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Dynamical flexibility of the free-free damped rod in transportation

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Analysis and modelling

ABSTRACT

Purpose: of this thesis is derivation of dynamical flexibility of the free-free rod system in transportation. The well-known problem of dynamical analysis of systems in rotational transportation was developed in this work to systems with taking into consideration damping forces.

Design/methodology/approach: The dynamical flexibility method was used to analysis of the free-free rod's vibrations. Mathematical models derived in previous articles were used to derivation of the dynamical flexibility. Considerations were done by the Galerkin's method.

Findings: There were considered systems in rotational motion treated in this thesis as main transportation. Dynamical characteristics in form of dynamical flexibility as function of frequency and mathematical models were presented in this work.

Research limitations/implications: Analyzed systems were simple linear homogeneous not supported rods. Working motion was limited to plane rotational motion. Future works would consider complex systems and nonlinearity.

Practical implications: of derived dynamical characteristics can easily support designing process and can be put to use in stability analysis and assigning stability zones. Thank to derived mathematical models the numerical applications can be implemented and some calculations can be automated.

Originality/value: Analyzing models are rotating flexible free-free rods with taking into consideration the damping forces.

Keywords: Numerical Techniques; Computational Mechanics; Applied mechanics; Transportation effect

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1. Introduction

One of the very popular method of analyzing dynamics of systems is the dynamical flexibility method. This method applies to analysis of dynamical states of technical systems and gives opportunity to derive stability zones and especially zones of minimal amplitude of vibrations, modes of vibrations and zeros of dynamical characteristics. Many publications e.g. [1-4, 13-14, 20] concern the subject area of vibrating systems in transportation and as distinguished from ones concerning stationary systems [5-12, 15-19] is the widening of the dynamical analysis at all.

An approximate method, the Galerkin's method was decided to use to search the solution of analyzed system. Dynamical flexibilities of analyzed systems derived using the Galerkin's method were compared with dynamical flexibility of stationary systems derived by the exact method. The results confirm the high effectiveness of the Galerkin's method. Characteristics derived by this method overlap with characteristics derived by exact method both in case of rotational systems and in case of systems with the zero-value angular velocity. Based on these there was accepted the Galerkin's method as a sufficient method for objectives of analysis of systems in motion.

2. Model of the analyzed system

There is considered model of the free-free vibratory rod. The rod is being rotated and rotations are treated in this work as transportation motion. The rod is loaded by an axial harmonic force that provides its longitudinal vibrations.

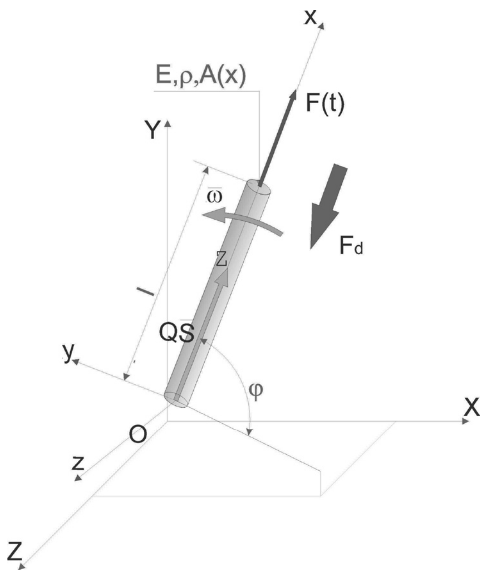

Fig. 1. Model of the analyzed system

In Figure 1 there is presented model of the analyzed system. A system was described in global reference fixed frame and local reference frame that is connected with rotation of the system.

2.1. Applications

The analyzed system can be put to use in analyzing of complex systems. A sample multibody system where the analyzed system (Fig. 1) can be put to use was presented in the Figure 2. It is well-known problem of longitudinal vibrations of rods and in this thesis the problem is extended to damped vibrations. The analyzed system is a part of the complex system and derived in this work dynamical flexibility can be used in algorithm of derivation of the complex system dynamical flexibility.

2.2. Description of the model

Model of the analyzed rod was described by symbols:

$Y(\Omega)$ – the dynamical flexibility in function of frequency of extorted force,
n – mode of vibrations of rod,
a – velocity of the wave propagation in the rod,

$$a = \sqrt{\frac{E}{\rho}}, \tag{1}$$

Ω – frequency of vibrations,
E – the Young modulus,
ρ – mass density of the rod,
ω – angular velocity of the rod,
φ – rotation angle,
x – the position of analyzed section.
A – the cross-section of rod,
l – length of the rod,
t – time,

a vector of linear displacement of the rod's section along center line of the bar in the local reference system:

$$\bar{\mathbf{u}} = \begin{bmatrix} u & 0 & 0 \end{bmatrix}^T, \tag{2}$$

a vector of linear displacement of the rod's in the global reference system:

$$\bar{\mathbf{u}} = \begin{bmatrix} u_X & u_Y & 0 \end{bmatrix}^T, \tag{3}$$

a position vector:

$$\bar{\mathbf{S}} = \begin{bmatrix} s & 0 & 0 \end{bmatrix}^T. \tag{4}$$

2.3. Equations of motion

In this section the equations of motion of analyzed system was presented.

The projection into the X axis of the global reference system:

$$\begin{aligned} &\frac{\partial^2 u_X}{\partial t^2} - \frac{E}{\rho}\cdot\frac{\partial^2 u_X}{\partial x^2} = \omega^2\cdot\left(s\cdot\cos\varphi + u_X\right) + \\ &+2\cdot\omega\cdot\frac{\partial u_Y}{\partial t} - \frac{b}{M}\cdot\frac{\partial u_X}{\partial t} + \frac{b}{M}\cdot\omega\cdot\left(s\cdot\sin\varphi + u_Y\right) \end{aligned} \tag{5}$$

The projection into the Y axis of the global reference system:

$$\begin{aligned} &\frac{\partial^2 u_Y}{\partial t^2} - \frac{E}{\rho}\cdot\frac{\partial^2 u_Y}{\partial x^2} = \omega^2\cdot\left(s\cdot\sin\varphi + u_Y\right) + \\ &-2\cdot\omega\cdot\frac{\partial u_X}{\partial t} - \frac{b}{M}\cdot\frac{\partial u_Y}{\partial t} - \frac{b}{M}\cdot\omega\cdot\left(s\cdot\cos\varphi + u_X\right) \end{aligned} \tag{6}$$

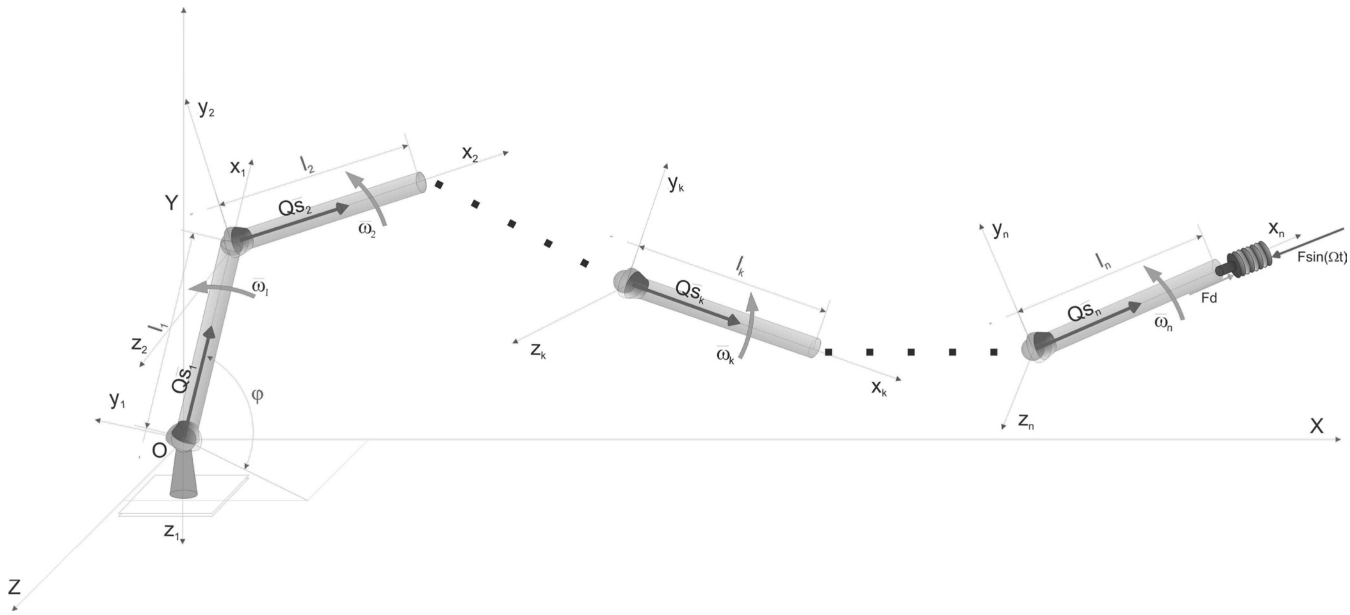

Fig. 2. The sample application of analyzed system

2.4. Boundary conditions

Boundary conditions of the rod were assumed as follow:

$$\begin{cases} E\cdot A\cdot\dfrac{\partial u(0,t)}{\partial x}=0, \\ E\cdot A\cdot\dfrac{\partial u(l,t)}{\partial x}=2\cdot\displaystyle\int_0^l F_0\cdot\delta(x-l)\cdot e^{j\Omega t}dx=1\cdot e^{j\Omega t}, \end{cases} \tag{7}$$

where the Dirac delta function was assumed as a distribution generalized function regarding to a point of application of force. Force was assumed as harmonic one with unitary amplitude up to the dynamical flexibility definition.

2.5. Eigenfunctions

The boundary problem was solved and the eigenfunction of displacement and eigenfunction of time variable were derived. The eigenfunction of displacement can be write:

$$X(x)=\cos(kx), \tag{8}$$

where:

$$k=\frac{n\cdot\pi}{l},\ n=0,1,2,3,... \tag{9}$$

In the Figure 3 there were presented six forms of vibrations of the free-free rod.

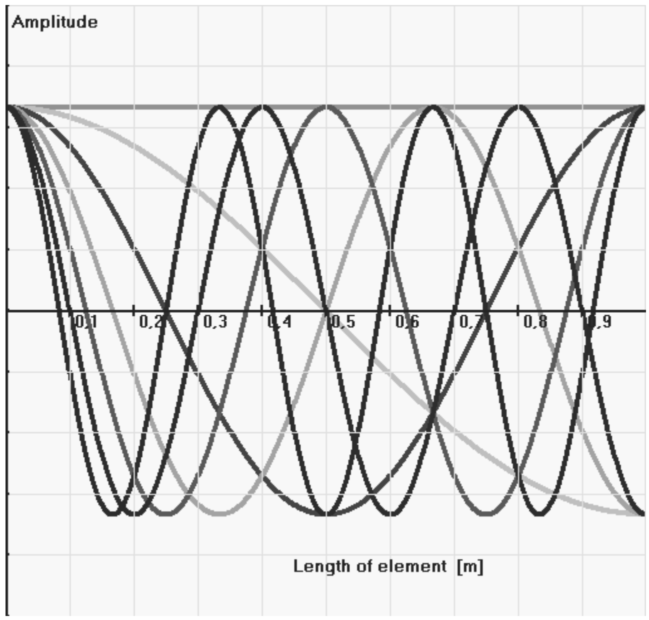

Fig. 3. Forms of vibrations of the free-free rod

3. Dynamical flexibility

In this section the dynamical flexibility of free-free rod in transportation was derived.

3.1. Searched displacement solution

Based on the eigenfuntions and up to the Galerkin's method the solution was assumed as:

For solution regard to X axis:

$$u_X=\sum_{n=0}^{\infty} A_X \cdot \cos(kx) \cdot e^{j\Omega t}, \tag{10}$$

and for displacement projected onto Y axis of the global reference frame:

$$u_Y=\sum_{n=0}^{\infty} A_X \cdot \cos(kx) \cdot e^{j\Omega t}, \tag{11}$$

so the displacement is searched as a sum of eigenfunctions of displacement and time.

3.2. Orthogonalisation

After orthogonalisation of equations of motion we can obtain equations:

$$\begin{aligned}
&\int_0^l \frac{\partial^2 u_X}{\partial t^2}\cdot\cos(kx)dx-\frac{E}{\rho}\cdot\int_0^l \frac{\partial^2 u_X}{\partial x^2}\cdot\cos(kx)dx=\\
&=\omega^2\cdot\int_0^l\left(s\cdot\cos\varphi+u_X\right)\cdot\cos(kx)dx+\\
&+2\cdot\omega\cdot\int_0^l \frac{\partial u_Y}{\partial t}\cdot\cos(kx)dx+\\
&-\frac{b}{M}\cdot\int_0^l \frac{\partial u_X}{\partial t}\cdot\cos(kx)dx+\\
&+\frac{b}{M}\cdot\omega\cdot\int_0^l\left(s\cdot\sin\varphi+u_Y\right)\cdot\cos(kx)dx
\end{aligned} \tag{12}$$

and for second axis:

$$\begin{aligned}
&\int_0^l \frac{\partial^2 u_Y}{\partial t^2}\cdot\cos(kx)dx-\frac{E}{\rho}\cdot\int_0^l \frac{\partial^2 u_Y}{\partial x^2}\cdot\cos(kx)dx=\\
&=\omega^2\int_0^l\left(s\cdot\sin\varphi+u_Y\right)\cdot\cos(kx)dx+\\
&-2\cdot\omega\cdot\int_0^l \frac{\partial u_X}{\partial t}\cdot\cos(kx)dx+\\
&-\frac{b}{M}\cdot\int_0^l \frac{\partial u_Y}{\partial t}\cdot\cos(kx)dx+\\
&-\frac{b}{M}\cdot\omega\cdot\int_0^l\left(s\cdot\cos\varphi+u_X\right)\cdot\cos(kx)dx
\end{aligned} \tag{13}$$

After integration by parts (12, 13) we can obtain:

$$\begin{aligned}
&-\frac{E}{\rho}\cdot\left[X(x)\cdot\frac{\partial u_X}{\partial x}-X'(x)\cdot u_X\right]_0^l+\\
&+\frac{E}{\rho}\cdot k^2\cdot\int_0^l u_X\cdot\cos(kx)dx+\int_0^l \frac{\partial^2 u_X}{\partial t^2}\cdot\cos(kx)dx=\\
&=\omega^2\cdot\int_0^l\left(s\cdot\cos\varphi+u_X\right)\cdot\cos(kx)dx+\\
&+2\cdot\omega\cdot\int_0^l \frac{\partial u_Y}{\partial t}\cdot\cos(kx)dx-\frac{b}{M}\cdot\int_0^l \frac{\partial u_X}{\partial t}\cdot\cos(kx)dx+\\
&+\frac{b}{M}\cdot\omega\cdot\int_0^l\left(s\cdot\sin\varphi+u_Y\right)\cdot\cos(kx)dx
\end{aligned} \tag{14}$$

$$\begin{aligned}
&-\frac{E}{\rho}\cdot\left[X(x)\cdot\frac{\partial u_Y}{\partial x}-X'(x)\cdot u_Y\right]_0^l+\\
&+\frac{E}{\rho}\cdot k^2\cdot\int_0^l u_Y\cdot\cos(kx)dx+\\
&+\int_0^l \frac{\partial^2 u_Y}{\partial t^2}\cdot\cos(kx)dx=\\
&=\omega^2\int_0^l\left(s\cdot\sin\varphi+u_Y\right)\cdot\cos(kx)dx+\\
&-2\cdot\omega\cdot\int_0^l \frac{\partial u_X}{\partial t}\cdot\cos(kx)dx-\frac{b}{M}\cdot\int_0^l \frac{\partial u_Y}{\partial t}\cdot\cos(kx)dx+\\
&-\frac{b}{M}\cdot\omega\cdot\int_0^l\left(s\cdot\cos\varphi+u_X\right)\cdot\cos(kx)dx
\end{aligned} \tag{15}$$

3.3. Displacements

Substitution of boundary conditions (7) into projected equations and after taking into consideration solutions (10, 11) gives the terms:

$$\begin{aligned}
&X(0)=1,\ \ X(l)=\cos(kl),\ \ X'(0)=0,\\
&X'(l)=-k\cdot\sin(kl)=0,\ E\cdot A\cdot\frac{\partial u_X(0,t)}{\partial x}=0,\\
&E\cdot A\cdot\frac{\partial u_Y(0,t)}{\partial x}=0,\ \ E\cdot A\cdot\frac{\partial u_Y(l,t)}{\partial x}=0,\\
&E\cdot A\cdot\frac{\partial u_X(l,t)}{\partial x}=2\cdot\int_0^l F_0\cdot\delta(x-l)\cdot e^{j\Omega t}dx,
\end{aligned} \tag{16}$$

after calculations:

$$
\begin{aligned}
&-\Omega^2\cdot A_X\cdot e^{j\Omega t}+a^2\cdot k^2\cdot A_X\cdot e^{j\Omega t}+\\
&-\frac{2\cdot\cos(kl)}{\rho\cdot A\cdot\gamma_n^2}\cdot\int_0^l F_0\cdot e^{j\Omega t}\cdot\delta(x-l)\cdot dx+\\
&+j\cdot\frac{b}{M}\cdot\Omega\cdot A_X\cdot e^{j\Omega t}=\\
&=\frac{1}{\gamma_n^2}\cdot\omega^2\cdot\int_0^l s\cdot\cos\varphi\cdot\cos(kx)dx+\omega^2\cdot A_X\cdot e^{j\Omega t}+\\
&+2\cdot j\cdot\omega\cdot\Omega\cdot A_Y\cdot e^{j\Omega t}+\\
&+\frac{1}{\gamma_n^2}\cdot\frac{b}{M}\cdot\omega\cdot\int_0^l s\cdot\sin\varphi\cdot\cos(kx)dx+\frac{b}{M}\cdot\omega\cdot A_Y\cdot e^{j\Omega t}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&-\Omega^2\cdot A_Y\cdot e^{j\Omega t}+a^2\cdot k^2\cdot A_Y\cdot e^{j\Omega t}+\\
&+j\cdot\frac{b}{M}\cdot\Omega\cdot A_Y\cdot e^{j\Omega t}=\omega^2\cdot A_Y\cdot e^{j\Omega t}\\
&+\frac{1}{\gamma_n^2}\cdot\omega^2\cdot\int_0^l s\cdot\sin\varphi\cdot\cos(kx)dx+\\
&-2\cdot j\cdot\omega\cdot\Omega\cdot A_X\cdot e^{j\Omega t}-\frac{b}{M}\cdot\omega\cdot A_X\cdot e^{j\Omega t}+\\
&-\frac{1}{\gamma_n^2}\cdot\frac{b}{M}\cdot\omega\cdot\int_0^l s\cdot\cos\varphi\cdot\cos(kx)dx
\end{aligned}
\tag{18}
$$

where the norm is equal:

$$\gamma_n^2=\int_0^l\cos^2(kx)dx=\frac{2\cdot k\cdot l+\sin(2kl)}{4\cdot k}=\frac{l}{2} \tag{19}$$

Further by writing (17 and 18) in matrix form and omitting s:

$$\begin{bmatrix}\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right) & \left(-2\cdot j\cdot\omega\cdot\Omega-\frac{b}{M}\cdot\omega\right)\\ \left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right) & \left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)\end{bmatrix}\cdot\begin{bmatrix}A_X\\A_Y\end{bmatrix}=\begin{bmatrix}\frac{2\cdot\cos(kl)\cdot F_0}{\rho\cdot A\cdot l}\\0\end{bmatrix} \tag{20}$$

Solving (20) regard to A factors we can write determinants:

$$W=\begin{vmatrix}\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right) & \left(-2\cdot j\cdot\omega\cdot\Omega-\frac{b}{M}\cdot\omega\right)\\ \left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right) & \left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)\end{vmatrix} \tag{21}$$

Determinant for A_X factor:

$$W_{A_X}=\begin{vmatrix}\left(\frac{2\cdot\cos(kl)\cdot F_0}{\rho\cdot A\cdot l}\right) & \left(-2\cdot j\cdot\omega\cdot\Omega-\frac{b}{M}\cdot\omega\right)\\ 0 & \left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)\end{vmatrix} \tag{22}$$

Determinant for A_Y factor:

$$W_{A_Y}=\begin{vmatrix}\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right) & \left(\frac{2\cdot\cos(kl)\cdot F_0}{\rho\cdot A\cdot l}\right)\\ \left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right) & 0\end{vmatrix} \tag{23}$$

where:

$$A_X=\frac{W_{A_X}}{W},\ A_Y=\frac{W_{A_Y}}{W} \tag{24}$$

We can now obtain factors:

$$A_X=\frac{2\cdot\cos(kl)\cdot F_0\cdot\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)}{\left[\rho\cdot A\cdot l\cdot\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)^2+\left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right)^2\right]} \tag{25}$$

and

$$A_Y=\frac{2\cdot\cos(kl)\cdot F_0\cdot\left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right)}{\left[\rho\cdot A\cdot l\cdot\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)^2+\left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right)^2\right]} \tag{26}$$

After substituting into searched solutions (10, 11) we can calculate the displacements:

$$u_X=\sum_{n=0}^{\infty}\frac{2\cdot\cos(kl)\cdot F_0\cdot\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)\cdot e^{j\cdot\Omega\cdot t}}{\rho\cdot A\cdot l\cdot\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)^2+\left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right)^2} \tag{27}$$

and

$$u_Y=\sum_{n=0}^{\infty}\frac{2\cdot\cos(kl)\cdot F_0\cdot\left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right)\cdot e^{j\cdot\Omega\cdot t}}{\rho\cdot A\cdot l\cdot\left(a^2\cdot k^2-\omega^2-\Omega^2+j\cdot\frac{b}{M}\cdot\Omega\right)^2+\left(2\cdot j\cdot\omega\cdot\Omega+\frac{b}{M}\cdot\omega\right)^2} \tag{28}$$

$$|Y| = \sum_{n=0}^{\infty} \left\{ \left(\cdot \frac{2 \cdot \cos(kl) \cos(kx)}{\rho \cdot A \cdot l \cdot \left[\left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right)^2 - \left(\frac{b}{M} \cdot \Omega \right)^2 + \left(\frac{b}{M} \cdot \omega \right)^2 - 4 \cdot \omega^2 \Omega^2 \right]^2 + \left[2 \cdot \frac{b}{M} \cdot \left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right) \cdot \Omega + 4 \cdot \frac{b}{M} \cdot \omega^2 \Omega \right]^2} \right)^2 \cdot \right.$$

$$\cdot \left\langle \left[\left[\left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right)^2 - \left(\frac{b}{M} \cdot \Omega \right)^2 + \left(\frac{b}{M} \cdot \omega \right)^2 - 4 \cdot \omega^2 \Omega^2 \right] \cdot \left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right) + \right. \right.$$

$$\left. + \left[2 \cdot \frac{b}{M} \cdot \left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right) \cdot \Omega + 4 \cdot \frac{b}{M} \cdot \omega^2 \Omega \right] \cdot \frac{b}{M} \cdot \Omega \right]^2 + \left[\left[\left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right)^2 - \left(\frac{b}{M} \cdot \Omega \right)^2 + \left(\frac{b}{M} \cdot \omega \right)^2 - 4 \cdot \omega^2 \Omega^2 \right] \cdot \frac{b}{M} \cdot \Omega + \right.$$

$$\left. \left. \left. - \left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right) \cdot \left[2 \cdot \frac{b}{M} \cdot \left(a^2 \cdot k^2 - \omega^2 - \Omega^2 \right) \cdot \Omega + 4 \cdot \frac{b}{M} \cdot \omega^2 \Omega \right] \right]^2 \right\rangle \right\}^{\frac{1}{2}} \tag{29}$$

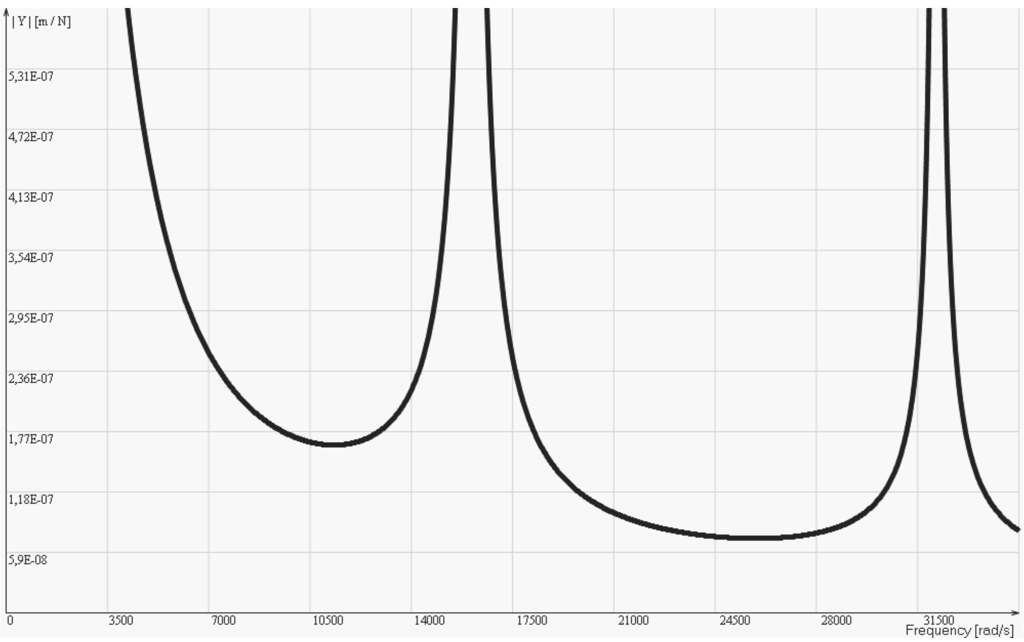

Fig. 4. Three modes of the free-free stationary rod (zero angular velocity)

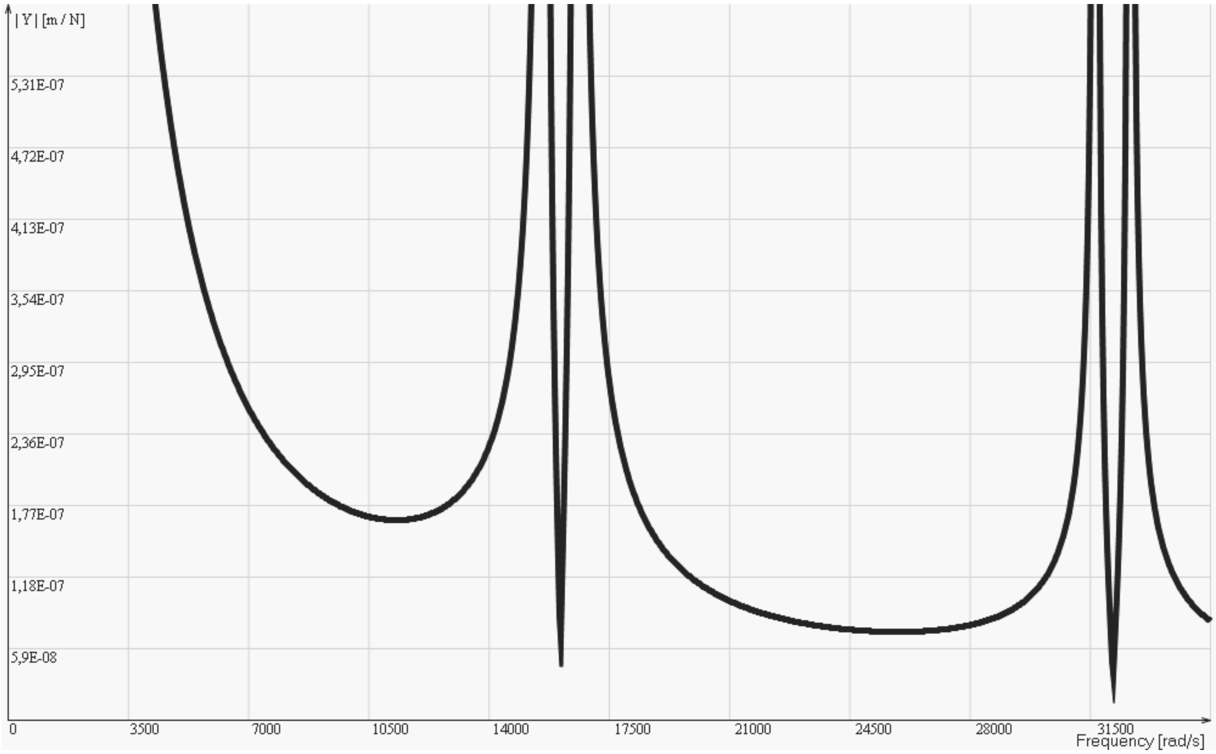

Fig. 5. Absolute of dynamical flexibility of the free-free rod rotating with angular velocity equals 500 rad/s without damping

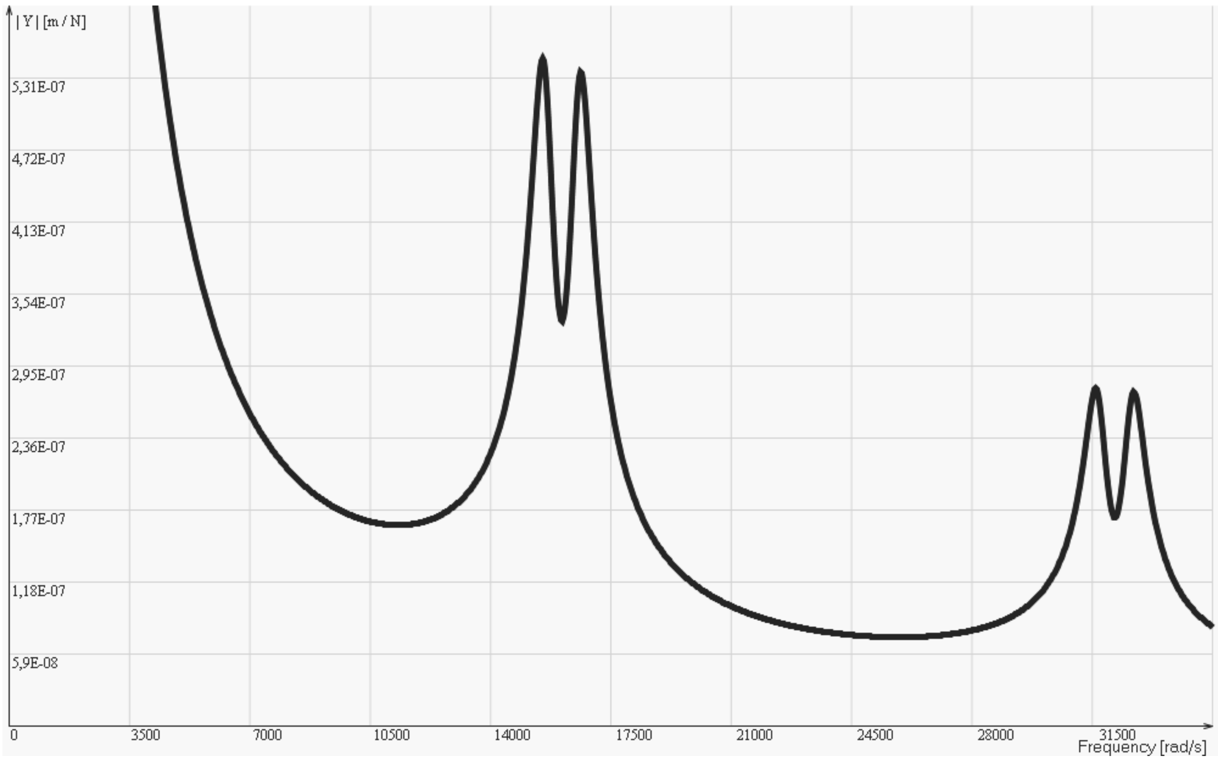

Fig. 6. Absolute of dynamical flexibility of the free-free rod rotating with angular velocity equals 500 rad/s with large damping

3.4. Dynamical flexibility mathematical form

Up to the definition there was derived the mathematical form of dynamical flexibility of the free-free rod in transportation with taking into consideration damping forces. In Figure 4 there is presented the dynamical flexibility of stationary system without damping, in Figure 5 there is presented the dynamical flexibility of rotating free-free rod without damping and in Figure 6 the dynamical flexibility of rotating rod with large damping.

4. Conclusions

Dynamical flexibility of the free-free rod in transportation was derived in this thesis. In the mathematical model damping forces were took into consideration and transportation effect was expressed. As a starting point of dynamical flexibility derivation algorithm was assumed the mathematical model in form of equations of motion. Considerations were done by the Galerkin's method. There were considered systems in rotational motion treated in this thesis as main transportation. Motion was limited to plane motion.

Dynamical characteristics in form of dynamical flexibility as function of frequency and mathematical models were presented in this work. Simple linear homogeneous not supported rods were analyzed in this thesis. Derived dynamical characteristics can easily support designing process and can be put to use in stability analysis and assigning stability zones.

Dynamical characteristics were generated by numerical application Modyfit (Modelling of dynamical flexible systems in transportation). Transportation effect relies on moving the zeros and modes of dynamical flexibility together with increasing of angular velocity of rotational systems. For suppression of longitudinal vibrations of the rod there are needful large damping forces.

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