately for first link $l_{01}$ and in second link $l_{12}$ and in third one $l_{23}$ like in Figure 4.

Rods were made from materials with Young's modulus $E_1$, $E_2$ and $E_3$ and mass densities $\rho_1$ and $\rho_2$ and $\rho_3$. The manipulator was loaded by a harmonic longitudinal force. The trend of increasing the motility of the mechanism is optional and can be increased up to the motility depends only on technical requirements.

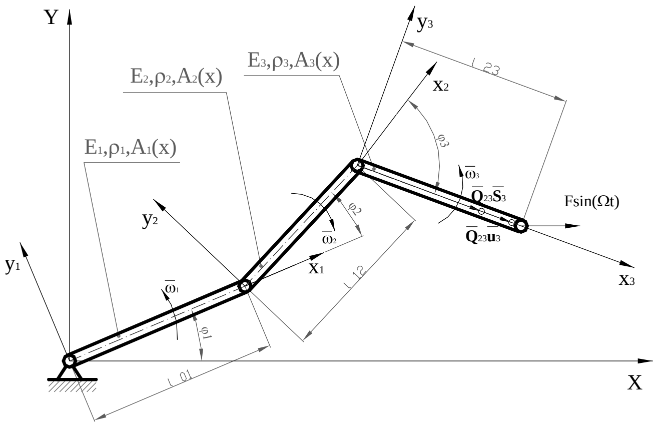


Fig. 4. The three-linked manipulator loaded by a longitudinal force in transportation

# 3. Mathematical model

In this section the mathematical models of the analyzed systems were presented. A vector of linear displacement of the rod's section along center line of the bar ($u$) in the local reference system (Fig. 1) can be expressed as:

$$\bar{\mathbf{u}} = \begin{bmatrix} u & 0 & 0 \end{bmatrix}^T. \tag{1}$$

Body mass of the rod that is made from material with mass density $\rho$ and a volume $V$ and a cross-section $A$:

$$M = \int_V \rho dV = \int_0^s \rho \cdot A ds, \tag{2}$$

where:

$$dV = A \cdot dx. \tag{3}$$

Vibrations of the rod were analyzed in places along the axis x of the local reference system ($s$), so a position vector in that system is as follow:

$$\bar{\mathbf{S}} = \begin{bmatrix} s & 0 & 0 \end{bmatrix}^T. \tag{4}$$

Vibrations of the rod in planar transportation is analyzed. Generalized coordinates and generalized velocities were assumed as orthogonal projections of coordinates ($r_X$, $r_Y$) and velocities of the rod to axes of the global reference frame:

$$q_1 = r_X, \quad q_2 = r_Y, \tag{5}$$

$$\dot{q}_1 = \frac{dq_1}{dt} = \dot{r}_X = v_X, \quad \dot{q}_2 = \frac{dq_2}{dt} = \dot{r}_Y = v_Y. \tag{6}$$

The generalized forces acting in the system were defined as internal forces in the rod as follows:

$$P_X = \frac{\partial N \cdot Q_{11} \cdot s_X}{\partial x}, \quad P_Y = \frac{\partial N \cdot Q_{21} \cdot s_Y}{\partial x}. \tag{7}$$

where the $Q_{11}$ and $Q_{21}$ are the components of the rotation matrix in the global reference system, $s_X$, $s_Y$, are projections of $s$ onto global reference axes and $N$ is an axial force.

A position vector of the vibrating points in the global reference system are presented as follow:

$$\overline{\mathbf{r}} = \overline{\mathbf{r}}_{\mathbf{X}} + \overline{\mathbf{r}}_{\mathbf{Y}} = \overline{\mathbf{i}} \cdot r_X + \overline{\mathbf{j}} \cdot r_Y = \mathbf{Q} \cdot \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right). \tag{8}$$

A linear velocity of the vibrating points in the global reference system:

$$\dot{\overline{\mathbf{r}}} = \overline{\mathbf{v}} = \overline{\mathbf{v}}_{\mathbf{X}} + \overline{\mathbf{v}}_{\mathbf{Y}} = \overline{\mathbf{i}} \cdot v_X + \overline{\mathbf{j}} \cdot v_Y = \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) + \mathbf{Q} \cdot \dot{\overline{\mathbf{u}}}. \tag{9}$$

An acceleration of the vibrating points in the global reference system is calculated from relationship (9) with taking into consideration the Coriolis acceleration and the normal acceleration and the tangential acceleration, so the following equation is obtained:

$$\begin{aligned} \ddot{\overline{\mathbf{r}}} &= \dot{\overline{\mathbf{v}}} = \dot{\overline{\mathbf{v}}}_{\mathbf{X}} + \dot{\overline{\mathbf{v}}}_{\mathbf{Y}} = \overline{\mathbf{i}} \cdot \dot{v}_X + \overline{\mathbf{j}} \cdot \dot{v}_Y = \\ &= \mathbf{Q} \cdot \dot{\overline{\boldsymbol{\omega}}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) + \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \overline{\boldsymbol{\omega}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) + \\ &+ \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \dot{\overline{\mathbf{u}}} + \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \dot{\overline{\mathbf{u}}} + \mathbf{Q} \cdot \ddot{\overline{\mathbf{u}}} = \\ &= \mathbf{Q} \cdot \dot{\overline{\boldsymbol{\omega}}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) + \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \overline{\boldsymbol{\omega}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) + \\ &+ 2 \cdot \mathbf{Q} \cdot \overline{\boldsymbol{\omega}} \times \dot{\overline{\mathbf{u}}} + \mathbf{Q} \cdot \ddot{\overline{\mathbf{u}}}. \end{aligned} \tag{10}$$

The rotation matrix is used to described the orientation of the rod with respect to the global reference frame and the matrix from the rotation round the Z axis of the global reference system is as follow:

$$\mathbf{Q} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{11}$$

the rotation matrix of second link with respect to global reference system:

$$\mathbf{Q}_{02} = \mathbf{Q}_{01} \cdot \mathbf{Q}_{12},$$

$$\mathbf{Q}_{02} = \begin{bmatrix} \cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2 & -\cos\varphi_1 \cdot \sin\varphi_2 - \sin\varphi_1 \cdot \cos\varphi_2 & 0 \\ \sin\varphi_1 \cdot \cos\varphi_2 + \cos\varphi_1 \cdot \sin\varphi_2 & \cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

## 3.1. Kinetic energy of the rod

Kinetic energy based on the Koenig's law is defined as a function of generalized coordinates and generalized velocities and can be written in the form:

$$\begin{aligned} T &= \frac{1}{2} \cdot M \cdot \overline{\mathbf{v}}^T \cdot \overline{\mathbf{v}} = \\ &= \frac{1}{2} \cdot M \cdot \mathbf{Q} \cdot \left[ \overline{\boldsymbol{\omega}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) \right]^T \cdot \left[ \overline{\boldsymbol{\omega}} \times \left( \overline{\mathbf{S}} + \overline{\mathbf{u}} \right) \right] + \\ &+ \frac{1}{2} \cdot M \cdot \mathbf{Q} \cdot \dot{\overline{\mathbf{u}}}^T \cdot \dot{\overline{\mathbf{u}}} = \frac{1}{2} \cdot M \cdot \left( \overline{\mathbf{i}} \cdot \dot{r}_X \right)^2 + \\ &+ \frac{1}{2} \cdot M \cdot \left( \overline{\mathbf{j}} \cdot \dot{r}_Y \right)^2 = \frac{1}{2} \cdot M \cdot \dot{r}_X^2 + \frac{1}{2} \cdot M \cdot \dot{r}_Y^2. \end{aligned} \tag{13}$$

where **i, j** are individual versors in the global reference system.

## 3.2. Equations of motion of the rod

The equations of motion of the rod were derived by using the classical methods and were presented as projections of individual values into axes of the global reference frame. The X axis of the global reference system projection is:

$$\frac{\partial^2 u_X}{\partial t^2} - \frac{E}{\rho} \cdot \frac{\partial^2 u_X}{\partial x^2} = \omega^2 \cdot \left( s \cdot \cos\varphi + u_X \right) + 2 \cdot \omega \cdot \frac{\partial u_Y}{\partial t} \tag{14}$$

The Y axis of the global reference system projection is:

$$\frac{\partial^2 u_Y}{\partial t^2} - \frac{E}{\rho} \cdot \frac{\partial^2 u_Y}{\partial x^2} = \omega^2 \cdot \left( s \cdot \sin\varphi + u_Y \right) - 2 \cdot \omega \cdot \frac{\partial u_X}{\partial t} \tag{15}$$

This system of equations can be put to use to derivation of the dynamical flexibility.

### 3.3. Forms of vibrations of the fixed rod

There are presented first three forms of vibrations for the rod fixed in the origin of the global reference frame (Fig. 5). The forms are consistent with a eigenfunction of displacement.

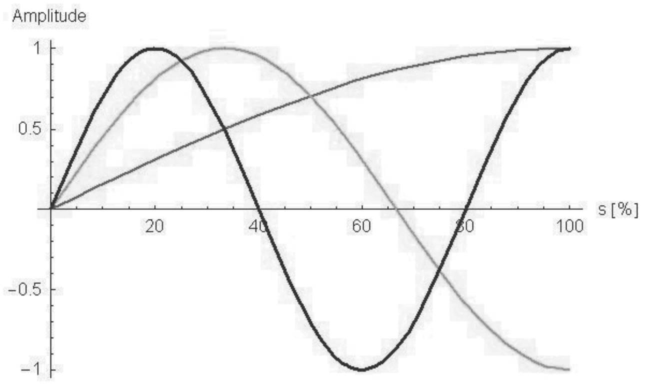


Fig. 5. The juxtaposition of first three forms of vibrations for the rod fixed in the origin of the global reference system

First form of vibrations was marked by a red line with only one node in place where the rod is fixed, the second one by a green line with two nodes and the third one by a blue line with three nodes.

### 3.4. Forms of vibration of the free rod

There are presented three forms of vibrations (Fig. 6) for the free rod loaded by harmonic longitudinal force. The figure does not contain first form for zero free vibration frequency that is a straight line.

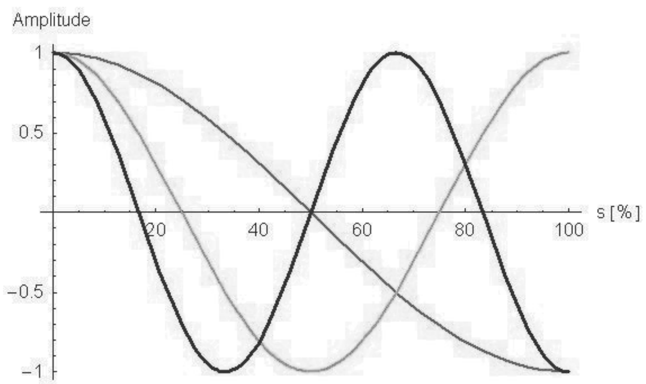


Fig. 6. The juxtaposition of three forms of vibrations for the free rod

First form of vibrations was marked by a red line with only one node in the middle of the rod, the second form of vibrations by a green line with two nodes and the third form of vibrations by a blue line with three nodes.

### 3.5. Equations of motion of the two-linked manipulator

The system of equations (16) presents the mathematical model of the manipulator in form of the equations of motion, there are equations bounded with first rod and second rod. Equations are projected into axes of the global reference system and are not coupled each other. The X axis of the global reference system projection is as follow:

$$\begin{cases} \frac{\partial^2 u_{1X}}{\partial t^2} - \frac{E_1}{\rho_1} \cdot \frac{\partial^2 u_{1X}}{\partial x_1^2} = \\ = \omega_1^2 \cdot \left(s_{1X} + u_{1X}\right) + 2 \cdot \omega_1 \cdot \frac{\partial u_{1Y}}{\partial t}, \\ \frac{\partial^2 u_{2X}}{\partial t^2} - \frac{E_2}{\rho_2} \cdot \frac{\partial^2 u_{2X}}{\partial x_2^2} = \\ = \left(\omega_1 + \omega_2\right)^2 \cdot \left(s_{2X} + u_{2X}\right) + 2 \cdot \left(\omega_1 + \omega_2\right) \cdot \frac{\partial u_{2Y}}{\partial t}. \end{cases} \quad (16)$$

The projection into the Y axis of the global reference system:

$$\begin{cases} \frac{\partial^2 u_{1Y}}{\partial t^2} - \frac{E_1}{\rho_1} \cdot \frac{\partial^2 u_{1Y}}{\partial x_1^2} = \\ = \omega_1^2 \cdot \left(s_{1Y} + u_{1Y}\right) - 2 \cdot \omega_1 \cdot \frac{\partial u_{1X}}{\partial t}, \\ \frac{\partial^2 u_{2Y}}{\partial t^2} - \frac{E_2}{\rho_2} \cdot \frac{\partial^2 u_{2Y}}{\partial x_2^2} = \\ = \left(\omega_1 + \omega_2\right)^2 \cdot \left(s_{2Y} + u_{2Y}\right) - 2 \cdot \left(\omega_1 + \omega_2\right) \cdot \frac{\partial u_{2X}}{\partial t}. \end{cases} \quad (17)$$

### 3.6. Equations of motion of the three-linked manipulator

The mathematical model of the three-linked manipulator in form of the equations of motion is presented as system of equations. The equations are not coupled each other and first equation into X and Y projection is appropriate to first link and analogically the rest. The projection into the X axis of the global reference system:

$$\begin{cases} \frac{\partial^2 u_{1X}}{\partial t^2} - \frac{E_1}{\rho_1} \cdot \frac{\partial^2 u_{1X}}{\partial x_1^2} = \\ = \omega_1^2 \cdot (s_{1X} + u_{1X}) + 2 \cdot \omega_1 \cdot \frac{\partial u_{1Y}}{\partial t}, \\ \frac{\partial^2 u_{2X}}{\partial t^2} - \frac{E_2}{\rho_2} \cdot \frac{\partial^2 u_{2X}}{\partial x_2^2} = \\ = (\omega_1 + \omega_2)^2 \cdot (s_{2X} + u_{2X}) + 2 \cdot (\omega_1 + \omega_2) \cdot \frac{\partial u_{2Y}}{\partial t}, \\ \frac{\partial^2 u_{3X}}{\partial t^2} - \frac{E_3}{\rho_3} \cdot \frac{\partial^2 u_{3X}}{\partial x_3^2} = \\ = (\omega_1 + \omega_2 + \omega_3)^2 \cdot (s_{3X} + u_{3X}) + \\ +2 \cdot (\omega_1 + \omega_2 + \omega_3) \cdot \frac{\partial u_{3Y}}{\partial t}. \end{cases} \tag{18}$$

The projection into the Y axis of the global reference system:

$$\begin{cases} \frac{\partial^2 u_{1Y}}{\partial t^2} - \frac{E_1}{\rho_1} \cdot \frac{\partial^2 u_{1Y}}{\partial x_1^2} = \\ = \omega_1^2 \cdot (s_{1Y} + u_{1Y}) - 2 \cdot \omega_1 \cdot \frac{\partial u_{1X}}{\partial t}, \\ \frac{\partial^2 u_{2Y}}{\partial t^2} - \frac{E_2}{\rho_2} \cdot \frac{\partial^2 u_{2Y}}{\partial x_2^2} = \\ = (\omega_1 + \omega_2)^2 \cdot (s_{2Y} + u_{2Y}) - 2 \cdot (\omega_1 + \omega_2) \cdot \frac{\partial u_{2X}}{\partial t}, \\ \frac{\partial^2 u_{3Y}}{\partial t^2} - \frac{E_3}{\rho_3} \cdot \frac{\partial^2 u_{3Y}}{\partial x_3^2} = \\ = (\omega_1 + \omega_2 + \omega_3)^2 \cdot (s_{3Y} + u_{3Y}) + \\ -2 \cdot (\omega_1 + \omega_2 + \omega_3) \cdot \frac{\partial u_{3X}}{\partial t}, \end{cases} \tag{19}$$

where:

$$\begin{aligned} \bar{\mathbf{u}}_{1XY} &= \bar{\mathbf{u}}_{1X} + \bar{\mathbf{u}}_{1Y} = \bar{\mathbf{i}} \cdot u_{1X} + \bar{\mathbf{j}} \cdot u_{1Y}, \\ \bar{\mathbf{u}}_{2XY} &= \bar{\mathbf{u}}_{2X} + \bar{\mathbf{u}}_{2Y} = \bar{\mathbf{i}} \cdot u_{2X} + \bar{\mathbf{j}} \cdot u_{2Y}, \\ \bar{\mathbf{u}}_{3XY} &= \bar{\mathbf{u}}_{3X} + \bar{\mathbf{u}}_{3Y} = \bar{\mathbf{i}} \cdot u_{3X} + \bar{\mathbf{j}} \cdot u_{3Y}, \end{aligned} \tag{20}$$

and

$$\begin{aligned} u_{1X} &= u_1 \cdot \cos\varphi_1, \\ u_{1Y} &= u_1 \cdot \sin\varphi_1, \\ u_{2X} &= u_2 \cdot \cos(\varphi_1 + \varphi_2), \\ u_{2Y} &= u_2 \cdot \sin(\varphi_1 + \varphi_2), \\ u_{3X} &= u_3 \cdot \cos(\varphi_1 + \varphi_2 + \varphi_3), \\ u_{3Y} &= u_3 \cdot \sin(\varphi_1 + \varphi_2 + \varphi_3). \end{aligned} \tag{21}$$

## 3.7. Numerical examples of the dynamical flexibility of the free rod

Numerical examples are presented as a dynamical characteristic as dynamical flexibility (Fig.7). The dynamical flexibility of the free rod in transportation is as follow:

$$Y = \frac{2}{\rho \cdot A \cdot l} \cdot \sum_{n=0}^{\infty} \frac{\cos(n\pi) \cdot \left( c^2 \cdot n^2 \cdot \frac{\pi^2}{l^2} - \Omega^2 - \omega^2 \right) \cdot \cos\left( n\pi \frac{x}{l} \right)}{\left[ \left( c^2 \cdot n^2 \cdot \frac{\pi^2}{l^2} - \Omega^2 - \omega^2 \right)^2 - 4 \cdot \omega^2 \cdot \Omega^2 \right]}. \tag{22}$$

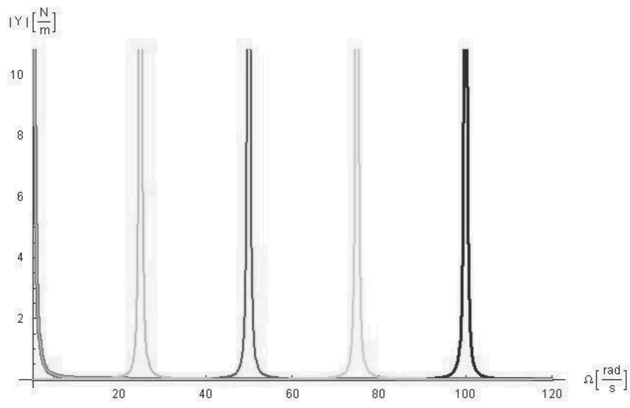


Fig. 7. Dynamical flexibility of the free rod vibrating longitudinally in transportation

In the Figure 7 there were presented numerical examples of dynamical flexibility of the rod in transportation. The dynamic flexibility of the rod with angular velocity equal $\omega_1$=0 rad/s was marked by a red line, $\omega_2$=10 rad/s by a green line, $\omega_3$=25 rad/s by a beige line, $\omega_4$=50 rad/s by a pink line, $\omega_5$=75 rad/s by a yellow line, $\omega_6$=100 rad/s by a blue line.

### 3.8. Numerical examples of the dynamical flexibility of the fixed rod

The dynamical flexibility of the rod fixed in the origin of the global reference system is as follow:

$$Y=\sum_{n=0}^{\infty}\frac{2\cdot\sin\left(\frac{2n+1}{2}\pi\right)}{\rho\cdot A\cdot l}\cdot$$
$$\cdot\sum_{n=0}^{\infty}\frac{\left[c^{2}\cdot\left(\frac{2\cdot n+1}{2}\right)^{2}\cdot\frac{\pi^{2}}{l^{2}}-\Omega^{2}-\omega^{2}\right]\cdot\sin\left(\frac{2n+1}{2}\pi\frac{x}{l}\right)}{\left[\left(c^{2}\cdot\left(\frac{2\cdot n+1}{2}\right)^{2}\cdot\frac{\pi^{2}}{l^{2}}-\Omega^{2}-\omega^{2}\right)\right]^{2}-4\cdot\omega^{2}\cdot\Omega^{2}}.\quad(23)$$

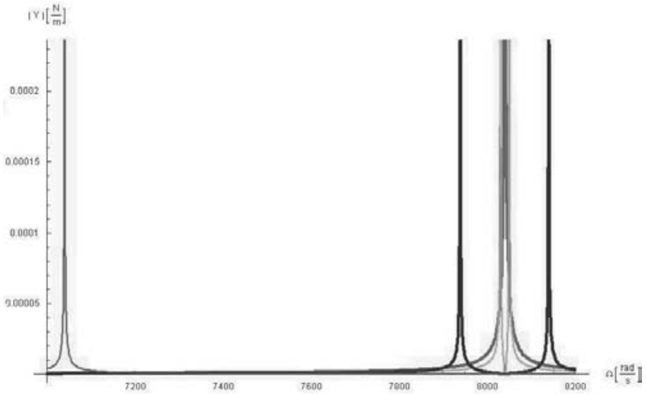


Fig. 8. Dynamical flexibility of the fixed rod vibrating longitudinally in transportation

The absolute value of the dynamical flexibility of the rod vibrating longitudinally with the angular velocity $\omega_1$=0 rad/s was marked by a red line, $\omega_2$=10 rad/s by a green line, $\omega_3$=100 rad/s by a blue line, $\omega_4$=1000 rad/s by a pink line (Fig. 8).

## 4. Conclusions

The equations of motion of system vibrating longitudinally in transportation were presented in this thesis. Movements of systems are two-dimensional motions. The derived model is the starting point of further more complicated dynamical analysis of those type systems and it can be put to use for the derivation of dynamical flexibility of manipulator. The following thesis's objectives will be the stability analysis and the derivation of attenuation-frequency characteristics.

The rotation matrix was used to determine of orientation of the system in space. The main rotation matrix was the product of the component rotation matrices with respect to individual axes of the global reference frame. The obtained model has the easy way of algorithmization, so it can be used to creating the numerical computer application.

The interactions between local displacements and transportation were emphasized. Occurrences of unbalanced forces lashed with transportation in the mathematical model were took into consideration. The Coriolis' force and the centrifugal force were took into account. The forces components were projected into the appropriate axes of the global reference system. The mathematical calculations of numerical examples were done assumed that the material of rods was the aluminum alloy and the length of the beam equals one meter. Equations of motion were derived by the classical methods such as the Lagrange equations or d'Alembert's method.

The longitudinally vibrating systems in terms of two-dimensional motion after derived the mathematical model in form of equations of motion can be put to use to derivation of the dynamical flexibility. We can also use those equations to deducing of the substitute dynamical flexibility of multi-body systems. Results of calculations after adopted and modified to appropriate models can be put to use into machines and mechanisms in transportation such as high speed turbines, rotors, wind power plant, manipulators and in aerodynamics issues, etc. Future problems of dynamical analysis such moving systems are the analysis of systems in non-planar transportation and systems loaded by transversal forces and also the nonlinear systems.

## Acknowledgements

This work was supported by Polish Ministry of Scientific Research and Information Technology (Polish State Committee for Scientific Research) Grant No. 4 T07A 009 28.

## References

[1] A. Buchacz, S. Żółkiewski, Dynamic analysis of the manipulator's mechanism with longitudinal vibrations of flexible links in transportation, Proceedings of the XLIII Symposium PTMTS „Modelling in Mechanics", Publishing house of Department of Applied Mechanics 34/04, Wisła, 2004, 23-24.

[2] A. Buchacz, S. Żółkiewski, Dynamic analysis of the manipulator's mechanism with longitudinal vibrations of flexible links in transportation, Proceedings of the XLIII Symposium PTMTS „Modelling in Mechanics", Publishing house of Department of Applied Mechanics 34/04, Wisła 2004, 57-62 (in Polish).

[3] A. Buchacz, S. Żółkiewski, Dynamical flexibility of the free rod with longitudinal vibrations in transportation, Proceedings of the XLV Symposium PTMTS „Modelling in Mechanics", Publishing house of Department of Applied Mechanics, Wisła, 2006 27-28.

[4] A. Buchacz, S. Żółkiewski, Longitudinal three-dimensional vibrations of the round rod with taking into consideration the transportation effect, Proceedings of the International Conference of Machine-Building and Technosphere of the XXI Century, vol. 5, Sevastopol, 2005 17-20.

[5] A. Buchacz, S. Żółkiewski, Equations of motion of the two-link system vibrating transversally and longitudinally in transportation, Proceedings of the International Conference of Machine-Building and Technosphere of the XXI Century, vol. 4, Sevastopol, 2006, 188-192.
[6] A. Buchacz, S. Żółkiewski, Mechanical systems vibrating longitudinally with the transportation effect, Journal of Achievements in Materials and Manufacturing Engineering 21/1 (2007) 63-66.
[7] A. Dymarek, The sensitivity as a criterion of synthesis of discrete vibrating fixed mechanical systems, Journal of Materials Processing Technology 157-158 (2004) 138-143.
[8] A. Dymarek, T. Dzitkowski, Modelling and synthesis of discrete-continuous subsystems of machines with damping, Journal of Materials Processing Technology 164-165 (2005) 1317-1326.
[9] T. Dzitkowski, Computer aided synthesis of discrete-continuous subsystems of machines with the assumed frequency spectrum represented by graphs, Journal of Materials Processing Technology 157-158 (2004) 1317-1326.
[10] G. Wszołek, Modelling of mechanical systems vibrations by utilization of grafsim software, Journal of Materials Processing Technology 164-165 (2005) 1466-1471.
[11] G. Wszołek, Vibration analysis of the excavator model in grafsim program on the basis of a block diagram method, Journal of Materials Processing Technology 157-158 (2004) 268-273.
[12] R. Solecki, J. Szymkiewicz, Rod and superficial systems, Dynamical calculations, Arcades, Building Engineering, Art, Architecture, Warsaw, 1964 (in Polish).
[13] S. Woroszył, Examples and tasks of the theory of vibrations. Continuous systems. PWN, Warsaw, 1979 (in Polish).
[14] K. Żurek, Active mechanical systems in reduction of vibration, Buletinul Institutului Politehnic Diu Iasi, Romania (2005) 480-484.
[15] K. Żurek, Design of reducing vibration machatronical systems, Proceedings of the 7th Conference "Computer Integrated Manufacturing", CIM'2005, Gliwice – Wisła, 2005, 292-297.
[16] G. Szefer, Dynamics of elastic bodies undergoing large motions and unilateral contact, Journal of Technical Physics XLI/ 4 (2000) 343-359.
[17] G. Szefer, Dynamics of elastic bodies in terms of plane frictional motion, Journal of Theoretical and Applied Mechanics 39/2 (2001) 395-408.
[18] J.J. Craig, Introduction to robotics, Mechanics and Control, WNT, Warsaw, 1995 (Polish edition).