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Difference in acceleration of electrons, protons and deuterons in a laser beam

M.J. Małachowski*, A. Dubik
Department of Applied Physics, K. Pulaski Technical University of Radom, ul. J. Malczewskiego 20A, 26-600 Radom, Poland
* Corresponding author: E-mail address: malachowski.m.j@interia.pl

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Methodology of research

ABSTRACT

Purpose: The aim of this paper is to find in a numerical way the main differences in the trajectories and kinetic energies of electrons, protons and deuterons accelerated in the laser or maser beams propagating in a vacuum, with an additionally applied external static co-axial magnetic field. The accelerated particles to the well defined energies are of interest in many applications, among others in medicine or in processing of different materials.
Design/methodology/approach: Due to differences in masses the comparison between the acceleration processes of electrons, protons and deuterons is possible to perform after appropriate parameters of radiation of a laser, maser and a static magnetic field have been designed.
Findings: The quantitative illustrations of the calculation results in a graphical form enable to discuss the main differences in the acceleration process of electrons, protons and deuterons. It was found that the rate at which a particle gains the energy depends not only on the particle's mass but also on the stage of the process. Due to the mass differences, in order to keep a particle inside the radiation beam, significantly different static magnetic fields should be used to each kind of a particle. The authors have found an answer to the question why the rate at which particles energy increases in time, is different for different particles and why the difference depends on a stage of the acceleration process.
Research limitations/implications: Limits in the energy of accelerated particles are caused by the limits of laser or maser beam energy or power available at present and the static magnetic fields.
Originality/value: The authors show, in an exact numerical way, the values of the acceleration equipment parameters which should be selected to obtain the desired energy of the accelerated particles. It is explained why the rate at which a particle gains the energy depends on the stage of the process and on the particle's mass.
Keywords: Acceleration of charged particles; Laser; Maser; Relativistic dynamics

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1. Introduction

Many experiments and theoretical works have been shown the possibility of particles acceleration as a result of their interaction with the laser or maser beams of different parameters [1-4]. The accelerated particles to the high energies are of interest in many applications, among others in fast ignition in inertial confinement fusion [5], proton cancer therapy, proton imaging and ion beam radiography [6], laser nuclear physics [7], or materials processing [8] etc. Recent years have seen important achievements in the development of laser particle accelerators [9]. Now there are accessible lasers showing radiation power density of the level 10^{22} W/cm^2, which corresponds to the electric field intensity amplitude 10^{14} V/m [10]. Such parameters of a laser radiation allows to generate electrons and ions to GeV and possibly TeV energies The main purpose of this paper is to find in a numerical way the basic differences in the trajectories and kinetic energies of electrons, protons and deuterons accelerated in the lossless conditions in the laser or maser beams propagating in a vacuum, with an additionally applied external static co-axial magnetic field [11, 12].

On the basis of the equation of motion of the charged particles in the laser beam and additionally applied magnetic field [13-15], the further evolution of the acceleration model is shown. The results of numerical calculation are presented in a graphical form illustrating the course of the electron, proton and deuteron trajectories and theirs kinetic energies behavior under various parameters of a laser beam and magnetic fields.

2. Equations for trajectory and energy of a charged particle in a laser beam and a co-axial static magnetic field

The dynamical relativistic equation and the continuous equation for charged particle in the electromagnetic and the static magnetic fields defining the trajectory and normalized energy γ in the lossless condition [13, 16] have the following form

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\left[\vec{V} \times \left(\vec{B} + \vec{B}_z\right)\right], \qquad \frac{d\gamma}{dt} = \frac{q}{m_0 c^2}\vec{V} \cdot \vec{E} \tag{1}$$

where p, V, q and m_0 are the momentum, velocity, electric charge and the rest mass of the particle, E is the electric field intensity of an electromagnetic wave, B is the magnetic field induction of the wave, c is the velocity of the electromagnetic wave and B_z is the external static magnetic induction in the direction along the z coordinate (the laser beam axis) and

$$\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}}, \quad \vec{\beta} = \frac{\vec{V}}{c}, \quad \vec{p} = \gamma \cdot m_0 c\vec{\beta},$$

$$\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2, \quad \beta_{x,y,z} = \frac{V_{x,y,z}}{c}$$

Assuming the electromagnetic wave is propagating in the direction z, coordinates of electric and magnetic field vectors at (x, y) plane, in the case of circularly polarized wave of circulation to the left side are as follows

$$\vec{E} = \vec{i}E_x + \vec{j}E_y = \vec{i}E_0 \sin\phi + \vec{j}E_0 \cos\phi, \qquad \vec{B} = \vec{j}B_y + \vec{i}B_x = \vec{j}\frac{E_0}{c}\sin\phi - \vec{i}\frac{E_0}{c}\cos\phi \tag{2}$$

where the phase ϕ is defined as

$$\phi = \omega\left[t - \frac{z(t)}{c}\right], \tag{3}$$

where ω is the frequency of a laser wave acting on the particle in the laboratory frame of reference. In a complex system consisting of an electromagnetic field and a static axial magnetic field the following vectors are essential

$$\vec{E} = \left[E_x, E_y, 0\right], \quad \vec{B} = \left[-B_x, B_y, B_z\right]$$

$$\vec{p} = m_0\gamma V_x\vec{i} + m_0\gamma V_y\vec{j} + m_0\gamma V_z\vec{k}$$

$$\frac{d\vec{p}}{dt} = \vec{i}m_0 c\frac{d(\gamma\beta_x)}{dt} + \vec{j}m_0 c\frac{d(\gamma\beta_y)}{dt} + \vec{k}m_0 c\frac{d(\gamma\beta_z)}{dt}$$

and

$$E_x = E_0 \sin\phi, \quad E_y = E_0 \cos\phi$$

$$B_x = \frac{E_0}{c}\cos\phi, \quad B_y = \frac{E_0}{c}\sin\phi, \quad B_z = \pm aB_0$$

where ϕ is the phase, E_0 and B_0 are the amplitudes of an electric and the magnetic induction fields, respectively. We can introduce the parameter b, which was found to be useful for determining the B_z value suitable in obtaining some interesting forms of the particle trajectories.

$$b = 1 - qB_z / m\omega \tag{4}$$

The impact of the static magnetic field on the particle is expressed through the parameter $a = B_z/B_0$ for the field B_z directed along the z coordinate, if '-' sign is chosen that means the magnetic field is directed contrary with respect to the direction of the laser beam propagation. We assume that the laser produces continuous, monochromatic, coherent, circularly polarized and plane wave propagating in vacuum and in the same or contrary directed, additional applied static axial magnetic field. Using the not complicated procedure, from the dynamical relativistic equation (1) we can obtain differential equations describing dynamics of a charged particle in a relativistic case in the form:

$$\begin{aligned}
\frac{d(\gamma\beta_x)}{dt} &= -f\left(1-\beta_z\right)\sin\phi - fa\beta_y, \\
\frac{d(\gamma\beta_y)}{dt} &= -f\left(1-\beta_z\right)\cos\phi + fa\beta_x, \\
\frac{d(\gamma\beta_z)}{dt} &= -f\beta_x \sin\phi - f\beta_y \cos\phi, \\
\frac{d\gamma}{dt} &= -f\beta_x \sin\phi - f\beta_y \cos\phi, \\
f &= -\frac{qE_0}{m_0 c}, \\
\alpha &= -\frac{f}{\omega}
\end{aligned} \tag{5}$$

Analytical solution of equations (5) for the dependant on time the trajectories coordinates, as it has been showed in [13-15], has the following form:

$$x = \eta \sin\phi + (\eta - \xi)\sin\chi\phi,$$
$$y = \eta\cos\phi - (\eta - \xi)\cos\chi\phi - \xi, \tag{6}$$

$$z = \frac{c}{2\omega}\left\{2\vartheta^2\phi - \frac{2\vartheta^2}{1+\chi}\sin[(1+\chi)\phi]\right\}$$

where

$$\eta = \frac{\chi\xi}{1+\chi},\quad \chi = \frac{a\alpha}{\gamma_0(1-\beta_{0z})},\quad \xi = \frac{c}{a\omega},\quad \vartheta = \frac{\alpha}{1+\chi}$$

The solutions (6) are suitable for calculation of the trajectory coordinates for a charged particle initially at rest. The reduced velocity components of the particle can be found from the following equation [13, 14, 15]:

$$\beta_x = \frac{\vartheta}{\gamma}(\cos\phi - \cos\chi\phi),$$
$$\beta_y = -\frac{\vartheta}{\gamma}(\sin\phi + \sin\chi\phi), \tag{7}$$
$$\beta_z = \frac{1 + 2\vartheta^2[1-\cos(1+\chi)\phi]}{2\gamma_0\gamma(1-\beta_{0z})} - \frac{1}{2\gamma}$$

One of the most required quantity is the kinetic energy of the particle. Having the total reduced velocity β which can be calculated using equation (7), the kinetic relativistic energy can be found from the following relativistic formula

$$E_k = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \tag{8}$$

The derived analytical relations have enabled us to obtain the quantitative picture of the impact of various parameters on the charged particle trajectory, its shape, size and direction of the motion and the kinetic energy of the particle.

As the electric component acts on the particle its velocity starts to increase in a transverse direction. Due to the circular polarization, the trajectory of the particle is deviated in a plane perpendicular to the axis of the laser beam. At the same time, due to the magnetic component of the laser beam field, the Lorentz force acts on the particle. It causes a deviation of the trajectory in the direction of the beam propagation. As a result of these forces, the particle moves along a helical trajectory. If the velocity of the particle increases, then the radius of the spiral rises.

The application of an additional static magnetic field in the co-axial direction causes a reduction of the radius of the helix. This qualitative description holds during the first half of the period of the electromagnetic wave oscillation. At the next half the electric field begins to reduce the particle velocity. This explains the oscillatory shape of the velocity and energy of the particle. In the relativistic region where due to the increase of the particles mass combined with the optical Doppler effect the oscillatory shape of the velocity variation no longer exists.

The acceleration model makes it possible to predict qualitative trends in the behavior of the parameters of the particle motion in the laser and static magnetic fields.

3. Trajectory and kinetic energy of accelerated particles

Let us start with acceleration of an electron. The electrons or other particles can be injected into the laser beam. The electron source can be the well known electron gun or other sources for example the electron emissive layers such as SnO_2 or ITO [17]. Set of differential Eqn. (6-8) allows us to perform the process of solution.

The analytical equations (6-8) were used to calculate the time dependence of the particle's trajectory, velocity and kinetic energy. According to the above analysis, the particles motion is affected by the circularly polarized laser beam and the static magnetic field. In order to perform verification of the obtained analytical solutions (6-8), the Runge-Kutta solving procedure has been applied to solve the original differential equations (5). The satisfactory results have been obtained.

The acceleration process of an electron, proton and deuteron was numerically analyzed. As the initial conditions for velocity, the V_z component of velocity has been chosen zero as well as the remaining velocity and position components were chosen to be zero in all presented cases. During this process the largest component of the particles velocity was found to be in the direction of the laser beam (the coordinate z). We start with the illustration of the trajectory shape along which the particle moves if it is subjected to the acceleration process. We have chosen an electron as an example (Fig.1).

a) b)

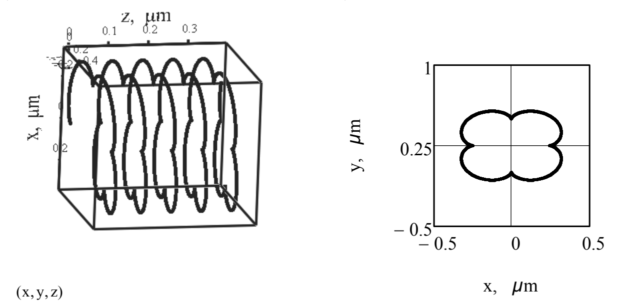

Fig. 1. (a) The stable form of the electron trajectory in 3D space, (b) the projection of the trajectory (epicycloid) onto the (*x, y*) plane as a result of 0.85 ps lasted acceleration in a laser beam and a constant co-axial magnetic field. Parameters: E_0 = 10^{10}V/m, B_z = -214 T, λ = 10 µm, b = 4/5

It should be admitted that other particles present qualitatively the similar trajectories. The shape of the helical trajectory is not smooth. The regularly located arcs appear on the helical trajectory (4 arcs in Fig. 1). At specific values of the static magnetic field B_z the stable in time form of the trajectory appears. Particular attention is given to these cases of fixed-line trajectories drawn as projections of the three-dimensional trajectory onto the plane (x, y). The stable in time geometric figures (hypocycloids or epicycloids) are obtained. Fig. 1 shows one of the examples of these shapes in the epicycloid form with $b = 4/5$ used in the equation (4).

The hypocycloid or the epicycloid curves appear at certain precise values of the parameter b defined by Eq. (4). When the fractural parameter values b = 3/2, 5/4, 7/6 … the hypocycloid curves appear and when b = 2/3, 4/5, 6/7 … we have the epicycloid curves, and the curves have 3, 5, 7 … concave or 2, 4, 6… convex arcs, respectively. These values of b determine B_z at which the hypocycloid, or epicycloid as a closed curves appear, which can be calculated from the expression (4). At other values of b the projection does not show any regularity. In Fig. 1b is shown the projection of the trajectory on to the (x, y) plane.

The projection of the trajectory onto the plane (x, y) is so important because it shows how many times the particle achieves its maximum kinetic energy during a single rotation around the laser beam axis. Since a single arc corresponds to the process of gaining the energy and loosing it by the particle. This example shows for electron the shape for right-handed polarization, and the static magnetic field directed contrary to the direction of propagation of the laser beam ($B_z <$ 0). The change of the B_z direction to the opposite one ($B_z >$ 0) results in another form of the drawings. For positive charged particle the opposite directions of B_z should be used.

Fig. 2 illustrates the projection of a stable trajectory of the electron on the (x, y) plane in the form of hypocycloid. It is obtained by using positive values of B_z found from the formula (4) with $b = 5/4$. However, in this case the acceleration was found to be less effective than for the negative B_z values.

a) b)

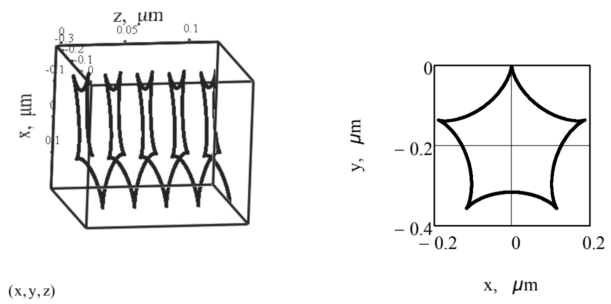

Fig. 2. (a) The stable form of the electron trajectory in 3D space and (b) the projection of the trajectory (hypocycloid) onto the (x, y) plane as a result of 0.637 ps lasted acceleration in a laser beam and a constant co-axial magnetic field. Parameters: $E_0 = 10^{11}$V/m, B_z = +268 T, λ = 10 μm, b = 5/4

By increasing the power of the laser beam the size of the trajectory can be increased in the case of hypocycloid or epicycloid. This change is proportional to the change in the amplitude of the electric field.

a) b)

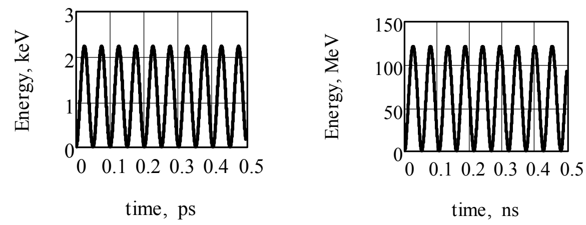

Fig. 3. Oscillations of (a) the electron and (b) proton kinetic energy in the electromagnetic field of laser radiation λ = 10 μm with the parameters: p = 2/3 (a) $E_0 = 10^{10}$V/m, B_z = -357 T, and (b) $E\text{-}_0 = 10^{11}$V/m, B_z = +656 T

Before reaching the area of resonance, the duration of the particle acceleration does not essentially affect the value of the kinetic energy. Each time the energy reaches the same maximum in the oscillation manner, then decreases almost to zero and so on. This effect is illustrated in Fig. 3 for an electron and a proton. The maximum energy increases with the magnetic field B_z and with the increase of the electric field amplitude E_0. Both the amplitude E_0 and the magnetic field induction B_z affect the rate of growth of the energy of the accelerated particles.

4. Comparison of the acceleration process of an electron, proton and a deuteron

Due to the difference in the masses of the considered particles the velocities rise in a different manner and especially at a different rate (not shown in figures). The main difference in acceleration process between an electron, proton and a deuteron lays in the rate at which velocity is increasing. In the acceleration process the most important item to discuss is the process of the kinetic energy gaining by the charged particles. The Fig. 4 shows variation in time of the kinetic energy of an electron and proton during acceleration. As it follows from Fig.4, due to the inertia a proton initially gains the energy at a smaller rate than an electron. At the used amplitude E_0 the energies of the electron and proton become nearly equal after 13 ps of the acceleration process.
When the acceleration continues the proton energy advances at a grater rate than the electron energy, which is rather unexpected. Partly it can be explained on the basis of inertia differences. The same laser intensity causes an electron to gain energy at a higher rate than the gaining rate of the heavier particles.

However, due to the rapid approaching by an electron the velocity of light and rising its mass the further rate of gaining the energy starts to decrease. While the velocity of a proton rises in much slower rate than it is for electron. The energy of these particles starts to the quick rise later than for electron and this rapid grow of energy period for heavier particles occurs later. This explains the higher rate of energy gaining by an electron than by a proton and deuteron at the beginning of acceleration. When the acceleration is advancing the opposite holds. Fig. 5 presents the oscillatory shape of the energy gained by a proton under action of smaller wavelength maser. Lower maximal energy than shown in Fig. 4 is connected with the shorter wavelength in spite of larger B_z.

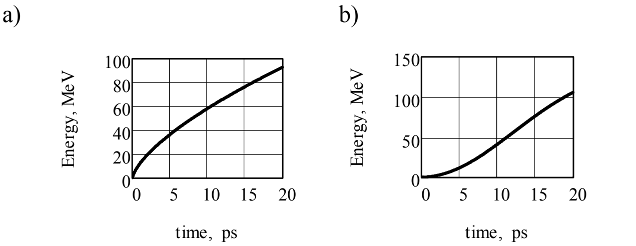

Fig. 4. Comparison of the acceleration process of (a) electron and (b) proton under the resonance conditions in the electromagnetic field of maser radiation with the parameters: $E_0 = 10^{11}$V/m, $\lambda = 1$ cm, $p = 2/3$ a) $B_z = -0.36$ T and b) +656 T

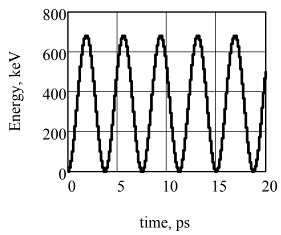

Fig. 5. Variation of a proton kinetic energy in the electromagnetic field of the maser radiation and a constant magnetic field, parameters; $\lambda = 1$ mm, $E_0 = 10^{11}$V/m, $B_z = 2185$ T

5. The resonance

For epicycloids an interesting range was found near the saturation region (the parameter p approaches zero) of the constant magnetic induction B_z calculated from the formula (4). This is the region of the resonance condition manifesting the most effective transfer of energy from the electromagnetic field to the accelerated particles [23]. The change in the constant magnetic field in the area close to the resonance value significantly affects the growth rate of the absorbed energy by the accelerated particle. Such a magnetic field approaches the cyclotron magnetic field B_c. In this case an efficient transfer of the energy from the electric field of an electromagnetic wave to a particle takes place. In a state of resonance the maximum absorption of energy by the particle occurs. The hypocycloids case will not be discussed. The energy gain by the charged particle at this condition is much less efficient due to the contrary action at the particle of electric and magnetic fields.

The electron trajectories subjected to the radical changes occurring in the conditions of resonance are accompanied by a huge increase of the energy transferred from the laser or maser radiation. In Fig. 6 the trajectory shape and the course of the energy gaining by an electron under maser radiation are presented. There is an apparent difference compared to the results obtained at the conditions before the resonance range. First of all the 3D trajectory does not show any arcs, the trajectory appears to be smooth. Secondly, the spiral has no longer a constant radius. It starts from zero and then continues to rise during the entire acceleration time. The energy as it was shown above indicates no oscillations.

Under resonance conditions a constant magnetic field B_z was found to be a relatively high for short wavelengths and a much lower for a maser beam. Now the kinetic energy of the electron increases continuously and shows no oscillations and no saturation.

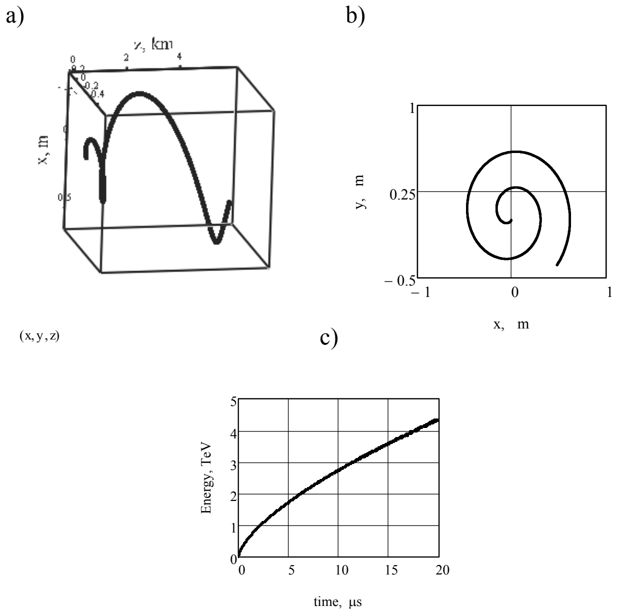

Fig. 6. (a) The electron trajectory in 3D space, (b) the projection of the trajectory onto the (x, y) plane and (c) the kinetic energy gained as a result of 20 µs lasted acceleration in a maser beam and a constant co-axial magnetic field under the resonance conditions. Parameters: $E_0 = 10^{12}$V/m, $B_z = -10.7$ T, $\lambda = 1$mm, $p = 0.001$

According to Fig.6 the combined effects of the maser electric field of the strength $E_0 = 10^{12}$V/m, and the constant longitudinal magnetic field of the induction -10.7 T, transfers the kinetic energy equal to 4.3 TeV to the electron during 20µs. One could state that this acceleration duration appears to be a bit too long and the field E_0 seems to be to high. Unfortunately it is not possible to reduce this time at the parameters shown in this example. The improvement appears to be possible with the application of maser with a longer wavelength.

A similar illustration concerning an electron acceleration in the laser beam is shown in Fig. 7. Due to the much larger field B_z the helix radius appears to be significantly reduced (not shown in Figure).

When lasers with electromagnetic fields of different intensities are used the changes in energy of the particle and in the shapes of trajectories are expected. Using the masers instead of lasers appears to be the most interesting parameter of the accelerator from the application point of view.

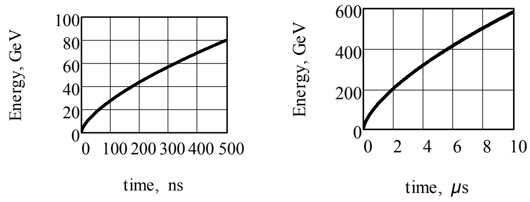

Fig. 7. The increase in electron kinetic energy under the resonance conditions in the electromagnetic field of laser radiation with the parameters: $E_0 = 10^{11}$V/m, $B_z = -1071$ T, $\lambda = 10$ μm, $p = 0.0001$.

In order to indicate what can be expected from the acceleration of protons using the masers, in Fig. 8 the impact of longer wavelength on the energy gained by a proton at two different durations of the process of acceleration is illustrated. A possibility to achieve a proton of a very large energy is apparent. However, for parameters used in Fig. 8 some experimental difficulties in the laboratory conditions can arise concerned among others with the rather large the maser beam diameter or large acceleration distance.

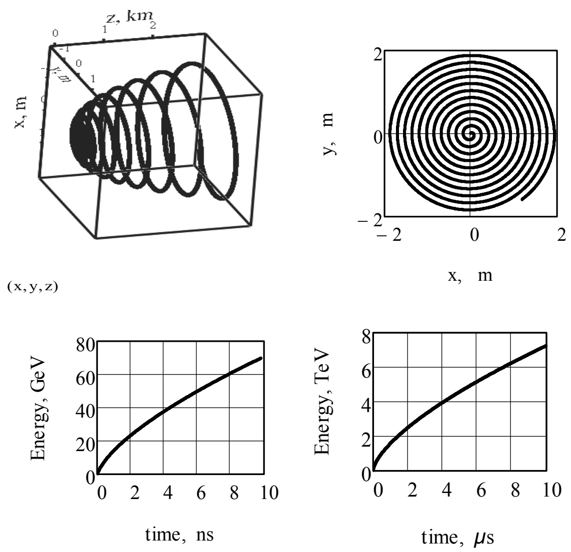

Fig. 8. The proton trajectory in 3D space, the projection of the trajectory onto the (x, y) plane (after 10 μs) and the kinetic energy gained as a result of acceleration in a maser beam and a constant co-axial magnetic field under the resonance conditions. Parameters: $E_0 = 10^{11}$ V/m, $B_z = +197$ T, $\lambda = 10$ cm, $p = 0.001$

Generally, in order to approach resonance conditions an increase of the intensity of the constant co-axial magnetic field B_z is required. In the case of an electron, the field B_z should be directed oppositely to the direction of a laser beam propagation. With the increase of its intensity the number of trajectory arcs increases and they eventually shrink and disappear. Thus, the frequency of the kinetic energy oscillation also decreases.

The main differences of each particle in the process lies in the moment the energy starts to grow rapidly. Dependence of acceleration ratio on wavelengths can be attributed to the elongation of the oscillation period of the electromagnetic wave. The longer the period the longer the electric field interacts with a particle.

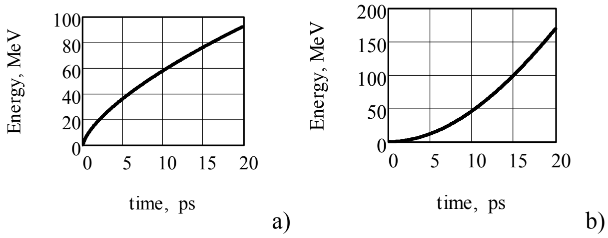

Fig. 9. Comparison of the acceleration process of (a) electron and (b) proton under the resonance conditions in the electromagnetic field of maser radiation with the parameters: $E_0 = 10^{11}$V/m, $\lambda = 1$ cm, a) $B_z = -0.107$ T and b) +1966 T

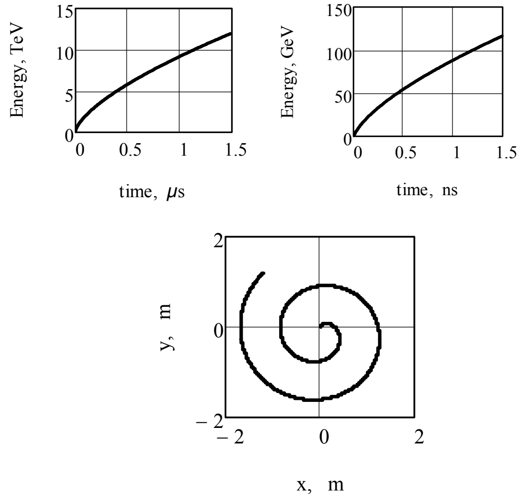

Fig. 10. Variation of a deuteron kinetic energy in the electromagnetic field of the maser radiation and a constant co-axial magnetic field for two acceleration times, parameters: $\lambda = 10$ cm, $E_0 = 10^{12}$V/m, $B_z = +396$ T

In this paper it is not examined the acceleration process with the use of the visible or ultraviolet lasers. The above results justify it since lower energies of the accelerated particles are expected. The intensity of the electric field affects the rate at which the particle gains the energy, while maintaining the co-axial magnetic field intensity close to the resonance value. The possibility of acceleration of charged particles using the electromagnetic radiation from the maser source, instead of the laser one, has been confirmed. For example, the maser operating in the wavelength of 1 mm, with the amplitude of electric field $E_0 = 10^{10}$V/m, can result in obtaining the accelerated particles of a relatively high kinetic energy. Particularly interesting is the relatively small value of the constant longitudinal magnetic induction B_z required to cause significant changes in the energy in the area close to the resonance conditions of the maser beam. A qualitative nature of the course in time of the trajectory, velocity or energy of the accelerated particles are similar to that obtained in the case of a laser. In the quantitative terms the changes in time or dependence on the field parameters of the particle trajectory or its energy are considerably different, which can be easily seen from the figures quoted. Under the conditions of resonance, as in the case of lasers, a very high kinetic energy of a particle can be achieved, however, it requires a relatively long time of the acceleration process (Figs. 9, 10).

6. Discussion of the features of the acceleration process

The acceleration process of charged particles by the laser or maser beams and an additional co-axial constant magnetic field has been analyzed. The object of a single electron, proton or deuteron of the charge q and mass m is subjected to the impact of a flat monochromatic wave of the laser radiation propagating in the positive direction of the z-axis and in the additional longitudinal and constant magnetic field B_z. The analytical equations discussed above were derived and the acceleration process of electrons, protons and deuterons have been discussed. The most important results in the discussion have been derived on the basis of the graphical presentations of the trajectories, and kinetic energies of the accelerated particles. Verification of the obtained analytical equations was carried out using the well-known Runge-Kutta procedure commonly used for solving differential equations. The results obtained by using the both methods proved to be identical. It can be concluded that when using the analytical formulas the obtained results are reliable. The analytical expressions are obtained for arbitrary values defining the parameters of laser radiation intensity and a constant co-axial magnetic field.

The analysis concerned the particle behavior in laser and maser beams during the acceleration process. The time dependent trajectories shapes and the kinetic energy variation during the acceleration process were shown in the graphical form. The obtained results indicate the differences in the acceleration process for electrons, protons and deuterons as well as the differences resulted from the application of different laser and maser as the sources of radiation. The quantitative illustrations of the calculation results enable the discussion of the influence of many parameters on the acceleration process of these particles. It is shown that the discussed values of the laser and magnetic fields parameters should be properly adjusted in order to obtain the desired energy of the accelerated particles.

Eqs (6, 7, 8) completely describe all the components of a position and velocity of the accelerated particle, as well as its total velocity and the kinetic energy under the interaction of the laser and a static magnetic field. Due to the complexity of the mechanism of the electromagnetic wave interaction with a charged particle, the presented here results describing the acceleration process are not easy to interpret. The interpretation can start with the observation that the results differ only in the kind of the accelerated particle, more precisely, in its mass. A very high electric field must be applied in order to increase the proton and deuteron velocity in a very short time compared to the oscillation period of an electromagnetic wave. A low electric field gives a proton and a deuteron relatively low velocity which changes according to the change in time of the electric field.

The analysis of the problem shows that the laser radiation of high intensity in the presence of constant co-axial magnetic field results in the trajectory of a particle being a helix circulating around the direction of the axis and a constant magnetic field. The trajectory is not smooth, it is compound of concave or convex arcs of different spatial frequencies. In the early stage of the electron acceleration caused by the action of the electric field of circularly polarized laser beam, the particle will move along the arc. The acceleration acts a half of the period, followed by a further half of a period of slowing down to zero velocity. Then the process repeats. At the same time as the electric field, the variable component of the magnetic field acts on the particle. As it is easy to find the Lorentz force results in the advancing of the electron along the z-axis.

An additional Lorentz force acts on the electron from the constant longitudinal magnetic field B_z. Taking into account its direction, it is easy to find that it will bend the electron path in the direction towards the centre. This force is responsible for the particle not escaping outside the laser beam. When the field B_z is greater, the force attracting the electron towards the centre of the helix is greater and the helix radius shrinks. Due to the high-frequency oscillations of the laser fields the particle does not keep pace with changing direction of the electromagnetic field. It is obvious that this effect increases with increasing amplitude E_0 and the mass of the accelerated particles. The influence of these factors has not been shown on the drawings.

While tracking the movement of the particles on the basis of the graphs in the (x, y) plane it should be kept in mind that the actual motion of the particles takes place in three dimensional space (x, y, z). The figures on the (x, y) plane are the projection of the trajectory onto this plane. Some examples of three dimensional trajectories are also shown. An analysis of the electron trajectories was performed on the basis of the trajectory projection onto the (x, y) plane. The projections which we discuss are stable regardless of the acceleration time. Their shape depends on the constant magnetic field B_z which is calculated on the basis of the formula (4) after substi-tuting the proper fractional value of the parameter p.

As it is shown in the figures the trajectory shapes appear to be of epicycloid or hypocycloid type. To sum it up the magnetic field B_z maintains the stability of the trajectory as well as its size. The trajectory size also depends on the amplitude of the electric field E_0. The amplitude of electric field E_0 has an impact on the duration that the charged particle gains the defined energy. The larger the amplitude the less time is required to obtain the defined level of the kinetic energy.

By changing the value of the magnetic induction, $\pm B_z$, it is possible not only to change the size of the trajectory but also to change their shapes, expressing the change in the number of arcs forming the trajectories. By changing the parameters it is possible to change not only the trajectories of the accelerated particles but also the velocity and the acceleration course of the particles. Generally speaking, the increase in the laser beam intensity results in the increase of the particle's energy and its trajectory dimension. However, the increase in the external magnetic field causes shrinking of the helical trajectories. It enables keeping the particle inside the laser beam if a sufficiently high magnetic field is applied.

Limits in the achievable energy of the accelerated particles arise from the limits in the laser beam power, available at present, the laser beam diameters, the pulse duration and its focusing and on the static magnetic field intensity. The presented figures show the examples of these parameters which are found to be different for electrons, protons and deuterons. The main differences in the acceleration process between these particles are expected due to the difference in their masses. It should be expected that for a proton it takes a longer time to achieve a defined kinetic energy than for an electron. However, as it has been shown, the opposite holds, with an exception for acceleration by very short pulses.

Achieving high kinetic energy of the particle does not necessarily require the use of magnetic field of high intensity. Kinetic energies of particles at the level of TeV can be achieved by using magnetic fields of intensities of a few Tesla, however, it requires the use of high-intensity maser radiation. In the case of the laser it is possible to reach high electron kinetic energies, but this requires the application of a constant magnetic field of rather high intensity.

As it was already mentioned, the resonance phenomenon has been associated with the equalization of two magnetic inductions, the first one being the cyclotron magnetic field of magnetic induction B_c, produced by a charged particle moving along a circular trajectory with a frequency ω, and the second being the constant longitudinal magnetic field of the magnetic induction B_z. In the case when the magnetic induction B_z approaches B_c, the frequency of oscillation of the total particle velocity and hence the oscillation frequency of the kinetic energy were found to be reduced to zero.

The charged particles acceleration process is closely connected with the distance which the particle has to surmount in order to get a desired level of kinetic energy. Since the component V_z of the electron velocity is close to the light velocity c through almost the whole acceleration process the distance $z(t)$ increases almost independently of the magnetic field or the laser beam intensity. The simple product ct is approximately equal to $z(t)$ for an electron. It is not the case for a proton, since its velocity remains rather far away from the light velocity through a significant part of the acceleration time. However, at high kinetic energies the product ct can be used to determine approximately the penetration distance of heavier particles. Of course, to achieve a lower kinetic energy the acceleration of the particle can be carried out on shorter distances. It is better for the acceleration process that a radius of the trajectory is not larger than the beam radius. The former can be controlled by the intensity of the static magnetic field. The larger the field the smaller the radius is.

In the resonance region, the projection of the particle trajectory on the (x, y) plane no longer shows the regular epi- or hypocycloid shapes but it is a rather smooth helix curve. The particle in the resonance conditions suffers a strong longitudinal acceleration gaining the kinetic energy at a significantly increased rate.

In order to maintain the acceleration, one should be sure that during the circulation of the particle in its helical motion it should not escape from the laser beam. This can be achieved by the application of the above mentioned co-axially directed static magnetic field indicating a proper intensity. The larger the field intensity the smaller the circulation radius of the particle. Especially for protons at large velocities, the magnetic field intensity should be extremely high in order to keep the particle inside the maser beam, if the acceleration process is to be continued. If such a high field is not available, the particle will leave the beam with the energy which may be sufficient for some purposes.

The amplitude the electric field E_0 has an impact on the duration at which the charged particle gains energy. The larger the amplitude the less time is required to obtain the defined level of the kinetic energy. The fact that at the boundary region of the beam the amplitude E_0 of a laser beam can be expected to be declining, could be of some help. This would result in stopping the rise of the trajectory radius. However, at this moment the acceleration process will be minimized or even interrupted.

The impact of the laser wavelength on the acceleration process has been also discussed. It has the influence on the acceleration process. The obtained results show that the longer the wavelength the more efficient acceleration is expected at the same acceleration time. The acceleration process with the use of the visible or ultraviolet lasers has not been examined in this paper. The above remarks justify it since lower energies of the accelerated particles are expected.

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