

Modelling of fracture process in concrete reinforced structures under steel corrosion

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Analysis and modelling

ABSTRACT

Purpose: Corrosion of reinforcement in concrete is one of the main causes of structural deterioration. The cracking process of concrete matrix induced by corrosion swelling of rebars is analyzed numerically and experimentally with aim to propose a methodology of corroding state monitoring based on the crack opening width.

Design/methodology/approach: The direct finite element simulation of the initiation and propagation of macro-cracks in concrete caused by steel corrosion are performed for the representative volume element. A range of various inelastic constitutive equations for concrete is used in modeling. Of these the damage models in elastic media, the holonomic theory of plasticity, the associated and non-associated flow theory of plasticity with Drucker-Prager and CAP yield surfaces are considered.

Findings: The results allow establishing the phases of crack initiation and propagation, the shape and trajectory of cracks.

Research limitations/implications: Analysis shows that application of the linear elastic model of concrete in general underestimate the values of local stress and strain at the crack initiation location.

Practical implications: Based on results of investigation the recommendations may be suggested aimed at assessment of necessary thickness of the concrete cover; apart from that, the technique of in-service monitoring the reinforcement condition may be further defined.

Originality/value: The original methodology of corroding rebar state monitoring based on the surface crack opening has been proposed. The obtained results and suggested approaches can be interesting for researchers working in the areas of mechanics of materials, computational mechanics and civil engineers.

Keywords: Corrosion; Concrete; Fracture mechanics; Crack propagation

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1. Introduction

Development of non-destructive methods of rebar corrosion control in a reinforced concrete is essentially important for a

practice. Of the known approaches the most prospective are regarded those based on examination of the surface protective layer of the reinforced concrete [1-4]. The oxides generated during active corrosion of steel bars induce a pressure on the surrounding concrete, which leads to crack initiation at the bar surfaces and

then at the outer surface of cover. Vasiliev in [3] suggested a correlation between the outer crack opening and the volume of corrosion products as a base of control method; further it was developed in [4]. However, in these analyses concrete matrix was assumed a linear elastic media; also, the beam theory was applied.

In present study a two-dimensional finite element (FE) analysis of a concrete cracking under reinforcement corrosion is carried out. The analysis is aimed at evaluation of the existing approximate approaches [3,4] and at further development of the crack growth prediction due the corrosion products expansion on the reinforcement. By applying direct mathematical modeling the process of corrosive swell stressing and failure of concrete matrix is examined considering the non-linear mechanical behavior of concrete and crack initiation and propagation stages. The results of investigation allow to suggest the principal mechanisms of cracking and the failure scenario and also to define the shape and trajectory of cracks. Apart from that, based on the present results the necessary concrete cover thickness may be recommended and technique of reinforcement damage control by the surface crack opening assessment may be developed.

2. The problem formulation

A reinforced concrete structural component behavior caused by expansion of corrosion products was modeled by a plate with periodical holes corresponding to the component cross-section. The model, single cell of the holed plate (Fig. 1), was assumed in the plane strain condition loaded by the internal uniform pressure p , applied along the hole circumference to simulate the corrosion products swell. As shown in [5] the volume of corrosive material is about 2.0-2.5 times of the reduction of the bar cross-section; respectively, high pressure on the concrete is generated resulting in origination of cracks and their propagation through the concrete cover.

The single cell dimensions are (Fig. 1 b): $c=R$, $W=5R$, $H=10R$, $R=8$ mm; the boundary conditions of the cell conform the symmetry and periodicity, as shown in Fig. 1 b.

In the first phase of investigation the concrete media was assumed free of macro-cracks. The analysis provided evaluation of the potential crack initiation locations and the critical pressure necessary to initiate cracks at the hole. Further, the crack system growth was assessed; it was found also, that along with the increasing pressure the new cracks were generated at the free surface of cover and extended towards the hole.

The reliability of results was tested by multi-model and multi-variant analyses in which the inelastic behavior of concrete was simulated using several constitutive models.

The alternative approaches are considered in [6-10].

3. Modelling inelastic behavior of concrete

Several material models were applied in analyses: successively, concrete was assumed first the linear elastic media; further, it was

considered holonomic theory of plasticity and flow theory of plasticity with various hardening laws. Also, the elasto-damage model was applied based on the continuum damage mechanics concept.

When elasto-plastic behavior was considered, the Drucker-Prager criteria were applied with linear and nonlinear envelope [11], different for compression and tension meridians of the yield surface, together with CAP model [12] ensuring the closed yield surfaces.

The plasticity criterion for the CAP model attributed to the three-invariant class is the generalization of the Drucker-Prager criterion by considering the J_3 invariant and double-sided truncation of conical surface by elliptic CAP at tension and compression. The criterion is written as follows:

$$Y(I_1, J_2, J_3) = J_2 \Gamma^2(J_2, J_3) - Y_s^2(I_1) Y_c(I_1) Y_t(I_1) = 0, \quad (1)$$

where $I_1 = \sigma \cdot \mathbf{1}$, $J_2 = \frac{1}{2} \mathbf{s} \cdot \mathbf{s}$, $J_3 = \text{dets}$, the conical envelope shape function is:

$$Y_s(I_1) = k(\varepsilon_i^p) - \alpha I_1 - A e^{\beta I_1}, \quad (2)$$

the functions defining truncation of the conical surface in compression and tension areas:

$$Y_c(I_1) = 1 - H(K_0 - I_1) \left[\frac{I_1 - K_0}{R_c Y_s(K_0)} \right]^2, \quad (3)$$

$$Y_t(I_1) = 1 - H(I_1) \left[\frac{I_1}{R_t Y_s(0)} \right]^2, \quad (4)$$

and $\Gamma(J_2, J_3)$ characterizes sensitivity of the criterion to the stress state:

$$\Gamma(J_2, J_3) = \frac{1}{2} \left[1 - \frac{3\sqrt{3}J_3}{2J_2^{3/2}} + \frac{1}{\psi} \left(1 + \frac{3\sqrt{3}J_3}{2J_2^{3/2}} \right) \right]. \quad (5)$$

In equations (1)-(5) $\alpha, A, \beta, K_0, R_c, R_t, \psi$ are material constants, function $k(\varepsilon_i^p)$ is defined by the tensile diagram, and $H(\dots)$ is the Heavyside's function.

The yield surface (1) is composed of three parts: conical and two ellipsoidal components. When truncation of the conical component may be neglected (on assumption $Y_c(I_1) = Y_t(I_1) \equiv 1$), equation (1) can be simplified as:

$$\bar{Y}(I_1, J_2, J_3) = \sqrt{J_2} - Y_s(I_1) / \Gamma(J_2, J_3) = 0. \quad (6)$$

Further, if the sensitivity of the criterion (1) to the stress state may be insignificant ($\Gamma(J_2, J_3) \equiv 1$) and the envelope of conical surface is a linear one ($A \equiv 0$), then (1) yields to the Drucker-Prager criterion:

$$\tilde{Y}(I_1, J_2) = \sqrt{J_2} + \alpha I_1 - k(\varepsilon_i^p) = 0. \quad (7)$$

In the following analyses several approximations of the concrete tensile diagram were applied shown in Fig. 2: bilinear approximation (1), three-linear (2), parabolic (6), polynomial of the 3, 4 and 5 order (3, 4, 5), Eurocode approximation (7) in the

$$\zeta = \frac{k\eta - \eta^2}{1 + (k-2)\eta}, \quad \text{where} \quad \zeta = \sigma / R_b', \quad \eta = \varepsilon / \varepsilon_{pick},$$

$k = E_b / (R_b' / \varepsilon_{pick})$ are dimensionless stress, strain, secant stiffness and last approximation (8) representing combination of power and fractionary-rational functions in the form

$$\zeta = \begin{cases} 1.2\eta - 0.2\eta^6, & \eta \leq 1 \\ \frac{\eta}{0.312\sigma_{pick}'(\eta-1)^{1.7} + \eta}, & \eta \geq 1 \end{cases}, \quad \text{suggested by Tuhr and}$$

Rack [13].

A series of uniaxial compression and tensile tests of cylindrical and prismatic concrete specimens was carried out to identify material characteristics. Also, plate specimens were tested at plane stress conditions and respective diagrams were recorded. Based on the test results, the following values of ultimate strength of concrete were obtained, $R_b = 18.5$ MPa and $R_b' = 1.55$ MPa, for compression and tension, respectively.

4. Crack initiation

First, potential locations of crack initiation and the pressure necessary for cracks to propagate were obtained. For the criterion of crack initiation was selected condition when the maximum principal strain would reach the value corresponding peak stress in tensile diagram. Fig. 3 shows the results of computations with

application of different material models. It can be seen the high sensitivity of stress field to selection of the material model; it is seen also that using the linear elastic model of material essentially underestimates the area affected by the stress raise.

Fig. 4 presents maximum principal strain ε_1 , depending on the corrosion pressure p , for various material models. Again, at a given pressure application of elastic model of material results, in general, in smaller strain than it is found with the elasto-plastic models. The difference between critical pressures obtained in elastic solution and elasto-plastic solution with CAP is more 30%.

5. Multiple cracks propagation

The following part of investigation was focused on analysis of extension of the system of curvilinear cracks. The crack growth was modeled in the step-wise sequence with automated definition of the extension step and direction based on the maximum tangential stress criterion. The extension process was assumed composed of several phases: initiation and growth of "horizontal" cracks (length a) slightly inclined with respect to the free surface of component (Figs. 5, a, b); birth of "vertical" crack at the free outer surface (Figs. 5, c, d); simultaneous growth of "horizontal" and "vertical" cracks (Figs. 5, e, f); "vertical" crack deceleration (Figs. 5, g, h) and further fragmentation or delamination of the concrete cover.

Comparison of results of analyses where the elastic or elasto-plastic material behavior was suggested reveals significant difference of the pressure values necessary to initiate a "horizontal" crack (Fig. 4), of the crack sizes corresponding to the above growth phases (Fig. 5) and in the respective stress and strain fields as well (Fig. 6).

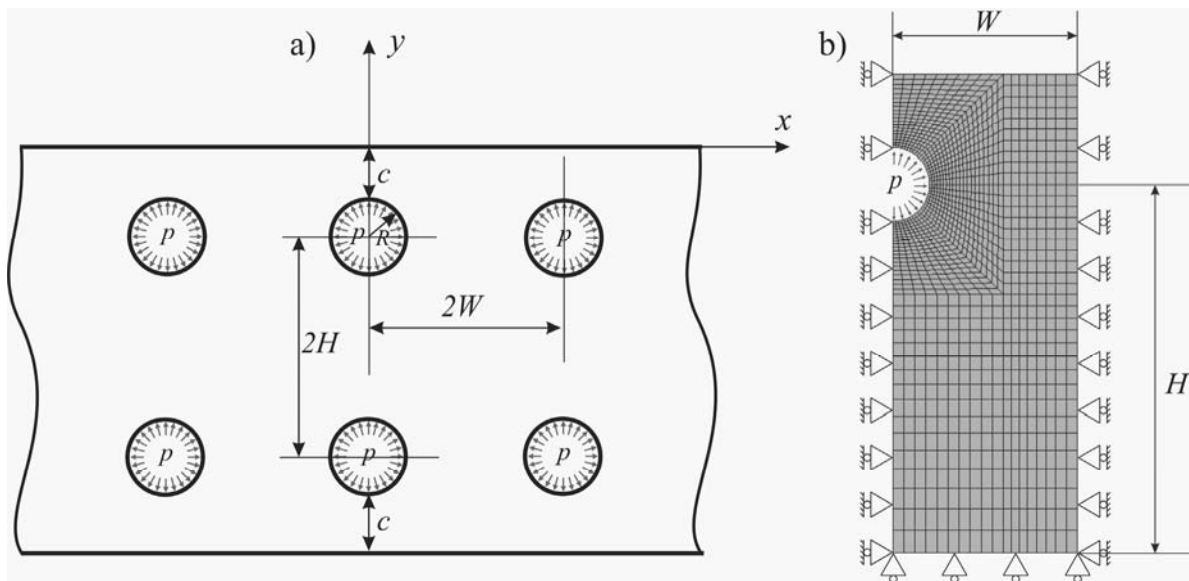


Fig. 1. Mechanical model of typical reinforced concrete structure a) and FE-model of a representative single cell b)

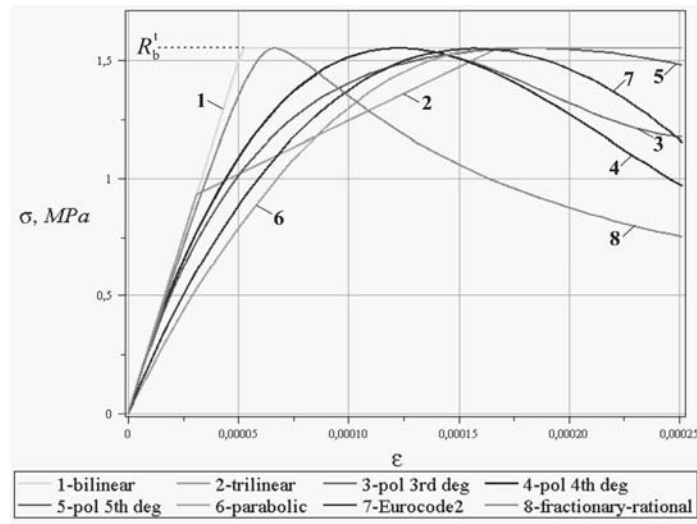


Fig. 2. Approximations of the concrete deformation diagram under tension

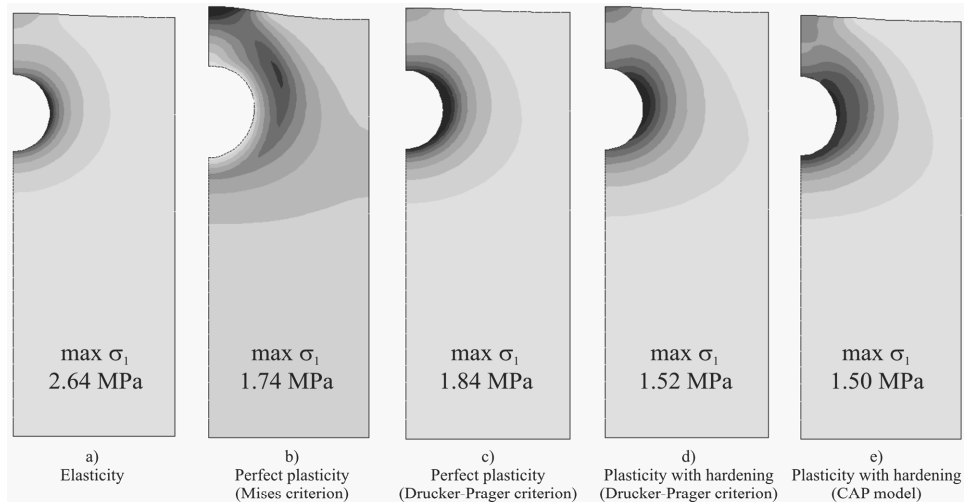


Fig. 3. Field distribution of maximum principal stress σ_1 , at $p = 2$ MPa, corresponding to different models of material behavior

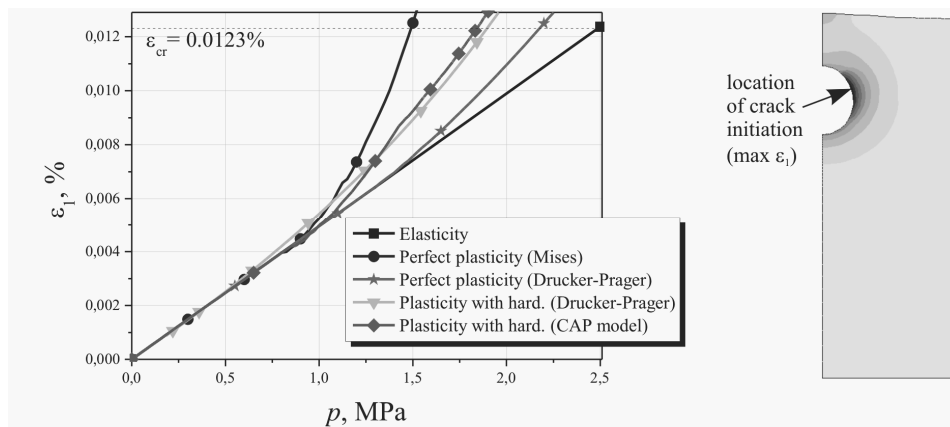


Fig. 4. Dependence of maximum principal strain ϵ_1 on the corrosion pressure p , corresponding to different models of material behavior

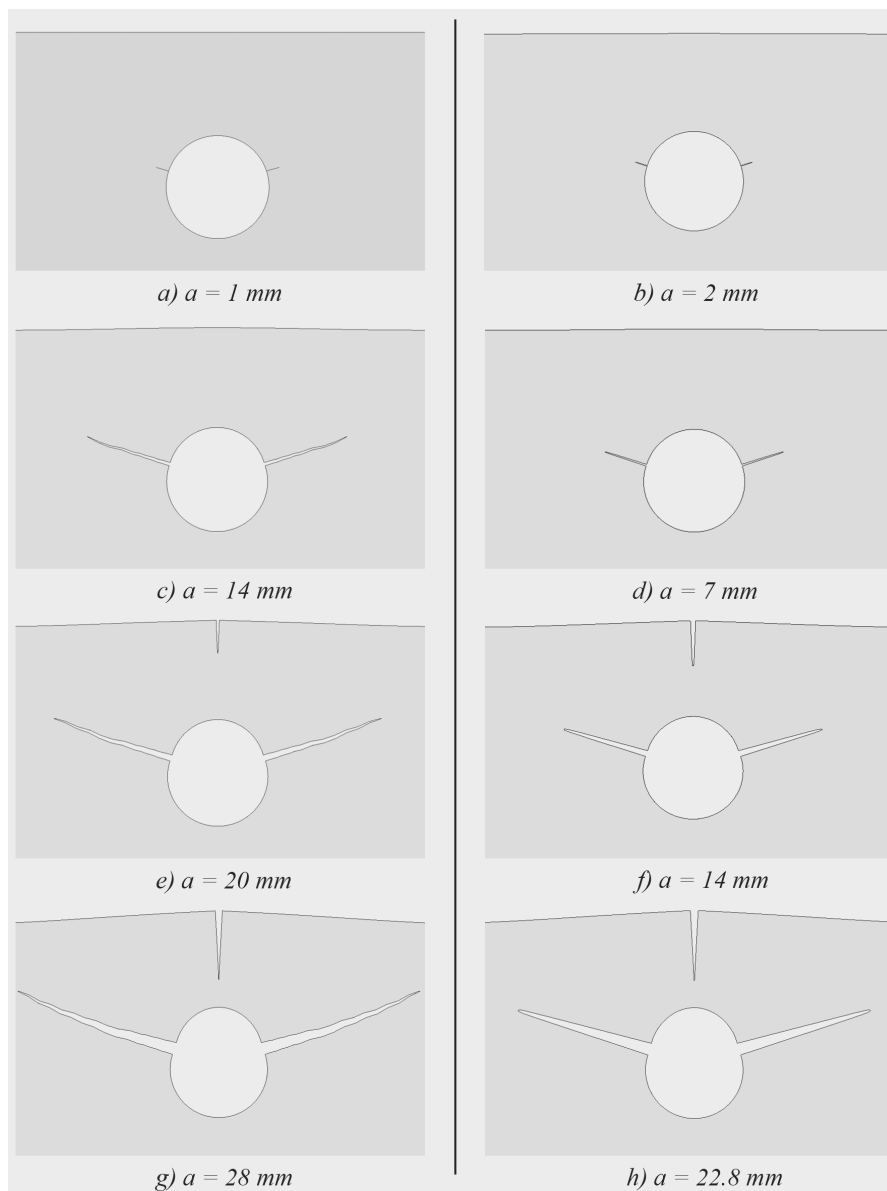


Fig. 5. Comparison of crack propagation for two material models: linear elastic (left) and elasto-plastic with Drucker-Prager criterion (right). a is the length of horizontal crack. Displacements are magnified 100 times for obviousness

6. Correlation between crack opening and corrosion degree

The present results are intended to assist in quantifying the service and residual life of concrete structures. Reinforcement damage can be evaluated by the opening of longitudinal crack along the rebar (see the vertical cracks in Figs. 5-7).

A simple analytical evaluation for the loss in steel cross-section due to corrosion on the base of the longitudinal crack opening has been proposed in [3]:

$$\Delta = \frac{\delta l}{8h}, \quad (8)$$

where Δ is the corrosion expansion (radial loss or attack penetration in the rebar, see Fig. 7 for details), δ is the crack width (vertical crack opening, see Fig. 7), h is a distance of horizontal crack start point from the free surface (Fig. 7), l is a distance between crack tips of two horizontal cracks (Fig. 7).

Expressions for parameters h and l , characterizing horizontal cracks, have been obtained in [4] for the linear elastic undamaged material and the beam approximation of the concrete cover upper horizontal cracks. In this case h and l are determined only by the rebar radius R and the cover thickness c :

$$h = c \left(1 + \frac{1}{c/R+1} \right), \tag{9}$$

$$l = 2R \sqrt[3]{\pi \cdot (1+\nu)(c/R+1)(c/R)^2 \left(1 + \frac{1}{c/R+1} \right)^2} \tag{10}$$

The aim of present investigations is to evaluate and to improve relations (8)-(10) by means of taking into account an inelastic material behavior and an applying of two-dimensional FE model for analysis. Numerical solutions are obtained for the two case of corrosion loading. Variants with the given uniform pressure p and with the given uniform dilatation Δ are considered.

The results of numerical analysis in a form of dependency of the radial loss Δ on the crack opening δ are presented in Fig. 8. Comparison of the results shows satisfied agreement between the prediction by (8) and linear elastic FE solution with the given dilatation Δ . The plastic solution with Drucker-Prager criterion

demonstrates nonlinear dependence of the radial loss on the crack opening. In the last case the attack penetration Δ increases in comparison with elastic solution under the same crack opening δ .

The obtained numerical results demonstrate a satisfactory agreement with experimental data [14, 15], where it was shown, that the radius losses of 15-50 μm are necessary to generate the crack width of $\sim 50 \mu\text{m}$ (see Fig. 8).

The case with corrosion simulation by uniform pressure p has been also considered with aim to obtain the lower boundary for δ . A dependence of the radial loss on the corrosion pressure is presented in Fig. 9 for the linear and non-linear cases. The comparison of the solution for the given corrosion pressure p with the solution for the given radial loss Δ are presented in Fig. 10. The contact between flanks of the vertical crack is taken into consideration in the case of the given pressure because of its considerable influence on results. Both the elastic and the elasto-plastic solutions with the given dilatation demonstrate more stiff behavior then solutions with given corrosion pressure.

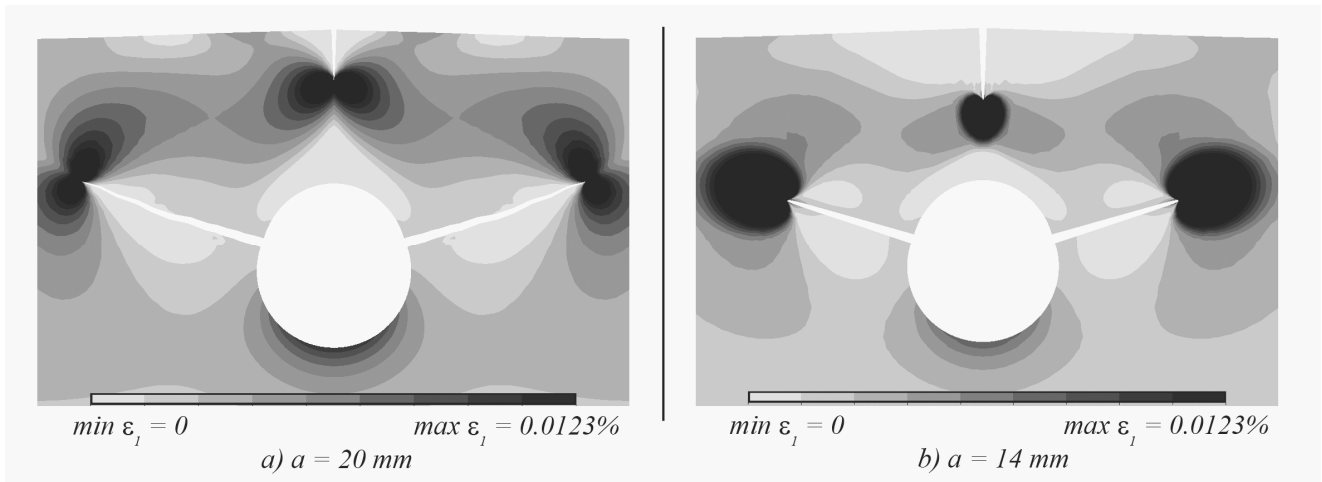


Fig. 6. Field distribution of maximum principal strain ε_1 : a) linear elastic material, b) elasto-plastic material with Drucker-Prager criterion

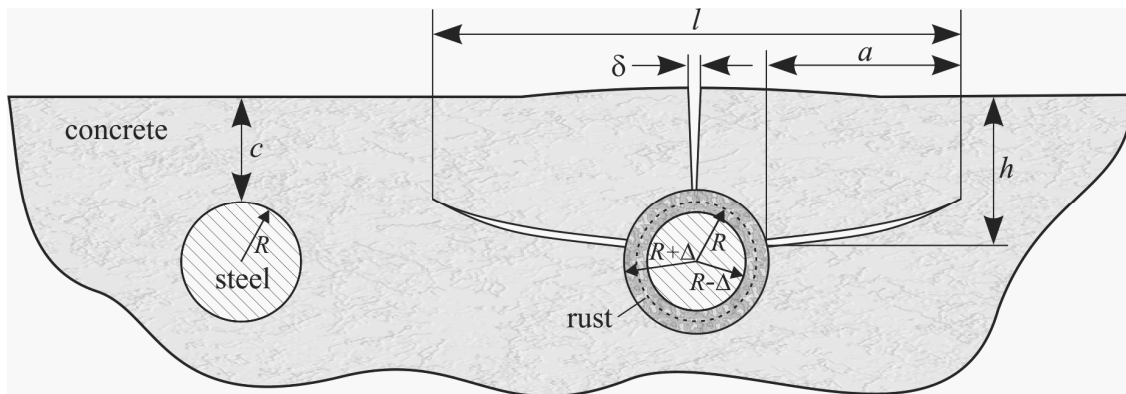


Fig. 7. Geometrical parameters, characterizing of concrete cover cracking

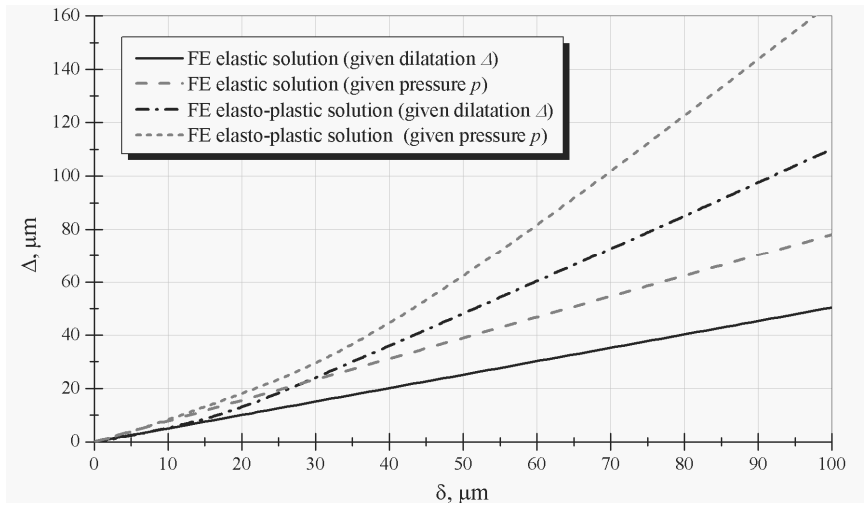


Fig. 8. Corrosion expansion Δ versus crack opening δ

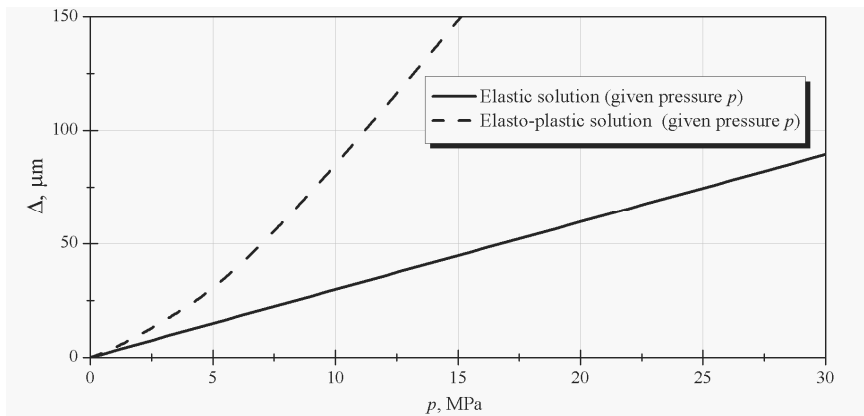


Fig. 9. Corrosion expansion Δ versus corrosion pressure

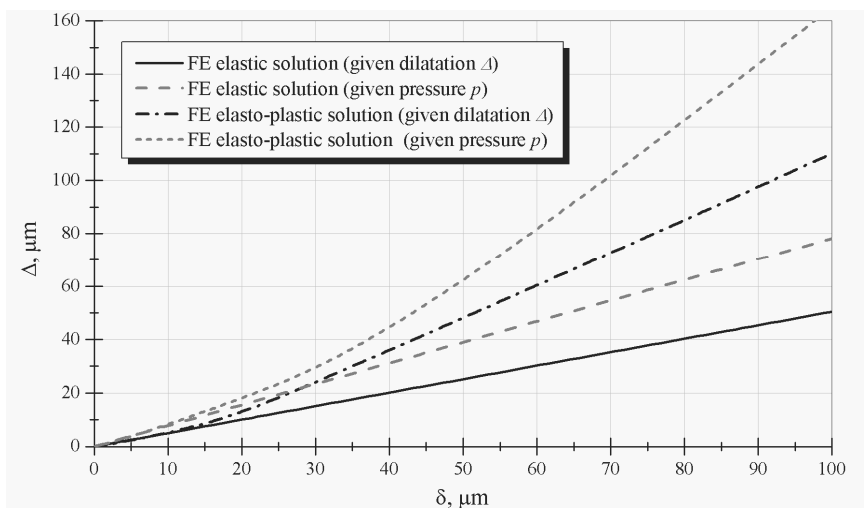


Fig. 10. Comparison of solutions with the given corrosion pressure p and with the given dilatation Δ

7. Conclusions

The process of corrosion driven cracking in concrete reinforced structures is simulated by applying the FE modeling. The results allow establishing the phases of crack initiation and propagation, the shape and trajectory of cracks. Also, the above analysis shows that application of the linear elastic model of concrete in general underestimate the values of local stress and strain at the crack initiation location.

Based on results of investigation the recommendations may be suggested aimed at assessment of necessary thickness of the concrete cover; apart from that, the technique of in-service monitoring the reinforcement condition may be further defined.

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