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Crack arrest saturation model under combined electrical and mechanical loadings

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Analysis and modelling

ABSTRACT

Purpose: The investigation aims at proposing a model for cracked piezoelectric strip which is capable to arrest the crack.
Design/methodology/approach: Under the combined effect of electrical and mechanical loadings applied at the edges of the strip, the developed saturation zone is produced at each tip of the crack. To arrest further opening of the crack, the rims of the developed saturation zones are subjected to in-plane cohesive, normal uniform constant saturation point electrical displacement. The problem is solved using Fourier integral transform method which reduces the problem to the solution of Fredholm integral equation of the second kind. This integral equation in turn is solved numerically.
Findings: The expressions are derived for different intensity factors and energy release rate. A qualitative analysis of the parameters affecting the arrest of opening of the crack and fatigue crack growth with respect to strip thickness and material constants are presented graphically.
Research limitations/implications: The investigations are carried out by considering the material electrical brittle. Consequently, the zones protrude along the straight lines ahead of the crack tips. And further, the small scale electrical yielding conditions are used.
Practical implications: Piezoelectric materials are widely getting used nowadays, even in day to day life like piezoelectric cigarette lighter, children toys etc. And, its advance used in technology like transducers, actuators has been already in progress. So, the aspect of cracking of piezoelectric materials are of great practical importance.
Originality/value: The piezoelectric material under the combined effect of electrical and mechanical loadings gives the assessment of electrical displacement which is required to arrest the crack. The various useful interpretations are also drawn from the graphs.
Keywords: Applied Mechanics; Energy release rate; Piezoelectric ceramics; Crack arrest model

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1. Introduction

The study of stress and electric displacement near a Griffith's type mode-III crack tips in piezoelectric media is carried out by Li [1]. Lynch [2] investigated the electric field induced cracking in ferroelectric ceramic. Gao and Barnett [3] established the local energy release rate for a piezoelectric crack. Gao [4] developed a strip saturation model for a finite crack perpendicular or parallel to the poling axis of an infinite poled piezoelectric ceramic medium. Using elastoplastic fracture mechanics approach for crack growth simulation in the presence of residual stress fields is presented [5] using a boundary element method. A numerical analysis was conducted [6] to establish basic toughening mechanism. A crack arrest model [7] proposed for a poled piezoelectric plate weakened by a finite hairline straight crack. The paper [8] described the principle of the process of micro-grinding using coated piezoelectric materials. A modified strip yield model is proposed [9]. The Acoustic Emission (AE) method was employed to study [10] the processes of surface crack initiation and evaluation in surface protective coatings. A finite element formulation [11] was developed for modelling the dynamic and static response of laminated plates. The saturation strip model for piezoelectric crack is re-examined by Li [12] in a permeable environment to analyse fracture toughness of a piezoelectric ceramic [13] analyzed the problem of a crack in a ferroelectric ceramic with perfect saturation under electric loading. A strip saturation model was employed by Beom et al. [14] to investigate the effect of the electrical polarization saturation on electric fields and elastic field for a cracked electrostrictive material under purely electrical loading.

2. Fundamental formulation

As it is well-known for a two-dimensional out-of-plane displacement u_i and in-plane electrical field E_i $(i=x,y,z)$ may be defined as $u_x = u_y = 0,\ u_z = w(x,y)$ and $E_x = E_x(x,y),\ E_y = E_y(x,y),\ E_z = 0$.

Constitutive equations for orthotropic piezoceramic poled in the z-direction may be written as

$$\sigma_{xz} = c_{44} w_{,x} + e_{15}\phi_{,x} \tag{1}$$

$$\sigma_{yz} = c_{44} w_{,y} + e_{15}\phi_{,y} \tag{2}$$

$$D_x = e_{15} w_{,x} - \varepsilon_{11}\phi_{,x} \tag{3}$$

$$D_y = e_{15} w_{,y} - \varepsilon_{11}\phi_{,y} \tag{4}$$

$$E_x = -\phi_{,x}\ ,\ E_y = -\phi_{,y} \tag{5}$$

Governing equation has the form of:

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = 0, \tag{6}$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0\ , \tag{7}$$

where ϕ is the electric potential, c_{44} is the elastic stiffness constant, ε_{11} is the dielectric constant and e_{15} is the piezoelectric constant. Comma implies the partial differentiation with respect to argument following it. ∇^2 denotes the two-dimensional Laplacian operator. Equations (6 and 7) are solved for the functions $w(x,y)$ and $\phi(x,y)$ using Fourier integral transform method and taking inverse; these may be written as

$$w(x,y) = \frac{2}{\pi}\int_0^\infty [A_1(\alpha)\cosh(\alpha y) + B_1(\alpha)\sinh(\alpha y)] \cos(\alpha x)\,d\alpha + a_0^i y, \tag{8}$$

$$\phi(x,y) = \frac{2}{\pi}\int_0^\infty [B_1(\alpha)\cosh(\alpha y) + B_2(\alpha)\sinh(\alpha y)] \cos(\alpha x)\,d\alpha - b_0^i y, \tag{9}$$

The arbitrary constants $a_0^i,\ b_0^i$ and arbitrary functions $A_i(\alpha), B_i(\alpha)\ \{i=1,2\}$ are determined from the boundary conditions of the problem.

3. Statement of the problem

A plane-strain problem is investigated for a narrow piezoelectric ceramic strip. The strip occupies the region $-\infty < x < \infty$ and $-h \le y \le h$ in xoy-plane and is thick enough in z-direction to allow for the anti-plane shear state. The strip is cut along a finite hairline straight crack lying on ox-axis and occupies the segment $-a \le x \le a$. The edges of the strip are subjected to uniform shear stress τ_0 and uniform electrical displacement D_0. Consequently, the rims of the crack yield electrically forming of the saturation zones S_1 and S_2 at the tips $-a$ and a of the crack respectively. These saturation zones S_1 and S_2 occupy the interval $-b \le x \le -a$ and $a \le x \le b;\ y = 0,$ respectively. To arrest the crack from further opening the rims of the saturation zones are subjected to a normal cohesive uniform saturation point at electrical displacement $D_y = D_S$. The entire configuration is depicted in Fig. 1.

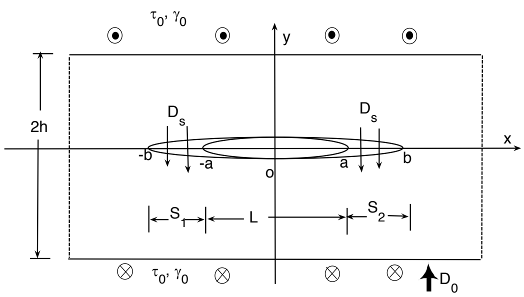

Fig. 1. Configuration of the problem

4. Mathematical model

A narrow piezoelectric strip of thickness for $2h$ occupies the region $-\infty < x < \infty$ and $-h \le y \le h$ in xoy-plane. It is cut along a hairline crack occupying the segment $-b \le x \le b;\ y = 0$. The boundary conditions of the problem are translated as:

(i) $\sigma_{yz}(x,0)=0$ for $0 \le x < a$

(ii) $w(x,0)=0$ for $a \le x < \infty$

(iii) $E_x(x,0)=E_x^V(x,0)$ for $0 \le x < a$

(iv) $\phi(x,0)=0$ for $a \le x < \infty$

(v) $D_y(x,0)=D_y^V(x,0)$ for $0 \le x < a$

(vi) $D_y(x,0)=D_s\,H(x-a)$ for $0 \le x < b$

(vii) Case I: $\sigma_{yz}(x,h)=\tau_0$, $D_y(x,h)=D_0$

(viii) Case II: $\gamma_{yz}(x,h)=\gamma_0$, $D_y(x,h)=D_0$

where superscript V denotes that the quantities refer to void inside the crack. $\mathrm{H}(\ )$ denotes Heaviside function and γ_0 is the uniform shear strain applied on the edge of the boundary.
The desired functions $w(x,y)$ and $\phi(x,y)$ for the problem are obtained by satisfying the boundary conditions (i to ix), obtained in equations (8 and 9).

5. Solution for Case I and Case II

Using values of $w(x,y)$ and $\phi(x,y)$ from equation (8 and 9) and edge boundary conditions (viii) one obtains $a_0^i, b_0^i\ (i = I, II)$. Superscript i denotes that the quantities refer to Case I or Case II.

Remaining boundary conditions (i to vi) yield following set of dual integral equation to determine $A_i(\alpha)$ and $B_i(\alpha)$

$$\int_0^\infty \alpha\, B_1(\alpha)\sin(\alpha x)d\alpha = 0 \qquad \text{for } 0 \le x < a \tag{10}$$

$$\int_0^\infty B_1(\alpha)\cos(\alpha x)d\alpha = 0 \qquad \text{for } a \le x < \infty \tag{11}$$

$$\int_0^\infty \alpha\, A_1(\alpha)\tanh(\alpha h)\cos(\alpha x)d\alpha = \left(d_0 - D_s\,H(x-a)\right)\frac{\pi}{2e_{15}} \qquad \text{for } 0 \le x < b \tag{12}$$

$$\int_0^\infty A_1(\alpha)\cos(\alpha x)d\alpha = 0 \qquad \text{for } b \le x < \infty \tag{13}$$

where

$$d_0 = e_{15}\,a_0^i + \varepsilon_{11}\,b_0^i\ . \tag{14}$$

For convenience of the function $\Psi_1(\xi)$ and $\Psi_2(\xi)$ are introduced as

$$\mathrm{A}_1(\alpha) = \frac{\pi b^2}{2}\int_0^1 \sqrt{\xi}\ \Psi_1(\xi)J_0(b\alpha\xi)d\xi \tag{15}$$

$$B_1(\alpha) = \frac{\pi a^2}{2}\int_0^1 \sqrt{\xi}\ \Psi_2(\xi)J_0(a\alpha\xi)d\xi \tag{16}$$

where $J_0(\ )$ is Bessel's function of first kind and zero order.

Equations (10, 11 and 16) lead to

$$B_1(\alpha)=0 \tag{17}$$

Edge boundary conditions together with equations (15 and 16) give

$$A_2(\alpha)=-\tanh(\alpha h)A_1(\alpha) \text{ and } B_2(\alpha)=0 \tag{18}$$

To determine $A_1(\alpha)$, its value from equation (15) is substituted in equations (12-13) which yield to a Fredholm integral equation of second kind to determine $\Psi_1(\xi)$

$$\Psi_1(\xi) + \int_0^1 K(\xi,\eta)\Psi_1(\eta)\,d\eta = \begin{cases} \dfrac{d_0\sqrt{\xi}}{e_{15}}, & \xi < \dfrac{b}{a} \\[2ex] \dfrac{d_0\sqrt{\xi}}{e_{15}} - \dfrac{2}{\pi}\dfrac{D_s\sqrt{\xi}}{e_{15}}\left(\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{b}{a\xi}\right)\right)^{1/2}, & \dfrac{b}{a} < \xi < 1 \end{cases} \tag{19}$$

where

$$K(\xi,\eta) = \sqrt{\xi\eta}\int_0^\infty \alpha\left[\tanh\left(\frac{\alpha h}{a}\right) - 1\right]J_0(\alpha\xi)J_0(\alpha\eta)d\alpha \tag{20}$$

6. Applications

The electric displacement intensity factor at the tip $x=b$ is obtained using the definition

$$K_I^D = \lim_{x\to b^+}\left[\sqrt{2\pi(x-b)}\,D_y(x,0)\right] = e_{15}\sqrt{\pi b}\ \Psi_1(1)$$

Analogously, the electric field intensity factor K_I^E at the tip $x=b$ is obtained using the definition

$$K_I^E = \lim_{x\to b^+}\left[\sqrt{2\pi(x-b)}\,E_y(x,0)\right] = 0$$

Sliding mode stress intensity factor at the tip $x=b$ is obtained as

$$K_{III}^S = \lim_{x\to b^+}\left[\sqrt{2\pi(x-b)}\,\sigma_{yz}(x,0)\right] = c_{44}\sqrt{\pi b}\ \Psi_1(1)$$

Analogously, sliding mode strain intensity factor at the tip $x=b$ is obtained as

$$K_{III}^\gamma = \lim_{x\to b^+}\left[\sqrt{2\pi(x-b)}\ \gamma_{zy}(x,0)\right] = \sqrt{\pi b}\ \Psi_1(1)$$

Energy release rate for the anti-plane case is obtained as

$$J = \frac{1}{2}(K_{III}^S K_{III}^\gamma - K_I^D K_I^E) = \frac{\pi b}{2} c_{44} \Psi_1^2(1)$$

The energy release rate for a cracked piezoelectric infinite plate is obtained by taking limit $h \to \infty$ one obtains

$$J_\infty = \frac{\pi b}{2} \frac{c_{44}}{e_{15}^2} \left[D_0 - \frac{2}{\pi} D_s \cos^{-1}\left(\frac{a}{b}\right)\right]^2$$

The length of the saturation zone $|(b-a)|$ is obtained from the fact that the energy release rate becomes zero at the tip $x = b$ which yields following equation to determine b

$$\frac{a}{b} = \cos\left[\frac{\pi}{2} \frac{D_0 - e_{15} U(h/b)}{D_s}\right],$$

where $U(h/b) = \int_0^1 K(\xi,\eta)\Psi_1(\eta)\, d\eta$

7. Case study

The various material constants have been taken from [15]. Energy release rate is plotted against strip width to crack length ratio in Fig. 2. It is observed that the energy release rate decreases as the strip width is increased. It is independent on the ceramic of the strip. As the ratio D_0/D_S is increased the energy release rate is reduced, thus, the crack opening is arrested.

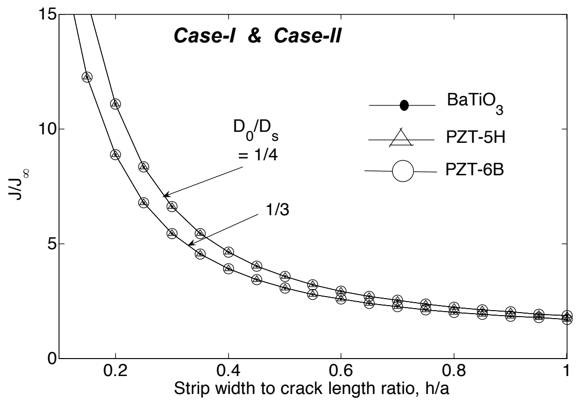

Fig. 2. Energy release rate versus strip width to crack length ratio

Fig. 3 shows the variation of energy release rate versus applied electrical displacement for different *h/a* ratio. It may be noted as *h/a* ratio increases i.e. the strip width becomes large there is less energy release. It is observed that $BaTiO_3$ and PZT-6B follow the behaviour of each other while PZT-5H lies farther away.

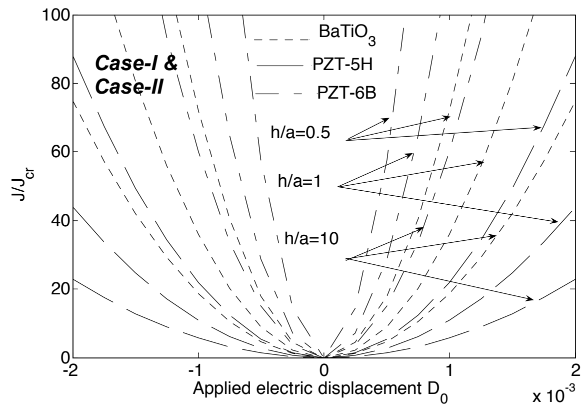

Fig. 3. Energy release rate versus applied electric displacement

8. Conclusions

The strip saturation model is proposed for a cracked piezoelectric ceramic strip. Energy release rate is calculated. And, its variation with respect to strip width and saturation zone length also shows that model is capable to arrest the crack opening.

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