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Exact and approximate analysis of mechanical and mechatronic systems

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Analysis and modelling

ABSTRACT

Purpose: of this paper is to compare the transients of characteristics obtained by the approximate method and exact one and to answer to the question – if the method can be used to nominate the characteristics of mechatronic systems.
Design/methodology/approach: was to nominate the relevance or irrelevance between the characteristics obtained by considered methods – especially concerning the relevance of the pole values of characteristics. The main subject of the research is the continuous torsionally vibrating bar considered as a mechanical subsystem of the mechatronic system.
Findings: this approach is fact, that approximate solutions fulfill all conditions for vibrating bars and some conditions only, particularly for flexibly vibrating beams.
Research limitations/implications: is that linear continuous torsionally vibrating bar is considered.
Practical implications: of this study is the main point is the analysis and the examination of torsionaly vibrating continuous mechatronic systems which characteristics can be nominated with similar methods only.
Originality/value: of this approach relies on the comparison of the compatibility of the characteristics of the mechatronic and mechanical systems with demanded accuracy, nominated with similar method.
Keywords: Applied mechanics; Approximate and exact methods; Continuous system; Vibrating shaft

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1. Introduction

The problems (other diverse problems have been modeled by different kind of graphs next they were examined and analyzed in e.g. Świder and Wszołek [15] of modelling, synthesis and analysis of a continuous bar system and selected class of discrete mechanical systems concerning the frequency spectrum has been made in the Gliwice research Centre in e.g.(Buchacz, Dymarek and Dzitkowski,) [1,2,6]. In the paper (Buchacz) [3] the continuous-discrete mechatronic system, were considered. To obtain the frequency -modal characteristics the approximate method of analysis called Galerkin's method has been used. In (Buchacz) [4] the exact method and Galerkin's method were used to comparison of obtaining dynamical characteristics – dynamical flexibilities only for mechanical subsystem as a part of complex mechatronic system (the problems concerned of piezoelectricity and electrostriction were presented for example in: (Callahan and Baruh, Friend and Stutts, Heimann et al., Heyliger and Ramirez, Ji-Huan He, Lu et al., Soluch, Song et al,) [5,7,8,9,10,12,14,16]. In this paper frequency analysis and frequency – modal analysis have been presented for the mechanical part of mechatronic system. In this aim three methods of analysis have been used - the exact method and two approximate methods.

2. Torsionally vibration shaft as the excitated continuous system

2.1. The dynamical flexibility of the shaft - solution with the exact method

The shaft with the constant cross section, clamped on left end and free on the right one is considered in Figure 1.

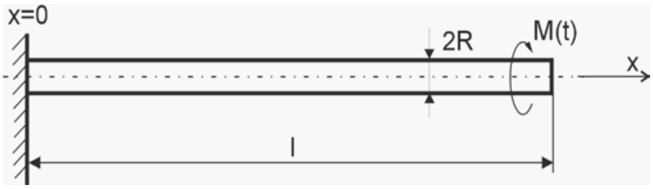

Fig. 1. Torsionally vibration shaft with excitation

The equation of motion of the shaft (Figure 1) takes form

$$c_\varphi \varphi_{,xx} = J\varphi_{,tt}, \tag{1}$$

where: $\varphi(x,t)$ - angle of torsion at the time moment t of the lining shaft section within the distance x from the beginning of the system, J the shaft inertia moment $c_\varphi = \frac{GI_0}{l}$ - torsional rigidity of the shaft, G - a transverse modulus ρ - mass density of material of the shaft, I_0 -polar inertia moment of the shaft cross section, l- length of the shaft.

The boundary conditions on the shaft ends are following

$$\begin{cases} \varphi = 0 \text{ when } x = 0, \\ c_\varphi \varphi_{,x} = M_0 \cos\omega t \text{ when } x = l. \end{cases} \tag{2}$$

Angle of torsion $\varphi(x,t)$ is the harmonic function because the excitation is harmonic one, that means

$$\varphi(x,t) = X(x)\cos\omega t. \tag{3}$$

Substituting expression (3) to (1) is obtained

$$c_\varphi X''(x) + J\omega^2 X(x) = 0. \tag{4}$$

Afterwards it searches the general solution of expression (4) in form

$$X(x) = A\cos\lambda x + B\sin\lambda x. \tag{5}$$

where: A and B are any real constants and

$$\lambda l = \omega\sqrt{\frac{I}{c_\varphi}}. \tag{6}$$

The solution (5) fulfills the boundary conditions (2) for:

$$A = 0,\ B = \frac{M_0}{c_\varphi \lambda \cos\lambda l}, \tag{7}$$

In the way the angle of torsion of the shaft takes form:

$$\varphi(x,t) = \frac{\sin\lambda x}{c_\varphi \lambda \cos\lambda l} M_0 \cos\omega t. \tag{8}$$

On the base of (8) the dynamical characteristic - dynamical flexibility Y_{xl} equals

$$Y_{xl} = \frac{\sin\lambda x}{c_\varphi \lambda \cos\lambda l}. \tag{9}$$

To enable the further analysis of the gained process, the absolute value of dynamical flexibility (9) is considered on the right end of the shaft; that means dependence (9), when $x=l$. will be given as:

$$|Y_{ll}| = \left|\frac{1}{c_\varphi \lambda}\operatorname{tg}\lambda l\right|. \tag{10}$$

The transient of expression (10) is shown in Figure 2. In the next figures the transient will be signed by different line and colour.

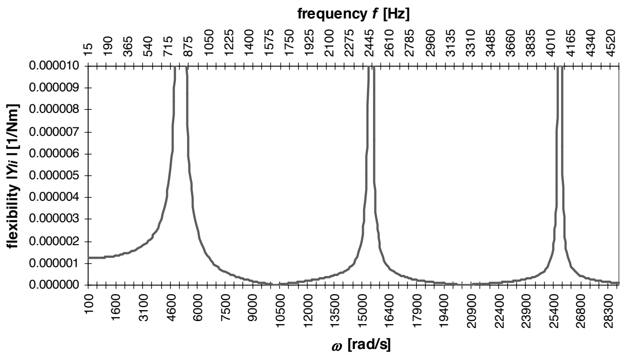

Fig. 2. The plot of dynamical flexibility of torsionally vibration continuous system

2.2. The dynamical flexibility of the considered system - solution with the approximate method

It has to be considered that if the shaft is under the action of moment with continuous factorization threw the shaft length with the value $M(x)\sin\omega t$ on the length unit – then the equation of motion of the element with length dx lining in the point x is:

$$J\frac{\partial^2\varphi}{\partial t^2}dx = c_\varphi \frac{\partial^2\varphi}{\partial x^2}dx + M(x)dx\sin\omega t \tag{11}$$

The translocation $\varphi(x,t)$ is a harmonic function, that it is considered to be:

$$\varphi(x,t) = X(x)\sin\omega t, \tag{12}$$

where $X(x)$ depends only on x.

Putting (12) to (11) the following equation is given:

$$-J\omega^2 X - c_\varphi \frac{\partial^2 X}{\partial x^2} = M(x). \tag{13}$$

Presuming that $M(x)$ can be showed as a series of infinite own functions $\varphi_n(x)$, which are defined with the dependence

$$\varphi_n(x) = \sin\frac{(2n-1)\pi x}{2l}, \tag{14}$$

that is:

$$M(x) = \sum_{n=1}^{\infty} P_n \varphi_n(x). \tag{15}$$

The equation (13) can be solved if:

$$X = \sum_{n=1}^{\infty} p_n \varphi_n(x). \tag{16}$$

Taking into consideration the fact that $\varphi_n(x)$ fulfills the equation of motion of the shaft, when $\omega = \omega_n$, then:

$$c_\varphi \frac{\partial^2 \varphi_n}{\partial x^2} + J\omega_n^2 \varphi_n = 0, \tag{17}$$

and

$$-c_\varphi \frac{\partial^2 X}{\partial x^2} = J \sum_{n=1}^{\infty} \omega_n{}^2 p_n \varphi_n(x). \tag{18}$$

Next putting the obtained expressions *X*, $\frac{d^2 X}{dt^2}$ and *M*(*x*) to (13) the equality is given:

$$J\left(\omega_n{}^2 - \omega^2\right) p_n = P_n, \tag{19}$$

therefore, if P_n factors of any excitation are known in (19), than the translocation of the shaft can be nominated.

Taking into consideration that it is acceptable to use series [5], the factors can be estimated as follows (16). Multiplying the dependence (15) with $\varphi_s(x)$ and integrating in the limits <0,*l*>, it is given:

$$\int_0^l M(x)\varphi_s(x)dx = \sum_{n=1}^{\infty} \int_0^l \varphi_n(x)\varphi_s(x)dx. \tag{20}$$

When now $n \neq s$, then:

$$\int_0^l \varphi_n(x)\varphi_s(x)dx = \int_0^l \sin\left[(2n-1)\frac{\pi}{2l}x\right]\sin\left[(2s-1)\frac{\pi}{2l}x\right]dx =$$

$$= \frac{1}{2}\int_0^l \left\{\cos\left[(n-s)\frac{\pi}{l}x\right] - \cos\left[(n+s-1)\frac{\pi}{l}x\right]\right\}dx = 0. \tag{21}$$

However if $n = s$, then

$$\int_0^l \varphi_n(x)\varphi_s(x)dx = \int_0^l \sin^2\left[(2s-1)\frac{\pi}{2l}x\right]dx =$$

$$= \frac{1}{2}\int_0^l \left\{1 - \cos\left[(2s-1)\frac{\pi}{l}x\right]\right\}dx = \frac{1}{2}. \tag{22}$$

after putting to the equation (20) and after transforming the factor can be appointed:

$$P_s = \frac{2}{l}\int_0^l M(x)\varphi_s(x)dx. \tag{23}$$

In order to (23) the factors P_s can be determined (in order to the general distortion factorization). To determine the dynamic flexibility Y_{xy} - the factors which are compatible to concentrate loading $M_0 \sin\omega t$ - which works in point *y* have to be found. The loading can be considered as a limit of concentrate loading threw the length- as follows:

$$M(x) = \begin{cases} \frac{M}{h} & \text{when } y-h \le x \le y, \\ 0 & \text{in other section,} \end{cases} \tag{24}$$

and therefore

$$P_s = \frac{2}{l}\lim_{h\to 0}\int_{y-h}^{y} \frac{M}{h}\varphi_s(x)dx = \frac{2}{l}M\varphi_s(y). \tag{25}$$

After putting the formula (25) into formula(19) equation (16) is given as:

$$X = \frac{2}{Jl}\sum_{n=1}^{\infty}\frac{\varphi_n(x)\varphi_n(y)}{\omega_n{}^2 - \omega^2}M_0. \tag{26}$$

On the base (26) the dynamic flexibility of the shaft is:

$$Y_{xy} = \frac{2}{Jl}\sum_{n=1}^{\infty}\frac{\varphi_n(x)\varphi_n(y)}{\omega_n{}^2 - \omega^2}. \tag{27}$$

In order to provide the flexibility results of equalization which has been determined by presented method with the dependence on the flexibility, which has been determined by exact method – still on the figures the module transients of these quantities were presented in form

$$\left|Y_{xy}\right| = \sum_{n=1}^{\infty}\left|Y_{xy}^{(n)}\right| = \left|\frac{2}{Jl}\sum_{n=1}^{\infty}\frac{\varphi_n(x)\varphi_n(y)}{\omega_n{}^2 - \omega^2}\right| =. \tag{28}$$

The dynamical flexibility for the first vibration mode at the end of the shaft, i.e. when *x*=*l* and *y*=*l* takes the following form

$$\left|Y_{ll}^{(1)}\right| = \left|\frac{2}{Jl}\frac{1}{\frac{G}{\rho}\left(\frac{\pi}{2l}\right)^2 - \omega^2}\right|. \tag{29}$$

The plot of expression (29) is shown in Figure3. In Figure 3 and in other figures the transients of flexibility for exact solution (10) are signed by thin line.

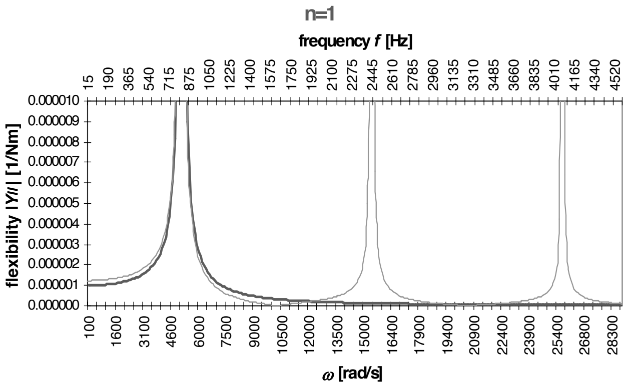

Fig. 3. The plot of dynamical characteristic for the first mode vibration

For the second vibration mode, i.e. when n=2, the dynamical flexibility (28) takes the form of

$$\left|Y_{ll}^{(2)}\right| = \left| \frac{2}{Jl} \frac{1}{\frac{G}{\rho}\left(\frac{3\pi}{2l}\right)^2 - \omega^2} \right|. \tag{30}$$

The plot of expression (30) is shown in Figure 4.

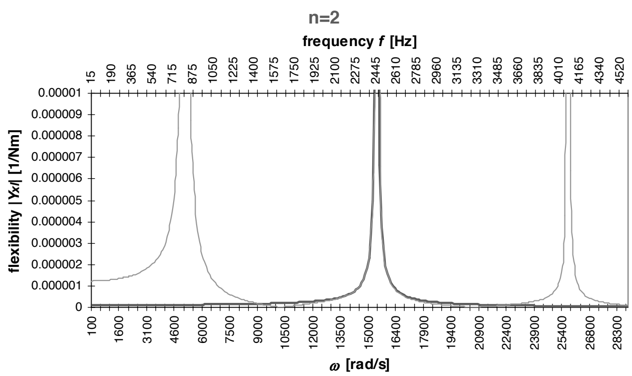

Fig. 4. The plot of dynamical characteristic for the second mode vibration

For the third vibration mode, i.e. when n=3, the characteristic (29) is given in shape

$$\left|Y_{ll}^{(3)}\right| = \left| \frac{2}{Jl} \frac{1}{\frac{G}{\rho}\left(\frac{5\pi}{2l}\right)^2 - \omega^2} \right|. \tag{31}$$

The plot of equation (31) is shown in Figure 5.

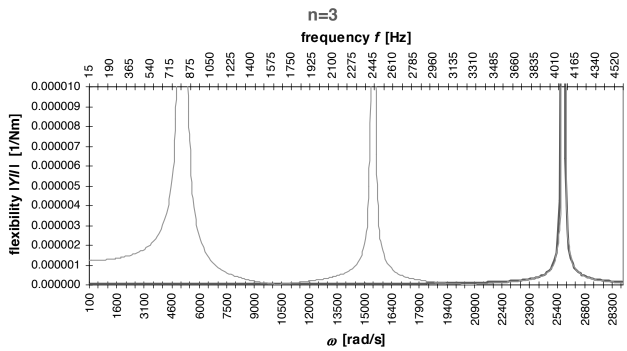

Fig. 5. The plot of dynamical characteristic for the third mode vibration

In global case the dynamical flexibility at the end of the shaft gets shape of

$$\left|Y_{xl}\right| = \left| \frac{2}{Jl} \sum_{n=1}^{\infty} \frac{\sin^2\left[(2n-1)\frac{\pi}{2}\right]}{\left[\sqrt{\frac{G}{\rho}}(2n-1)\frac{\pi}{2l}\right]^2 - \omega^2} \right|. \tag{32}$$

The plot of sum of n=1, 2, 3 for expression (32) is shown in Figure 6.

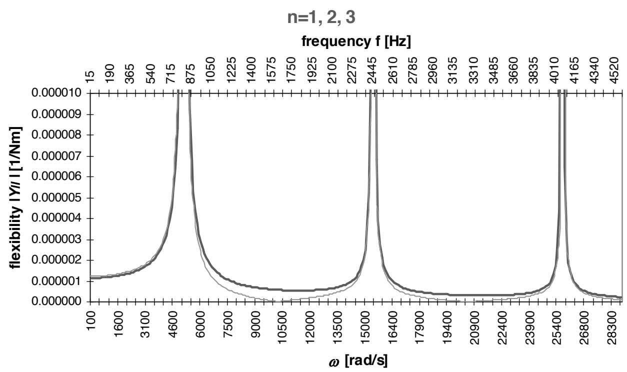

Fig. 6. The plot of the sum for n=1, 2, 3 mode vibration

2.3. The dynamical flexibility of the mechanical system - solution with the Galerkin's method

As it known [2-4] in Galerkin's method, the final solution is searched within the sum of own functions which will respond to the variables of the time and dislocation, which are strictly accepted and fulfill the boundary conditions, that means

$$\begin{cases} \varphi(0,t)=0,\ X(0)T(t)=0, \\ c_\varphi\, \varphi_{,x}\big|_{x=l}=M,\ c_\varphi X'(l)T(t)=M. \end{cases} \tag{33}$$

where: $M = M_0 \cos\omega t$.

The angle of torsion - the solution of (1) is given in shape of

$$\varphi(x,t)=\sum_{k=1}^{\infty}\varphi_k(x,t)=A_k\sum_{k=1}^{\infty}\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t \tag{34}$$

The solution of the differential equation (1) comes to fulfilling the appropriate derivatives of (34)

$$\begin{cases} \varphi_{k,t}=-A\omega^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\sin\omega t \\ \varphi_{k,tt}=-A\omega^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t, \\ \varphi_{k,x}=A_k\left[(2k-1)\frac{\pi}{2l}\right]\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t, \\ \varphi_{k,xx}=-A_k\left[(2k-1)\frac{\pi}{2l}\right]^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t. \end{cases} \tag{35}$$

Substituting the expression (34) to (1) is obtained

$$-c_\varphi A_k\left[(2k-1)\frac{\pi}{2l}\right]^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t+$$
$$+JA_k\omega^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\cos\omega t=M_0\cos\omega t. \tag{36}$$

After transformations, the amplitude value A_k of the vibrations takes form of

$$A_k=\frac{M_o}{J\omega^2-c_\varphi\left[(2n-1)\frac{\pi}{2l}\right]^2} \tag{37}$$

Using the equation (36) and putting it to (34) the dynamical flexibility equals

$$Y_{xl}=\sum_{k=1}^{\infty}Y_{xl}^{(k)}=\sum_{k=1}^{\infty}\frac{\sin\left[(2k-1)\frac{\pi}{2l}x\right]}{J\omega^2-c_\varphi\left[(2k-1)\frac{\pi}{2l}\right]^2} \tag{38}$$

For sum k=1,2,3 the plot of expression (37) is shown in Figure 7.

Comparing the plots in Figure 6 and 7 is shown that the poles of flexibilities have the same values.

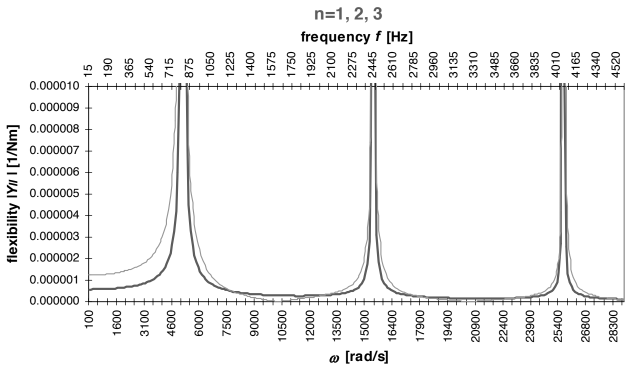

Fig. 7. The plot of expression (&&) for n=1, 2, 3 mode vibration

3. Last remark

On the base of the obtained formulas and plots, which were determined by the exact method and approximate methods, it is possible to make the analysis of the considered class vibrating mechanical and mechatronic systems using only approximate methods. In case of others of boundary conditions of mechanical or mechatronic systems and others kinds of their vibrations it is necessary to achieve offered researches in this paper. The problems shall be discussed in future research works.

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