Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 37
ISSUE 2
December
2009

Analytical calculation of the CNC machines servo drives position loop gain

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Received 15.09.2009; published in revised form 01.12.2009

Analysis and modelling

ABSTRACT

Purpose: One of the most important factors which influence on the dynamical behavior of the servo drives with rotary and linear motors for CNC machine tools is position loop gain or Kv factor.
Design/methodology/approach: From the magnitude of the Kv-factor depends tracking or following error. In multi-axis contouring the following errors along the different axes may cause form deviations of the machined contours. Generally position loop gain Kv should be high for faster system response and higher accuracy, but the maximum gains allowable are limited due to undesirable oscillatory responses at high gains and low damping factor. Usually Kv factor is experimentally tuned on the already assembled machine tool.
Findings: This paper presents a simple method for analytically calculation of the position loop gain Kv. A combined digital-analog models of the 6-th order (for rotary motors) and 4-th order (for linear motors) of the position loop are presented. In order to ease the calculation, the 6-th order system or 4-th order system is simplified with a second order model. With this approach it is very easy to calculate the Kv factor for necessary position loop damping. The difference of the replacement of the 6-th order system and 4-th order system with second order system is presented with the simulation program MATLAB. Analytically calculated Kv factor for the servo drives with rotary motors is function of the nominal angular frequency ω and damping D of the servo drive electrical parts (rotary motor and regulator), nominal angular frequency ωm and damping Dm of the mechanical transmission elements, as well as sampling period T. Kv factor for the servo drives with linear motors is calculated as function of the nominal angular frequency ωm and damping D of the linear motor servo drive electrical parts (motor and regulator) and sampling period T.
Research limitations/implications: The influence of nonlinearities was taken with the correction factor
Originality/value: Our investigations have proven that experimentally tuned Kv factor differs from analytically calculated Kv factor less than 10%, which is completely acceptable.
Keywords: Numerical techniques; CNC machine tool; Servo drive; Position loop gain

Reference to this paper should be given in the following way:
Z. Pandilov, V. Dukovski, Analytical calculation of the CNC machines servo drives position loop gain, Journal of Achievements in Materials and Manufacturing Engineering 37/2 (2009) 578-583.

1. Introduction

The most important variable, which describes the behaviour of a position control loop for CNC machine tools servo drives with rotary and linear motors, is position loop gain or Kv-factor. This is the ratio of the command velocity (feed rate) v to the position control deviation (following error, tracking error, lag) Δx [3,4,7,13,14]

$$Kv[s^{-1}] = \frac{v[mm/s]}{\Delta x[mm]} \quad \text{or} \quad Kv\left[\frac{m/min}{mm}\right] = \frac{v[m/min]}{\Delta x[mm]} \tag{1}$$

$$Kv[s^{-1}] = \frac{1000}{60} \cdot Kv\left[\frac{m/min}{mm}\right] \tag{2}$$

From the magnitude of the Kv-factor depends tracking or following error. In multi-axis contouring the following errors along the different axes may cause form deviations of the machined contours. Generally position loop gain Kv should be high for faster system response and higher accuracy, but the maximum gains allowable are limited due to undesirable oscillatory responses at high gains and low damping factor. Usually Kv factor is experimentally tuned on the already assembled machine tool [4,9,14]. This paper presents approach for analytically calculation of the position loop gain Kv. A combined 6-th order (for rotary motors) and 4-th order (for linear motors) digital-analog models of the position loop are presented. In order to ease the calculation, the 6-th order and 4-th order systems are simplified with a second order model. With this approach it is very easy to calculate the Kv factor for necessary position loop damping. The difference of the replacement of the 6-th order and 4-th order system with second order system is presented with the simulation program MATLAB. Analytically calculated Kv factor for servo drives with rotary motors is function of the nominal angular frequency ω and damping D of the servo drive electrical parts (motor and regulator), nominal angular frequency ω_m and damping D_m of the mechanical transmission elements, as well as sampling period T. Kv factor for the servo drives with linear motors is calculated as function of the nominal angular frequency ω and damping D of the linear motor servo drive electrical parts (motor and regulator) and sampling period T.

2. The combined digital-analog model of the servo drive with rotary motor position control loop and analytical calculation of the Kv factor

Fig. 1 presents digital-analog model of the CNC machine tool servo drive with rotary motor position control loop, where s represents Laplace operator.

Similar models are presented in [4,10,12], but the transfer function of mechanical transmission elements is not taken in consideration. Because of the existence of the digital part in the presented model we must use z-transformation for analysis. With some approximations and substitutions it is possible to analyze presented model in s-domain (with Laplace transformation). Digital-analog converter is substituted with zero order holder (z.o.h.) and sampler [5]. The new model is presented in Fig. 2.

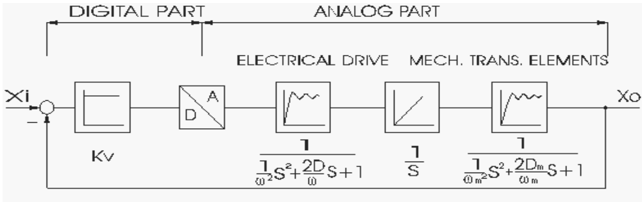

Fig. 1. Combined digital-analog model of the servo drive with rotary motor position control loop

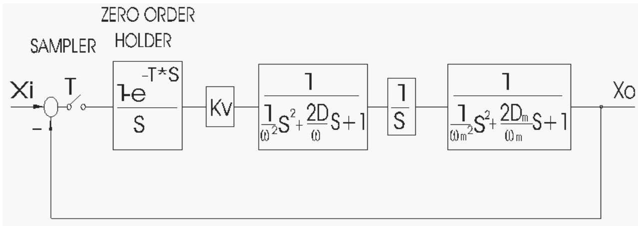

Fig. 2. Modified model of the servo drive with rotary motor position control loop presented in Figure 1

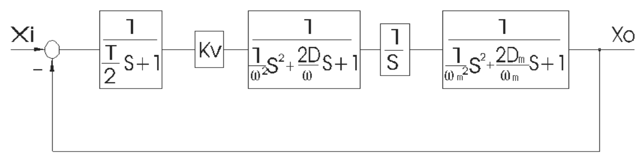

Fig. 3. Analog model of the servo drive with rotary motor position control loop

According [5] we can approximate sampler and zero order holder (z.o.h.) in Laplace domain with the following transfer function:

$$G(s) = \frac{1 - e^{-T \cdot s}}{T \cdot s} \tag{3}$$

With the Padè approximation of the first order for the $e^{-T \cdot s}$ we get:

$$e^{-T \cdot s} \approx \frac{1 - \frac{T}{2} \cdot s}{1 + \frac{T}{2} \cdot s} \tag{4}$$

where T is sampling time (period).

In that case G(s) becomes:

$$G(s)=\frac{1-e^{-T\cdot s}}{T\cdot s}\approx\frac{1}{1+\frac{T}{2}\cdot s} \tag{5}$$

With these simplifications servo drive with rotary motor position control loop may be presented with following model Fig. 3).

The model in Fig.3 may be analyzed in s-domain with Laplace transformation. The transfer function of the servo drive with rotary motor position control loop presented in Fig. 3 is:

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}\cdot\frac{1}{\frac{1}{\omega_m^2}\cdot s^2+\frac{2D_m}{\omega_m}\cdot s+1}}{1+Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}\cdot\frac{1}{\frac{1}{\omega_m^2}\cdot s^2+\frac{2D_m}{\omega_m}\cdot s+1}} \tag{6}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{T}{2}\cdot s+1\right)\cdot\left(\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1\right)\cdot\left(\frac{1}{s}\right)\cdot\left(\frac{1}{\omega_m^2}\cdot s^2+\frac{2D_m}{\omega_m}\cdot s+1\right)+Kv} \tag{7}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_6\cdot s^6+a_5\cdot s^5+a_4\cdot s^4+a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0} \tag{8}$$

where:

$$a_6=\frac{T}{2\omega^2\omega_m^2},$$

$$a_5=\left[\frac{1}{\omega^2\omega_m^2}+\left(\frac{2D_m}{\omega^2\omega_m}+\frac{2D}{\omega\omega_m^2}\right)\cdot\frac{T}{2}\right],$$

$$a_4=\left[\left(\frac{2D_m}{\omega^2\omega_m}+\frac{2D}{\omega\omega_m^2}\right)+\left(\frac{1}{\omega^2}+\frac{4DD_m}{\omega\omega_m}+\frac{1}{\omega_m^2}\right)\cdot\frac{T}{2}\right],$$

$$a_3=\left[\left(\frac{1}{\omega^2}+\frac{4DD_m}{\omega\omega_m}+\frac{1}{\omega_m^2}\right)+\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}\right)\cdot\frac{T}{2}\right],$$

$$a_2=\left[\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}\right)+\frac{T}{2}\right],\quad a_1=1,\quad a_0=Kv\quad \text{and}\quad b_0=Kv.$$

Having information's about the magnitude of the variables ω, D, ω_m, D_m and T in real servo drives with rotary motors position control loops, we can conclude that a_6, a_5, a_4, a_3 tends towards zero (a_6, a_5, a_4, a_3). So in that case we can simplify 6-th order system with rotary motor with the second order system [1,4]. We will present servo drive with rotary motor position control loop with the simplified transfer function:

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_2\cdot s^2+a_1\cdot s^1+a_0} \tag{9}$$

where $a_2=\left[\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}\right)+\frac{T}{2}\right]$, $a_1=1$, $a_0=Kv$ and $b_0=Kv$.

In that case

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)\cdot s^2+s+Kv} \tag{10}$$

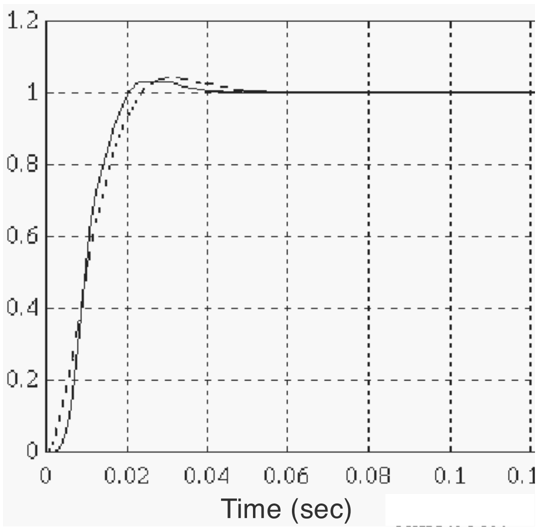

Fig. 4. (--------) Time response of the 6-th order system, (- - - - -) time response of the 2-nd order system

To check if it is correct to substitute 6-th order with second order system, we will simulate the system transfer function response on step function with simulation program MATLAB. Numerical values of the parameters of the examined system are: $\omega=1000\,s^{-1}$, D=0.7, $\omega_m=663\,s^{-1}$, $D_m=0.17$, T=0.006 s and $Kv=100\,s^{-1}$.

Fig. 4 gives responses of the position control with rotary motor loop transfer function of 6-th and 2-nd order on step function.

From Fig. 4 it is obvious that the differences caused by substitution are minimal. It makes substitution completely acceptable. For the second order system it is possible very easy and fast to calculate Kv-factor for necessary position control with rotary motor loop damping.

We can write the second order system transfer function in the following form:

$$\frac{Xo(s)}{Xi(s)}=\frac{1}{\frac{1}{\omega_n^2}\cdot s^2+\frac{2\zeta}{\omega_n}\cdot s+1} \tag{11}$$

where ζ is position control loop damping ($0<\zeta<1$), and ω_n is nominal angular frequency of the position control loop. We will transform equation (10) in the form of equation (11).

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)\cdot s^2+s+Kv}=$$

$$=\frac{1}{\frac{\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)}{Kv}\cdot s^2+\frac{1}{Kv}\cdot s+1} \tag{12}$$

Comparing (11) and (12) we can obtain:

$$\omega_n=\sqrt{\frac{Kv}{\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}}}\quad \text{and}\quad \zeta=\frac{1}{2}\cdot\sqrt{\frac{1}{Kv\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)}} \tag{13}$$

In order to have required position control loop damping ζ,Kv-factor should be calculated with the following equation:

$$Kv=\frac{1}{4\zeta^2\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)} \tag{14}$$

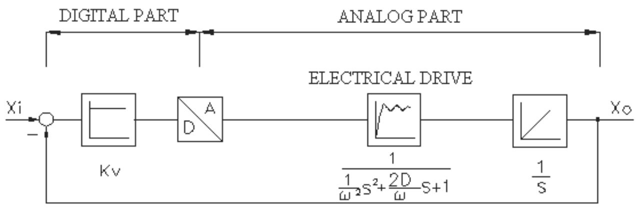

Fig. 5. Combined digital-analog model of the linear motor servo drive position control loop

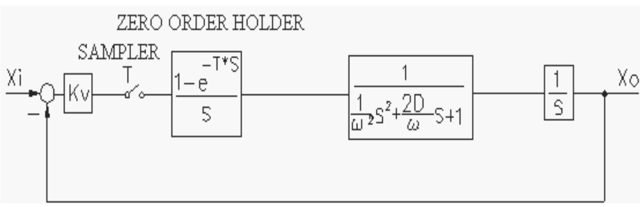

Fig. 6. Modified model of the linear motor servo drive position control loop presented in Figure 5

Equation (14) gives direct analytical relationship between Kv-factor and ω, D, ω_m, D_m, T and ζ, which are already known variables, or can be calculated very easy

With the equation (14) it is possible to estimate CNC machine tool servo drive with rotary motor position loop gain Kv without performing experiments.

We will check correctness of the equation (14) on real servo drive with rotary motor position control loop of CNC milling machine FGS 32-CNC. Position loop damping ζ=0.7 is preferable according [2,6,8,12,15]. That is the value, which gives minimal contouring errors. Other numerical values of the examined system are: ω=1000 s^{-1}, D=0.7, ω_m=663 s^{-1}, D_m=0.17 and T=0.006 s. With the substitution in the equation (14) the position loop gain value Kv=103.85 s^{-1} is calculated. Experimentally tuned value of Kv-factor on examined machine tool axis was Kv=100 s^{-1}. The difference between analytically calculated and experimentally obtained value of Kv-factor is around 4%, which is completely acceptable.

3. The combined digital-analog model of the linear motor servo drive position control loop and analytical calculation of the Kv factor

Fig. 5 presents digital-analog model of the CNC machine tool linear motor servo drive position control loop, where s represents Laplace operator.

Existence of the digital part in the presented model needs use z-transformation for analysis. Using some approximations and substitutions it is possible to analyze presented model in s-domain (with Laplace transformation). According [5] digital-analog converter is substituted with zero order holder (z.o.h.) and sampler. The new model is presented in Fig.6.

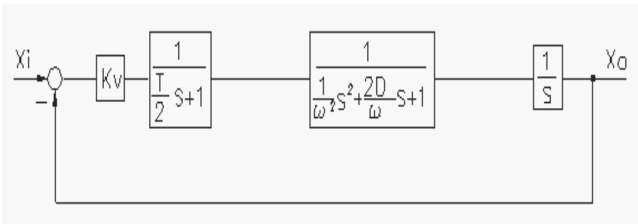

Fig. 7. Analog model of the linear motor servo drive position control loop

We can approximate sampler and zero order holder (z.o.h.) in Laplace domain with the transfer function G(s) given with equation (3).

With the Padè approximation of the first order for the $e^{-T\cdot s}$ we obtain equation (4):

With these simplifications linear motor servo drive position control loop may be presented with following model (Fig.7).

The model in Fig.7 may be analyzed in s-domain with Laplace transformation. The transfer function of the linear motor servo drive position control loop presented in Fig.7 is:

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}}{1+Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}} \tag{15}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{T}{2}\cdot s+1\right)\cdot\left(\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1\right)\cdot\left(\frac{1}{s}\right)+Kv} \tag{16}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_4\cdot s^4+a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0} \tag{17}$$

where:

$a_4=\frac{T}{2\omega^2}$, $a_3=\frac{1}{\omega^2}+\frac{D}{\omega}\cdot T$, $a_2=\frac{2D}{\omega}+\frac{T}{2}$, $a_1=1$, $a_0=Kv$ and $b_0=Kv$.

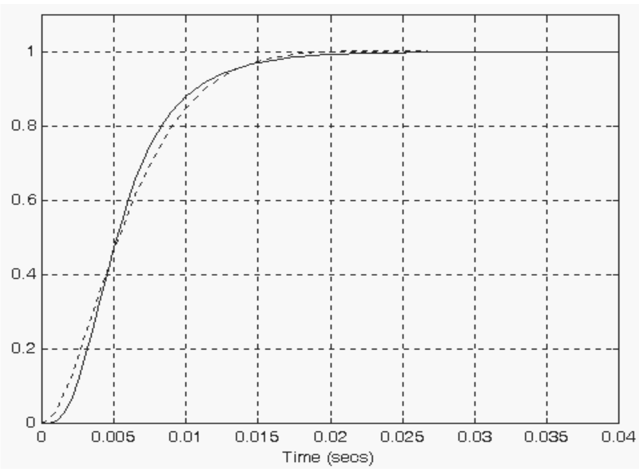

Fig. 8. (--------) Time response of the 4-th order system, (- - - - -) Time response of the 2-nd order system

Having information's about the magnitude of the variables ω, D, and T in real linear motor servo drive position control loops, we can conclude that a_4, a_3 tends towards zero ($a_4, a_3 \to 0$). So in that case we can simplify 4-th order system with the second order system [1,4]. We will present linear motor servo drive position control loop with the simplified transfer function:

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_2\cdot s^2+a_1\cdot s^1+a_0} \tag{18}$$

where $a_2=\frac{2D}{\omega}+\frac{T}{2}$, $a_1=1$, $a_0=Kv$ and $b_0=Kv$.

In that case

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{2D}{\omega}+\frac{T}{2}\right)\cdot s^2+s+Kv} \tag{19}$$

To check if it is correct to substitute 4-th order with second order system, we will simulate the system transfer function response on step function with simulation program MATLAB. Numerical values of the parameters of the examined system are: $\omega=1000\,s^{-1}$, D=0.7, T=0.001 s and $Kv=166.67\,s^{-1}$.

Fig. 8 gives responses of the position control loop transfer function of 4-th and 2-nd order on step function.

From Fig. 8 it is obvious that the differences caused by substitution are minimal. It makes substitution completely acceptable. For the second order system it is possible very easy and fast to calculate Kv-factor for necessary position control loop damping.

We can write the second order system transfer function in the following form:

$$\frac{Xo(s)}{Xi(s)}=\frac{1}{\frac{1}{\omega_n^2}\cdot s^2+\frac{2\zeta}{\omega_n}\cdot s+1} \tag{20}$$

where ζ is position control loop damping (0< ζ <1), and ω_n is nominal angular frequency of the position control loop.

We will transform equation (19) in the form of equation (20).

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{2D}{\omega}+\frac{T}{2}\right)\cdot s^2+s+Kv}==\frac{1}{\frac{\left(\frac{2D}{\omega}+\frac{T}{2}\right)}{Kv}\cdot s^2+\frac{1}{Kv}\cdot s+1} \tag{21}$$

Comparing (20) and (21) we can obtain:

$$\omega_n=\sqrt{\frac{Kv}{\frac{2D}{\omega}+\frac{T}{2}}} \quad\text{and}\quad \zeta=\frac{1}{2}\cdot\sqrt{\frac{1}{Kv\left(\frac{2D}{\omega}+\frac{T}{2}\right)}} \tag{22}$$

In order to have required position control loop damping ζ, Kv-factor should be calculated with the following equation:

$$Kv=\frac{1}{4\zeta^2\left(\frac{2D}{\omega}+\frac{T}{2}\right)} \tag{23}$$

Due to the presence of nonlinearities in reality the value of Kv must be decresesed up to 40% [4].

$$Kv = \frac{0.6}{4\zeta^2\left(\frac{2D}{\omega}+\frac{T}{2}\right)} \tag{24}$$

Equation (24) gives direct analytical relationship between Kv -factor and ω, D, T and ζ which are already known variables, or can be calculated very easy.

With the equation (24) it is possible to estimate CNC machine tool linear motor servo drive position loop gain Kv without performing experiments.

We will check correctness of the equation (24) on real linear motor servo drive position control loop of high speed cutting CNC linear motor machine (HSC 11) developed on the Institute for Production Engineering and Machine Tools (PTW) at Technical University Darmstadt, Germany. Position loop damping ζ=0.7 is preferable according [2,6,8,12,15]. That is the value, which gives minimal contouring errors. Other numerical values of the examined system are: $\omega=1000\,s^{-1}$, D=0.7, and T=0.001 s. With the substitution in the equation (24) the position loop gain value Kv=157.89 s^{-1} is calculated. Experimentally tuned value of Kv-factor on examined machine tool axis was Kv=166.67 s^{-1}. The difference between analytically calculated and experimentally obtained value of Kv-factor is around 5.5%, which is completely acceptable.

4. Conclusion

The equation (14) for servo systems with rotary motors enables very fast, simple and precise analytical calculation of position loop gain Kv as a function of already known position control loop parameters (ω-nominal angular frequency of the rotary motor servo drive electrical parts, D-damping of the rotary motor servo drive electrical parts, ω_m -nominal angular frequency of the mechanical transmission elements, D_m -damping of the mechanical transmission elements and T-sampling time).

For the servo systems with linear motors the equation (24) we can determine precisely position loop gain Kv as a function of the position control loop parameters (ω-nominal angular frequency of the linear motor servo drive electrical parts, D-damping of the linear motor servodrive electrical parts, and T-sampling time).

In that way we can avoid long-time experimental tuning of the Kv-factor on machine tool. And of course analytical calculation of the Kv factor gives possibility to estimate the accuracy of the system in the design phase.

Acknowledgements

A part of this research was done during at the Institute of Production Engineering and Machine Tools (PTW), Faculty of Mechanical Engineering, Darmstadt University of Technology, Darmstadt, Germany, sponsored by the DAAD (German Academic Exchange Service).

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