Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 43
ISSUE 1
November
2010

Providing the reliability of technical systems during the production process

Yu. Bobalo*, L.M. Kiselychnyk
Theoretical Radioengineering and Radiomeasuring Department,
Lviv Polytechnic National University, 12 S.Bandera Str., Lviv, Ukraine
* Corresponding author: E-mail address: rashkev@polynet.lviv.ua

Received 13.09.2010; published in revised form 01.11.2010

Industrial management and organisation

ABSTRACT

Purpose: of the work concerns the theoretical generalization and solving of the scientific problem of increasing the efficiency processes of technical system planning and producing, providing of the necessary level of quality by the automated control based on the mathematical design and complex optimization.
Design/methodology/approach: used the methods of probability theory and mathematical statistics, theories of matrices, theory of prognostication, analysis and control of security, methods of design and optimization of production processes, as well as methods of analysis of systems and theory of systems.
Findings: The developed variant of through modeling of matrix stream formation and transformation processes with the use of universal probability quality criterion meets requirements as a new approach for the development of optimization models and programs. The variant has no significant structural and parameter limits. Experimental-statistical research of the suggested modeling methods proved their efficiency.
Practical implications: The gained results form the theoretical engineering methodology basis of enhancement the technico-economical parameters of technical systems using automated control, modeling and complex optimization of designed processes, production and operation due to the criteria of quality and total production.
Originality/value: Original models and methods of sophisticated technical system security by the way of complex multi-critirion production process optimization are suggested. The scientific base is the developed theory of production defect stream formation at all stages and their through estimation with the help of universal criteria.
Keywords: System quality; Optimization; Reliability; Matrix modeling

Reference to this paper should be given in the following way:
Yu. Bobalo, L.M. Kiselychnyk, Providing the reliability of technical systems during the production process, Journal of Achievements in Materials and Manufacturing Engineering 43/1 (2010) 15-26.

1. Introduction

High quality and security of technical system functioning is one of the topical scientific and practical problems. In today's circumstances of tense economic state, the problem solution strategic line lies in the further development of the scientific fundamentals of the complex enhancement of the new technical and economic technique efficiency as well as in the introduction of perspective technologies of automated design process and manufacturing control with the maximum usage of their potential possibilities and rational usage of the all resources[1,2].

Imperfective project documentation of the production technology and equipment quality control results in defects that might reach the operation stage and become a potential reason of losing working capacity. The factors, which cause defects, condition the diversity of their localization during the whole process of creation and operation of the equipment, during the period of influence upon the security indices, detection and it is of importance to conduct the analysis of formation and support processes of the given technical system characteristics at all stages of their life cycle taking into account the project, production and operation defectiveness and the theoretical basis development of mathematic modeling of processes. Difference in the kind of indices of the system quality at any life cycle stage practically excludes the possibility of the complex modeling of the defectiveness appearing processes. The conducted research [3] showed that in the real conditions the probability model of the process formalization, which enables adequacy, flexibility, universality and practical usefulness of the models for their automated control, is the most acceptable.

2. Defectiveness system essence and structure

At all stages of any technical system life cycle, while performing planned and non-planed procedures of design, the technological procedures of production and system maintenance with the appropriate level at the operation stage, there is always a real chance of the defectiveness, what is characterized with the deviation of the parameters from the system norms as established. The generalized and universal quality index is accepted to be the probable index of the relative defect content of the appropriate initial product in the proposed variant of the through modeling and optimization of the quality insurance process at the design stage, serial production and operation stages. Further, the index is denoted by P_{def}, which is added to the appropriate index.

There are defects at the stage of technical system design:

The production stage defects:

- defects of materials, semi-finished products and componentries;
- defects at the shaping technological operation stage;
- the structurization operation defects;
- the assemblage operation defects;
- the defects of block and complex adjustment;
- the defects in the technological final product adjustment.

The operation stage defects.

The defectiveness that appears in the process of design, production and product usage is described by the additive function of the type:

$$P_{def}=P(P_{def.des}, P_{def.pr}, P_{def.op}), \quad (1)$$

where P_{def} - defect existence probability;
$P_{def.des}$ - defect existence probability at the design stage;
$P_{def.pr}$ - defect existence probability at the production stage;
$P_{def.op}$ - defect existence probability at the operation stage.

Let us point out that the reasons of defects, which appear during the system life cycle, are not studied equally. During the recent years, scholars and professionals have conducted the thorough scientific research, what made possible the formation of the powerful scientific sphere directed to solve the issues concerning precision assurance and production defectiveness prevention of mechanical and instrument engineering. Scientific works, which are dedicated to the project work defectiveness issues as well as the problems of system-technical design, serial production and system operation, are less known. During the development period of the 4th and 5th generation apparatus, engineering acquires the characteristics of the system-technical design, losing the features of the traditional scheme-technical one, which was typical of the previous generation.

Further, the state of products, which are characterized by any type defect existence, is estimated by the quantitative indices below:

P_{pr} - probability of defect lead-in within the performance period of formation procedures of the given final product quality indices;
P_{def} - defect existence probability having the given quality index formation procedures conducted;
P_{det} - defect detection probability resulted from the performed control;
$P_{aft.contr}$ - defect existence probability at the coming stage due to the inefficient control performed;
P - correct control probability.

The exemplified quality indices are connected with the generally accepted index of defectlessness P_{dln} via the following dependencies:

$$P_{d\ln} = 1 - \frac{P_{det}}{P}; \quad (2)$$

$$P_{d\ln} = 1 - \frac{P_{aft.contr}}{1-P}; \quad (3)$$

$$P_{dln}=1- P_{def}; \quad (4)$$

$$P_{det}= P_{def} \cdot P; \quad (5)$$

$$P_{aft.contr}=P_{def}(1-P). \quad (6)$$

Formalized subsystems of design, production and operation of the systems might be described via dependencies of their parameters from the parameters of the lower level.

Let S_{des} be the design subsystem and S_{pr} – the production subsystem:

$$\begin{aligned} &P_{prS_{pr}} = F_{prS_{pr}} (P_{pr.to1}, P_{pr.to2}, \ldots, P_{prton}),\\ &P_{\det S_{pr}} = F_{\det S_{pr}} (P_{det.to1}, P_{det.to2}, \ldots, P_{det.ton}),\\ &P_{aft.contrS_{pr}} = F_{aft.contrS_{pr}} (P_{aft.contr.to1}, P_{aft.contr.to2}, \ldots, P_{aft.contr.ton}), \end{aligned} \quad (7)$$

and S_{op} be the operation subsystem:

$$\begin{aligned} &P_{prS_{op}} = F_{prS_{op}} (P_{pr.ne}, P_{pr.rep}, P_{pr.cor}),\\ &P_{\det S_{op}} = F_{\det S_{op}} (P_{det.ne}, P_{det.rep}, P_{det.cor}),\\ &P_{contrS_{op}} = P_{contrS_{op}} (P_{contr.ne}, P_{contr.rep}, \ldots, P_{contr.cor}). \end{aligned} \quad (8)$$

The probability quality indices being used in the dependencies above are dimensionless values, what enabled the development of the mathematical modeling apparatus suitable for the optimization task solution and for the through quality assurance modeling processes at the main life cycle stages.

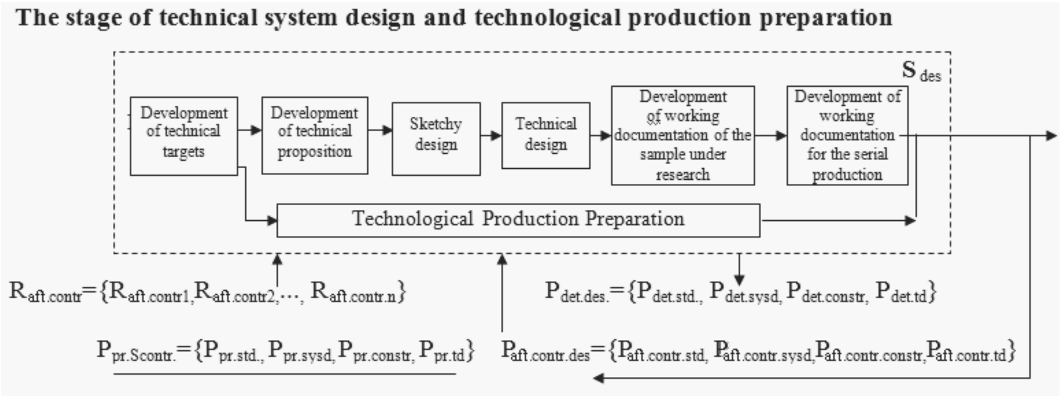


Fig. 1. The subsystem of system design and technological production preparation

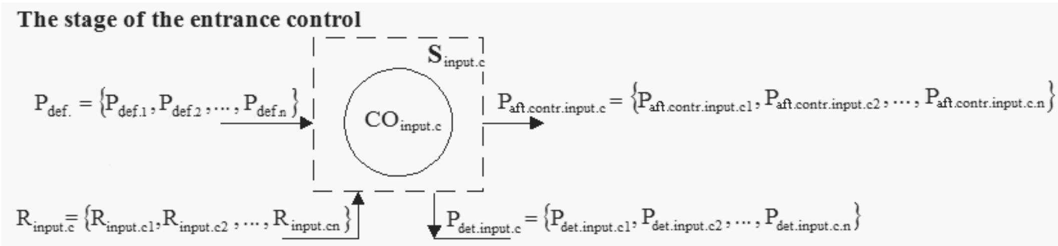


Fig. 2. The subsystem of entrance control of materials and semi-finished products

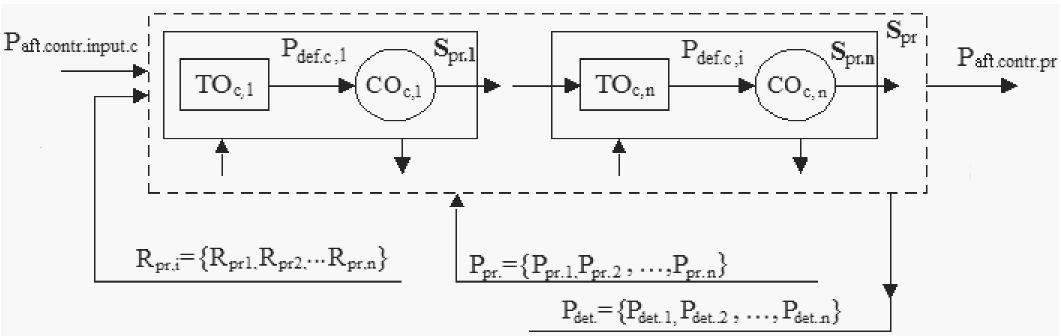


Fig. 3. Step subsystem S_{pr} of the technological $TO_{c,n}$ and control $CO_{c,n}$ procedures

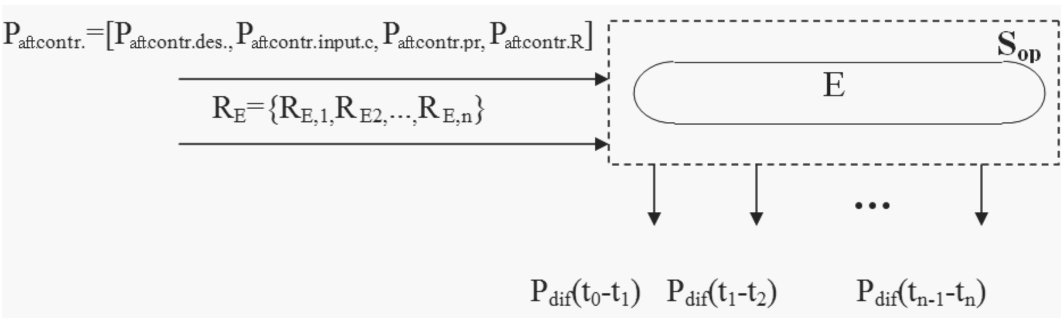


Fig. 4. The subsystem of the system operation

3. Probable process formalization of the defect detection and omission at the system life cycle stages efectiveness

The formalized structures of the design subsystem S_{des}, the entrance control $S_{input.c}$, the step subsystems of the technological and control procedures $S_{c,i}$ and the product operation subsystems S_{op} are illustrated in the figures below.

The design product quality (the subsystem $S_{aft.contr}$) is determined by the quality of scheme-technical, system-technical, constructor and technological design. The design product quality could be detected at the stages of production and product operation. The matrix of the omitted defects of the design $P_{aft.contr.des}$ takes the look of the column matrix:

$$P_{aft.contr.des} = \left\| P_{aft.contr.1} \quad P_{aft.contr.2} \quad \cdots \quad P_{aft.contr.n} \right\|^T \quad \text{(the matrix A)} \quad (9)$$

The matrix of the detected defects of the design $P_{det.des}$ is calculated analogically:

$$P_{det.des} = \left\| P_{det.1} \quad P_{det.2} \quad \cdots \quad P_{det.n} \right\|^T \quad \text{(the matrix B)} \quad (10)$$

The matrix of omission $P_{contr.input.c}$ and the matrix of detection $P_{det.input.c}$ during the entrance control of materials, semi-finished products and product components (the subsystem $S_{input.c}$) are the diagonal matrixes of the type:

$$P_{aft.contr.input.c} = \left\| \begin{array}{ccccc} P_{aft.contr.input.c.01} & & & & \\ & P_{aft.contr.input.c.12} & & & \\ & & P_{aft.contr.input.c.23} & & \\ & & & \cdots & \\ & & & & P_{aft.contr.input.c.nn} \end{array} \right\|$$

(the matrix C) (11)

$$P_{det.input.c.} = \left\| \begin{array}{ccccc} P_{det.input.c.01} & & & & \\ & P_{det.input.c.12} & & & \\ & & P_{det.input.c.23} & & \\ & & & \cdots & \\ & & & & P_{det.input.c..nn} \end{array} \right\|$$

(the matrix D) (12)

where the initial numeral index determines the technological process step and the second numeral index determines the ordinal number of the material and semi-finished product under control.

The **production stage** is considered as the set of the technological and control procedure step subsystems (Fig. 3).

The technical system production, which comprises the set of technological and control procedure combinations (the subsystem S_{pr}), is reflected by the probability matrixes of omitted and detected defects $P_{aft.contr.pr,i}$ and $P_{det.pr,i}$, which have upper triangle structure, i.e.:

$$P_{aft.contr.pr} = \left\| \begin{array}{ccccc} P_{aft.contr.pr11} & P_{aft.contr.pr21} & P_{aft.contr.pr31} & \cdots & P_{aft.contr.prn1} \\ & P_{aft.contr.pr22} & P_{aft.contr.pr32} & \cdots & P_{aft.contr.pr2} \\ & & P_{aft.contr.pr33} & \cdots & P_{aft.contr.pr3} \\ & & & & \vdots \\ & & & & P_{aft.contr.prnn} \end{array} \right\|$$

(the matrix E) (13)

$$P_{det.pr} = \left\| \begin{array}{ccccc} P_{det.pr11} & P_{det.pr21} & P_{det.pr31} & \cdots & P_{det.prn1} \\ & P_{det.pr22} & P_{det.pr32} & \cdots & P_{det.pr2} \\ & & P_{det.pr33} & \cdots & P_{det.pr3} \\ & & & & \vdots \\ & & & & P_{det.prnn} \end{array} \right\|$$

(the matrix F) (14)

The subsystem of the operation system is illustrated in Fig.4. It is possible to consider the refuse appearance process caused by the defects of design and production, which are received from the *n* step technological process.

The reasons for a product refuse during the operation process are inadequate quality of resources as well as the violation from the modes and rules of prevention and repair.

At the operation stage, the probability of defect omission $P_{aft.contr.op}$ is the main index to determine the parameters of the system security, in particular the probability of its refuses:

$P_{dif}(t)=\xi(P_{aft.contr.des}, P_{aft.contr.pr})$.

The probability matrixes of the defect omission and detection at the operation stage are following:

$$P_{aft.contr.op} = \left\| P_{aft.contr.des} \quad P_{aft.contr.input.c} \quad P_{aft.contr.pr} \quad P_{aft.contr.R} \quad P_{aft.contr.p} \quad P_{aft.contr.prof} \right\|^T$$

(the matrix G) (15)

$$P_{det.op} = \left\| P_{det.des} \quad P_{det.input.c} \quad P_{det.pr} \quad P_{det.R} \quad P_{det.p} \quad P_{det.prof} \right\|^T$$

(the matrix H) (16)

The matrixes A, C, E, G and B, D, F, H can be considered as blocks of synthesized matrixes of defect omission probabilities at the stage of design, entrance control, production and operation of the system $P_{aft.contr}$ (the matrix M) and the matrixes of defect detection probabilities at the stages P_{det} (the matrix N):

$M = A + C + E + G$
$N = B + D + F + H$

The synthesized matrixes of omitted and detected defects at all life cycle stages are of the following type:

$$P_{aft.contr}=\left\| \begin{array}{llllll} P_{aft.contr.pr.1} & & & & & \\ P_{aft.contr.2} & & & & & \\ \cdots & & & & & \\ P_{aft.contr.n} & & & & & \\ & P_{aft.contr.input.c.01} & P_{aft.contr.pr.11} & P_{aft.contr.pr.21} & P_{aft.contr.pr.31} \cdots & P_{aft.contr.pr.n1} \\ & & P_{aft.contr.input.c.12} & P_{aft.contr.pr.22} & P_{aft.contr.pr.32} \cdots & P_{aft.contr.pr.n2} \\ & & & P_{aft.contr.input.c.23} & P_{aft.contr.pr.33} \cdots & P_{aft.contr.pr.n3} \\ & & & & \vdots & \\ & & & & P_{aft.contr.input.c.n-1n} & P_{aft.contr.pr.nn} \\ & & & & & P_{aft.contr.e} \end{array} \right\|$$

(the matrix M) (17)

$$P_{det}=\left\| \begin{array}{llllll} P_{det1} & & & & & \\ P_{det2} & & & & & \\ \cdots & & & & & \\ P_{det.n} & & & & & \\ & P_{det.input.c.01} & P_{det.pr.11} & P_{det.pr.21} & P_{det.pr.31} \cdots & P_{det.pr.n1} \\ & & P_{input.pr.12} & P_{det.pr.22} & P_{det.pr.32} \cdots & P_{det.pr.n2} \\ & & & P_{input.pr.23} & P_{det.pr.33} \cdots & P_{det.pr.n3} \\ & & & & \vdots & \\ & & & & P_{input.pr.n-1n} & P_{det.pr.nn} \\ & & & & & P_{det.e} \end{array} \right\|$$

(the matrix N) (18)

The conducted research resulted in the determination of the reasons and sources of the defects, which bring down the general level of quality and security of the technical system. The approach to formalize the processes of defectiveness creation and their modeling is original, because is based upon the first proposed defect structuring, which determines and estimates the quantity of defect additive and multiple constituents. On the one side, it provides more adequate mathematical models of defectiveness in comparison with the existed ones, and, on the other side, it enables localization of the sources of defects and draws a line of control around them.

4. Analysis and modeling of the defectiveness development processes during the technical system production

Any product is produced under the conditions of the permanent destabilization factor influence upon the technological processes. In such a way, the conditions for defect emerging are created, what can be qualified as spoilage. The factors mainly are accidental and that is why the production defect emergence is an accidental event. In the paper, the defect occurrence probability serves as the evaluation of the technological process quality during the technical operation performance. The content of this kind of valuation is obvious under the condition that:

$$P_{pr.k,i}=P(X_{pr,i}>X_{k,i}^{acc}), \qquad (19)$$

where:

$X_{k,i}$ - the value of an error, which appears during the technological operation performance;

$X_{k,i}^{acc}$ - the acceptable value of the error;

$P_{pr.k,i}$ - the probability of the fact that the production error exceeds its acceptable value.

The point of the production errors is diverse. Mainly the errors made at the previous technological process steps are not fully detected and removed and move to the next stages of the technological process. For instance, all technological form creation methods are characterized with the errors of different dimensions and with the deviations of the produced element forms. Heat treatment results in deformations and lowering of the accuracy of the dimensions which are determined at the previous stages of the technological process. Furthermore, the electric and magnetic material characteristics can be changed and regenerated only partially.

While being galvanically covered, the errors made during the previous operations of preparation the high roughness surface can be diminished by the metal layer of the appropriate thickness, which is put onto the rough surface. Nevertheless, the metal creates mechanical tension and other negative effects.

The quality of any process of the formation of the given product characteristics (product elements, a node, a block and the system in general) is always characterized with the medium defectiveness level, which quantitatively is estimated by the probability of the defect $P_{pr.k,i}$ entrance during the *i* technological procedure. The defects entrance probability can be accepted as an objective quality measure of the product and of the technological operation as well as of the route and technological process in general.

The general scheme of the defectiveness formation by the subsystem S_{TOk} is illustrated in Fig. 5.

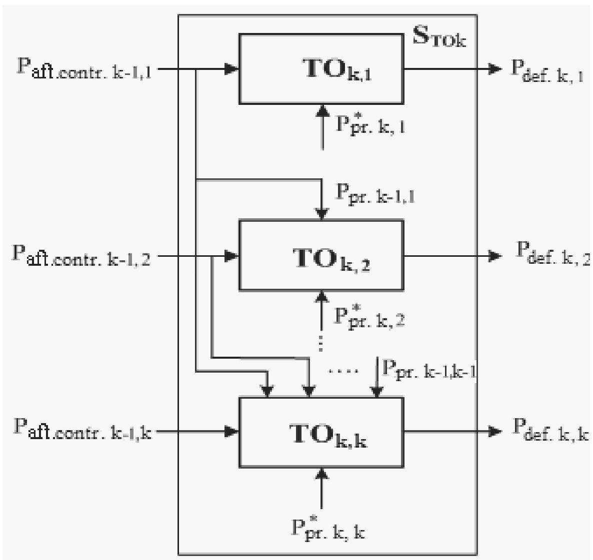


Fig. 5. The defectiveness formation scheme during the *k* technological operation performance at the *k* step of the technological process (the subsystem S_{TOk})

During the formation of the *k* parameter at the *k* technological process step, the defect entrance phenomenon is described by the function below:

$$P_{pr.k,i}=\varphi(P^{*}_{pr.k,i}\ ,\ P_{aft.contr.k-1,i}) \tag{20}$$

where:

$P^{*}_{pr.k,i}$, $i=\overline{1,k}$ - the defect entrance probability during the *i* technological operation performance at the *k* technological process step resulted from the technological equipment imperfectness, the defectiveness of the materials and other resources as well as from the staff's mistakes.

$P_{aft.contr.k-1,i}$, $i=\overline{1,k}$ - the probability of the omission of defects made and not removed at the previous technological process stages, which reached the *k* step and became the defectiveness reason at the step.

The formation scheme of the product defectiveness omitted at the *k* technological process step is illustrated in Fig.6.

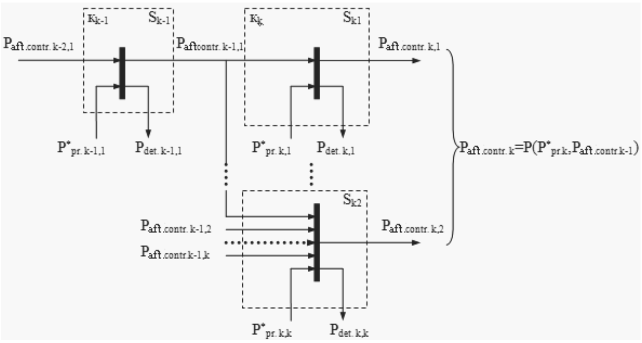


Fig. 6. The formation scheme of the product defectiveness omitted at the *k* technological process step

The total value of the defect omission probability $P_{aft.contr.k}$ at the technological process *k* step is illustrated in the Fig. above as the Petri's network and is calculated by the probability summing up of the partial probabilities of the omitted defects accordingly to every formed parameter.

$$P_{aft.contr.k} = P_{aft.contr.k,1} \oplus P_{aft.contr.k,2} \oplus \ldots \oplus P_{aft.contr.k,k}, \tag{21}$$

where:

$\oplus$ - the symbol of the probability summing up.

Analyzing the defectiveness formation processes, it is essential to consider one more reason of their emergence. Numerous materials used in the system production technological processes influence in different ways upon the defectiveness emergence. The largest part of them forms the group of the main and substantial materials, which are used for the production of the product elements and for the bearing structure nodes, mechanisms and commutation devices. The quality of the products depends directly on the quality of the materials used. The alloys of iron, copper, aluminium, nickel, cobalt, chromium and of other metals as well as plastic, ceramics, ferrite, semi-conductors and other materials belong to the group of materials.

The other part of the materials, which are traditionally called the additional materials, is used to ensure the normal technological process mode and to gain the given product characteristics. They are gases, reagents for cleaning and etching, electrolytes, menstrua, suspensions, etc. Though the materials don't create the material base for the product elements, they determine the formation process of their characteristics and impact substantially upon the product defectiveness emergence.

Taking into consideration the exemplified above, the probability $P_{aft.contr.k-1}$ is the complex function, which arguments are partial probabilities of the omission of the defects, which appeared during the performance of the technological operations at the previous production process stages, as well as the probabilities of the omission of the defects of the main and additional materials, which are used at the *k* step:

$$P_{aft.contr.k-1}=P(P_{aft.contr.k-1,1};\ P_{aft.contr.k-1,2};\ \ldots,\ P_{aft.contr.k-1,k-1};P_{aft.contr.o.k-1};P_{aft.contr.add.k-1}) \tag{22}$$

In the dependency above $P_{aft.contr.o.k-1}$ is the probability of the omission of the defects of the materials, semi-finished products and of the product elements, which are used during the *k* technological operation at the *k* technological process step.

$P_{aft.contr.add.k-1}$ stands for the probability of the omission of the additional material defects.

The formation scheme of the product defectiveness with the probability $P_{def.k.k}$ of the k technical operation ($TO_{k,k}$) at the *k* technological process step is illustrated in Fig.7.

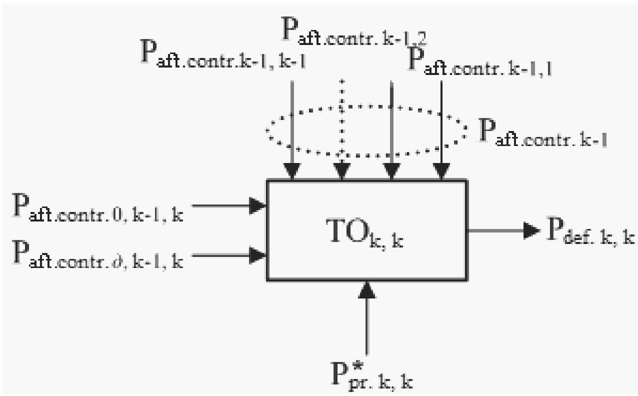


Fig. 7. The defectiveness formation scheme of the *k* technological operation performance

The probability $P_{def.k.k}$ is mainly the increasing dependency, which change character is determined by the partial influence of the omitted defects from the previous technological process steps upon the *k* step technological operation performance quality.

Taking into consideration the conducted research, it is established that the impact of the previously made defects upon the next technological operation performance quality, which causes the phenomenon of emergency, can be considerable and essentially differs.

It is necessary to note that the impact is insufficiently studied today from the theoretical and practical view point, what keeps back the creation of the through mathematical model of the product quality security and the optimization of the processes in compliance with the quality criterion and the contemporary demands.

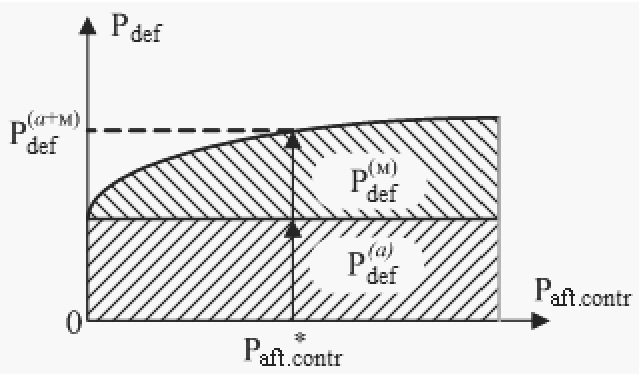


Fig. 8. The product defectiveness structure after *k* technological operation performed

The examples of forming defectiveness with the additive and multiplicative constituents are illustrated in Fig. 9 and Fig. 10. These dependencies are the results of the conducted active and passive experiments at different stages of the technological process of the technical system production.

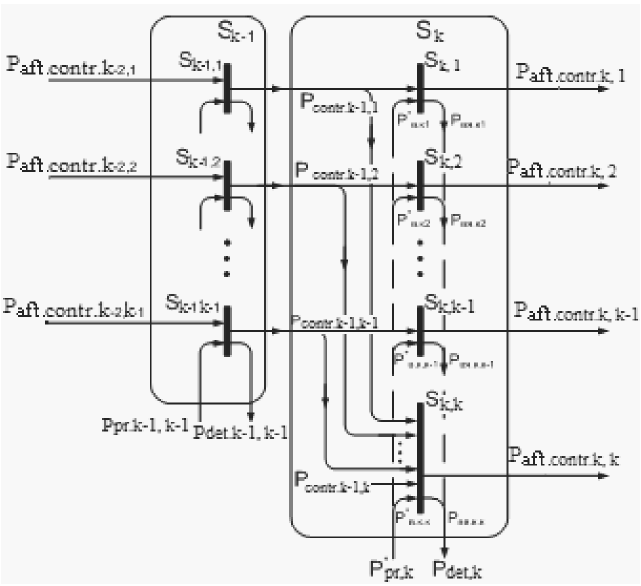
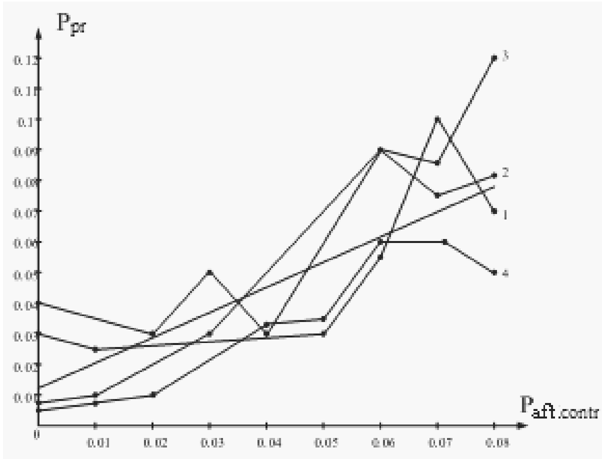
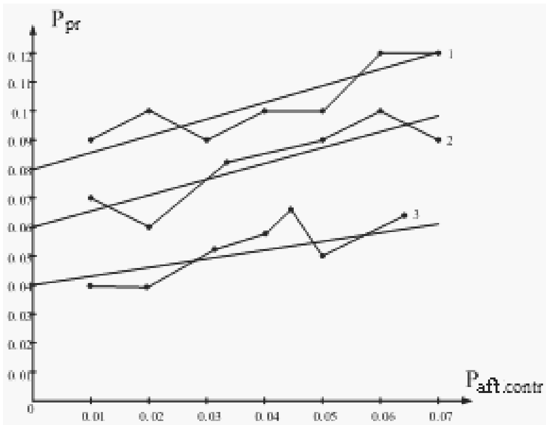


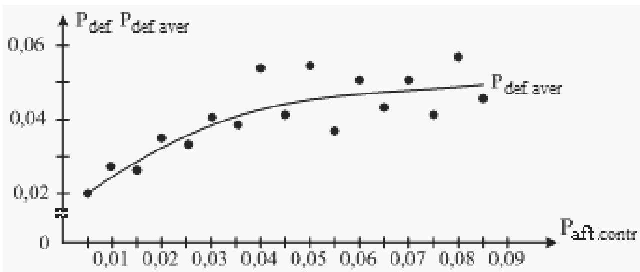
Fig. 9. The defectiveness formation scheme at the level of subsystems S_{k-1} and S_k



Pв defect entrance probability dependence of the galvanic copper building-up upon the probability of the chemical copper coating defect omission



Pв defects entrance probability dependence of the galvanic zinc covering upon the defect omission probability of the chemical surface cleaning



Galvanic copper covering defectiveness dependence upon the activation defectiveness

Fig. 10. Experimental research defectiveness formation

The defectiveness formation scheme at the level of the subsystems S_{k-1} and S_k.

The additive defectiveness:

$P_{def.k(A)}=[F_{def.k1(A)}(P^*_{pr.k1}, P_{aft.contr.k-1,1}); F_{def.k2(A)}(P^*_{B.K2}, P_{aft.contr.k-1,2})\ldots$

$\ldots F_{def.k.k(A)}(P^*_{pr.k.k}; P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2}; \ldots P_{aft.contr.k-1,k})]^T$.

The multiplicative defectiveness:

$P_{def.k(m)}=[\psi_{def.k1(m)}(P^*_{pr.k1}, P_{aft.contr.k-1,1}); \psi_{def.k2(M)}(P^*_{pr.k2}, P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2})\ldots$

$\ldots\psi_{def.k.k(M)}(P^*_{pr.k.k}; P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2}; \ldots P_{aft.contr.k-1,k-1})]^T$.

The defectiveness indices of the S_k subsystem exit:

$P_{aft.contr.k(A)}=P_{def.k(A)}\cdot(1-P_{k(A)})$;
$P_{det.k,i(A)}=P_{def.k(A)}\cdot P_{k(A)}$;
$P_{aft.contr.k(M)}=P_{def.k(M)}\cdot(1-P_{k(M)})$;
$P_{det.k,i(M)}=P_{def.k(M)}\cdot P_{k(M)}$;
$P_{aft.contr.k(A+M)}=P_{aft.contr.k(A)}\oplus P_{aft.contr.k(M)}$;
$P_{det.k(A+M)}=P_{det.k(A)}\oplus P_{det.k(M)}$.

The defectiveness at the κ technological process step under the condition of the compatibility of the previous step defect omission events and the entrance of the defects at the k step is determined by the dependency below:

$$P_{def.k,k}=P_{aft.contr.k-1,k,o}+(1-P_{aft.contr.k-1,k,o})P_{pr.k.k}. \quad (23)$$

The product defectiveness additivity, which is presented by the formula, is determined by summing up the omission probability of the defects of the main materials and semi-finished products as well as of the product elements and other products from the previous technological process stages, and the probability $P^*_{pr.k,k}$ of the defect entrance during the κ technological operation performance.

The defectiveness multiplicativity is determined by the character of the probabilities $P_{pr.k,i}, i=\overline{1,k}$, which are described (3.53) by the following dependancies:

$$P_{pr.k,1}=\varphi_1(P^*_{pr.k,1}, P_{aft.contr.k-1,1});$$
$$P_{pr.k,2}=\varphi_2(P^*_{pr.k,2}, P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2}); \quad (24)$$
$$P_{pr.k,k}=\varphi_k(P^*_{pr.k,k}, P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2}, \ldots, P_{aft.contr.k-1,k-2})$$

The practical and theoretical aspects of these dependencies are not studied today, though it is obvious that they are necessary during the optimization problem solving.

The empiric functions that describe these dependencies should correspond to the conditions below:

$P_{pr.k,k}=P^*_{pr.k,k}$ при $P_{aft.contr.k-1,i}=0,\ i=\overline{1,k-1}$;
$P_{pr.k,k}=0$ при $P^*_{pr.k,k}=0,\ \forall P_{aft.contr.k-1}=[0,1]$;
$\lim P_{pr.k,k}=1,\ \forall P_{aft.contr.k-1,i}=[0,1]$;
$P^*_{pr.k,k}\to 1$.

As a result of the conducted experimental research it is found out that during the serial technical system production these dependences are described by the following formula

$$P_{pr.k,k}=1-(1-P^*_{pr.k,k})\exp[-K_a P^*_{pr.k,k}(1-P^*_{pr.k,k})P_{aft.contr.k-1,i}], \quad (25)$$

where K_a – the adaptive coefficient that is calculated as follows

$$K_a=\frac{\ln\left(\dfrac{1-P^*_{pr.k,k}}{1-P_{pr.k,k}}\right)}{P^*_{pr.k,k}(1-P^*_{pr.k,k})P_{aft.contr.k-1,k}}.$$

Taking all the above into consideration, the dependency (25) can be presented either via the total influence of the partial defectivenesses coming from the previous steps of the assumption about their equal ponderability or via the actual ponderability of every dependency, i.e.

$$P_{pr.k.k}=1-(1-P^*_{pr.k,k})\exp[-K_a P^*_{pr.k.k}(1-P^*_{pr.k.k})P_{aft.contr.k-1,\Sigma}], \quad (26)$$

where:

$$P_{aft.contr.k-1,\Sigma}=P_{aft.contr.k-1,1}\oplus P_{aft.contr.k-1,2}\oplus\ldots\oplus P_{aft.contr.k-1,k,\partial} \quad (27)$$

or:

$$P_{pr.k.\Sigma}=K_1P_{pr.k,1}\oplus K_2P_{pr.k,2}\oplus\ldots\oplus K_{k,o}P_{pr.k,k,o}\oplus P_{k,\partial}P_{pr.k.k,\partial}, \quad (28)$$

where:

$P_{aft.contr.k-1,\Sigma}$, $P_{pr.k.\Sigma}$, P_Σ - the total values of probabilities;
$\oplus$ - the operation symbol of the probability summing up, which is calculated by the scheme:

$$P_\Sigma=K_1P_1\oplus\ldots\oplus K_nP_n=\sum_i K_iP_i-\sum_{i,j}K_iP_iK_jP_j+ +\sum_{i,j,k}K_iP_iK_jP_jK_kP_k-\ldots+(-1)^{n-1}K_1P_1K_2P_2\ldots K_nP_n \quad (29)$$

$K_i,\ i=\overline{1,k-1}$ $K_{k,o}$, $K_{k,\partial}$ - the ponderability coefficients, which are calculated by the formulae:

$$K_i=\frac{P_{pr.k,\Sigma}dP_{pr.k,i}}{P_{pr.k,i}dP_{pr.k,\Sigma}}, i=\overline{1,k}; \quad (30)$$

$$K_{i,o}=\frac{P_{pr.k,\Sigma}dP_{pr.k,k,o}}{P_{pr.k,k,o}dP_{pr.k,\Sigma}}; \quad (31)$$

$$K_{i,\partial}=\frac{P_{pr.k,\Sigma}dP_{pr.k,k,\partial}}{P_{pr.k,k,\partial}dP_{pr.k,\Sigma}}. \quad (32)$$

Let consider one more detected peculiarity of the defectiveness formation during the multi-step technological process performance.

In spite of the fact that during the k technological operation at every step of the technological process one quality index (i.e. k

index) is formed as a rule, it is necessary to consider the possible influence of this procedure upon the quality indices, which are formed at the previous stages. These influences are illustrated in Fig.11 as probability sets of the defect entrance – $P_{pr.k,1}$, $P_{pr.k,2}$, ... , $P_{pr.k,k-1}$, during performance of the fictitious technological operations $TO_{k,1}$; $TO_{k,2}$; ... $TO_{k,k-1}$. Moreover,

$$P_{pr.k.k,i} = P_{pr.k,k} \cdot P_{pr.k}(i)\,, \tag{33}$$

where $P_{pr.k}(i)$ – the conditional probability of the defects entrance in accordance with the *i* parameter $i=\overline{1,k-1}$ under the condition that $P_{pr.k.k}=1$ is determined statically.

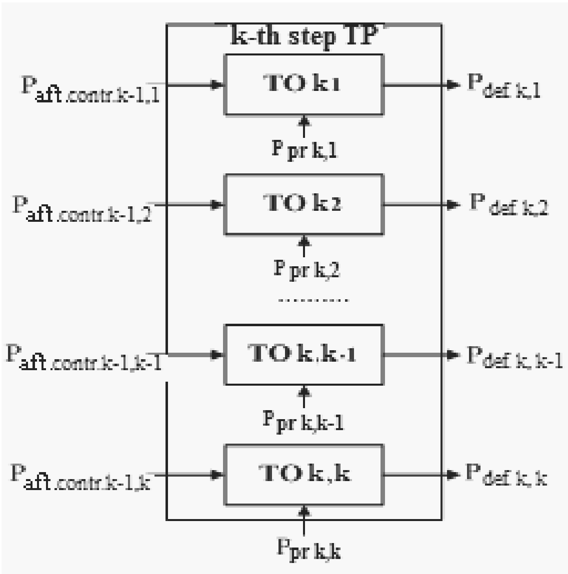


Fig. 11. The defectiveness formation scheme at the *k* step of the technological process

As a result of numerous active and passive experiments conducted under the conditions of the real production of technical system devices of different use purposes, it is indicated that taking into account the influence of defects, which emerged at the previous technological process stages and reached the *k* step, the $P_{pr.k}$ matrix of the defect introduction looks like the following:

$$P_{pr.k.} = \left\| \begin{array}{l} P_{pr.k,1} = f_{k,1}(P^{*}_{pr.k,1}, P_{aft.contr.k-1,1}) \\ P_{pr.k,2} = f_{k,2}(P^{*}_{pr.k,2}, P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2}) \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \\ P_{pr.k,k} = f_{k,k}(P^{*}_{pr.k,k}, P_{aft.contr.k-1,1}, P_{aft.contr.k-1,2}, \cdots, P_{aft.contr.k-1,k}) \end{array} \right\| \tag{34}$$

The defectiveness creation research during the separate technological operation performance proved the ambiguous influence upon the produced product defectiveness. It is claimed that some technological operations are not only the potential sources of the production defects but simultaneously they are procedures, which are characterized by the corrective characteristics concerning these defects. During the performance of such operations (e.g. chemical and galvanic covering of the normalized roughness surfaces), the defectiveness omitted at the previous technological process stages is decreased.

It is stated as well that the defectiveness of the products at any technological process step is not only determined by the possible entrance of the defect parameter, which is formed at this particular step. The poor quality technological operation performance results in creation of the defect emergence conditions because of some other parameters, in particular the parameters formed at the previous production stages. Such a group of operations comprises the technological form creation operations based upon the mechanical, thermal, chemical, electrophisical and vibrochemical processing of the materials as well as the production of any powder product components, etc. These operations form the basis for the updated technological processes of the product component production via castings under the pressure, cold and hot pressing, pressing and the production of the product elements with the use of ferrites, pyroceramics, ceramics, etc. The technological processes applying group production methods, when a set of products is processed at the *k* step, form the above group. E.g. the processes of the type are chemical and galvanic covering, processing of the silisic film plates, diffusion, thermal processing, etc. The processes are characterized by the fact that the poor quality performance of any operation can worsen the previously formed parameters of the products up to the final spoilage emergence.

Thus, the matrix $P_{pr.k}$ comprises the specified (corrected) values of the probabilities of the previous step defect omissions.
The entrance depth of the defects, which are characterized by the significant values of the defect entrance probabilities, is determined by the set of the imaginary technological processes of the *k* technological process step. The index of the defect entrance depth $A_{def.k}$ is determined under the condition:
$i \in A_{def.k}$ when $P_{pr.k}(i) \geq \nu_{def.k}$;
$i \notin A_{def.k}$ when $P_{pr.k}(i) < \nu_{def.k}$,
when $\nu_{def.k}$ - a priori established limit significant value of the probability $P_{pr.k}(i)$.

In the models of the $P_{pr.k}$ matrix product formation and control processes it is necessary to omit insignificant elements to shorten the machine time when solving the optimization problems and take into consideration the index $A_{def.k}$.

For the complete technological process of the group the defectiveness matrix P_{def} looks like the following:

$$P_{def} = \left\| \begin{array}{ccc} P_{def1,1}(P_{aftcont0,1}, P_{в.1,1}) & P_{def2,1}(P_{aftcont1,1}, P_{pr.2,1}) & \ldots\; P_{defn,1}(P_{aftcontn-1,1}; P_{pr.n,1}) \\ 0 & P_{def2,2}(P_{aftcont1,2}, P_{pr.2,2}) & \ldots\; P_{defn,2}(P_{aftcontn-1,2}; P_{pr.n,2}) \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & 0 & \ldots\; P_{defn,n}(P_{aftcontn-1,n}; P_{pr.n,n}) \end{array} \right\| \tag{35}$$

The *k* technological process step is characterized by the set of the probabilities of the existence of the defects $P_{def.k,i}$, $i=\overline{1,k}$:

$$P_{def.k,1} = P_{aft.contr.k-1,1} + (1 - P_{aft.contr.k-1,1})P_{pr.k,1};$$
$$P_{def.k,2} = P_{aft.contr.k-1,2} + (1 - P_{aft.contr.k-1,2})P_{pr.k,2}; \quad (36)$$
$$P_{def.k,k-1} = P_{aft.contr.k-1,k-1} + (1 - P_{aft.contr.k-1,k-1})P_{pr.k,k-1};$$
$$P_{def.k,k} = P_{aft.contr.k-1,k} + (1 - P_{aft.contr.k-1,k})P_{pr.k,k}.$$

When the κ technological operation is performed, the total probability of the $P_{def.k}$ defect existence is calculated according to the equation below

$$P_{def.k} = P_{def.k,1} \oplus P_{def.k,2} \oplus \ldots \oplus P_{def.k,k}.$$

Moreover, it is necessary to note that the every parameter effectiveness emergence events of the *k* technological operation process step is to be considered as a compatible and independent one. That is why the calculation of the total product defectiveness at the *k* technological process step can be made via calculating of the total probability of the appropriate quantity of the compatible and independent defect emergence events according to the product parameters:

$$P_{def.1} = P_{def.1,1};$$
$$P_{def.2} = P_{def.2,1} + P_{def.2,2} - P_{def.2,1} \cdot P_{def.2,2}; \quad (37)$$
$$P_{def.k} = P_{def.k,1} + P_{def.k,2} + \ldots + P_{def.k,k} - P_{def.k,1} \cdot P_{def.k,2} - P_{def.k,1} \cdot P_{def.k,3} - \ldots$$
$$\ldots - P_{def.k,k-1} \cdot P_{def.k,k} + P_{def.k,1} \cdot P_{def.k,2} \cdot P_{def.k,3} + \ldots$$
$$\ldots - P_{def.k,k-2} \cdot P_{def.k,k-1} \cdot P_{def.k,k} \cdot + \ldots$$
$$\ldots + (-1)^{k-1} P_{def.k,1} \cdot P_{def.k,2} \cdot \ldots \cdot P_{def.k,k}.$$

The peculiarity of the technological processes, which form the second group, is the fact that the formation of the product characteristics at the *k* step does not essentially influence the parameters, which are formed at the previous stages. (Fig.12).

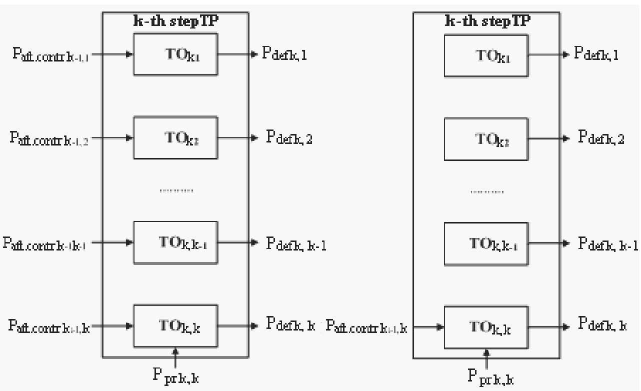


Fig. 12. Defectiveness formation scheme variants at the *k* technological process step

The group comprises the processes of chemical, galvanic and varnish-and-paint covering, pouring, steeping, pressurization, coating, etc. During these procedures the characteristics of the products, which are previously formed, do not worsen significantly.

Such processes as assembly, montage and regulation of the systems and their components are a good example of the technological processes of the type, e.g. an assembly of the system that consists of the separate devices, which are block performed. From a practical view point, the procedure of the system assembly and regulation does not influence upon the component parameters formed before. Moreover,

$$P_{aft.contr.k-1,i} \neq 0, \quad i = \overline{1,k}$$
$$P_{aft.contr.k} \neq 0;$$
$$P_{aft.contr.k,i} = 0, \quad i = \overline{1,k-1}.$$

When the *k* technological operation is performed, the constituents of the total product defectiveness are determined as follows:

$$P_{def.k,1} = P_{aft.contr.k-1,1};$$
$$P_{def.k,2} = P_{aft.contr.k1,2};$$
$$P_{def.k.k-1} = P_{aft.contr.k1,k-1};$$
$$P_{def.k,k} = P_{aft.contr.k-1,k} + (1 - P_{aft.contr.k-1,k})P_{pr.k,k}.$$

To counterbalance the 1st group, it is determined that this group of processes is characterized by the insignificant increase of the additive constituent according to the *i* parameter.

The defectiveness matrix of the 2nd group looks like the following:

$$P_{def} = \begin{Vmatrix} P_{def.1,1} = P_{aft.contr.0,1} & P_{def.2,1} = P_{aft.contr.1,1} & \cdots & P_{def.n,1} = P_{aft.contr.n-1,1} \\ 0 & P_{def.2,2} = P_{aft.contr.1,2} & \cdots & P_{def.n,2} = P_{aft.contr.n-1,2} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ 0 & 0 & \cdots & P_{def.n,n}(P_{aft.contr.n-1,n}; P_{pr.n,n}) \end{Vmatrix} \quad (38)$$

The 3rd group consists of processes, which are characterized by the little influence of the *k* technological operation upon the parameters formed previously (Fig.12). It is observed when the quality of the entrance materials, semi-finished products, product components as well as of the whole blocks is high enough not to perform the entrance control. In addition, the defectiveness permitted at the *k* technological process step is mainly determined by the quality of the *k* operation performance. The conditions of the defectiveness performance are

$$P_{aft.contr.k-1,i} = 0; \quad i = \overline{1,k-1},$$
$$P_{aft.contr.k-1,ik} \neq 0; \quad P_{pr.k,i} = 0, \; i = \overline{1,k-1},$$
$$P_{pr.k.k} \neq 0.$$

The constituents of the total defectiveness are:

$$P_{def.k,1} = 0;$$
$$P_{def.k,2} = 0;$$
$$P_{def.k,k} = P_{pr.des.k-1,k} + (1 - P_{aft.contr.k-1,k})P_{pr.k,k}.$$

$$P_{def} = \begin{Vmatrix} P_{def.1,1}=0 & P_{def.2,1}=0 & \dots & P_{def.n,1}=0 \\ 0 & P_{def.2,2}=0 & \dots & P_{def.n,2}=0 \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ 0 & 0 & \dots & P_{def.n,n}(P_{aft.contr.n-1,n};\ P_{pr.n,n}) \end{Vmatrix} \quad (39)$$

It is worth noting that from the formal view point the 1st exemplified scheme can be considered as the main universal one of the defectiveness emergence, and the 2nd and 3rd are the varieties of the first. The emergent esssence of every scheme is determined by the specific character of the technologies applied, by the characteristics of the production process organization as well as by other factors, which can differ essentially. The technological systems, which defectiveness is formed according to the 1st scheme, are characterized by the multiplicative constituent of the defectiveness in comparison with the systems, which functionality is based upon the 2nd and 3rd schemes.

5. Mathematical product defectiveness modeling of the n step technological process

The multiple step processes of the technical system production are considered as the local subsystem set $S_{k,i}$, $k=\overline{1,n}$, $i=\overline{1,n}$, which function at every step of the technological process providing the necessary characteristics of the products, which are characterized by the appropriate quality indices. The entrance parameters of the subsystems are the streams of the defects, which come from the previous steps with the probabilities $P_{aft.contr.k-1,i}$ and enter the products with the probabilities $P_{pr.k,i}$ during the performance of the technological procedures. The subsystem exit parameters are the streams of the omitted defects with the appropriate probabilities $P_{aft.contr.k,i}$ and the streams of the defects with the probabilities $P_{det.k,i}$ detected during the control process. The technological processes are mainly characterized by the consecutive, parallel and mixed system functionality.

The consecutive technological process structure is typical of the form creation operation performance, i.e. mechanical, thermal, and chemical, electroerosive and vibrochemical processing of materials, which form the base of the technological production of the major part of the contemporary technical system components.

The technological processes, which are based on the group production methods, i.e. when some set of the products is processed simultaneously at the one step, form the group of the parallel structure processes, e.g. the galvanic covering, group printed circuit card soldering, silicic plate processing, film spraying, epitaxy, diffusion, etc.

The mixed structure processes differ from the processes of a consecutive and parallel structure by the fact that they comprise the system of assembly and montage S_Σ, which functionality results in the production of the product with the general quality index, e.g. the generalized value of the defect omission probability $P_{aft.contrS_\Sigma}$ that is of the following functional view:

$$P_{aft.contrS_\Sigma} = P(P^{(1)}_{aft.contr.n,1}, P^{(2)}_{aft.contr.n,1}, \dots, P^{(m)}_{aft.contr.n,1}, P_{pr.s_\Sigma}).$$

Let us consider three variants of the typical process structure, which are described by the schemes of consecutive, parallel and mixed joints of the subsystems.

The subsystem S_{sav}, which mainly perform the function of the adder of the product quality indices formed at the previous stages of the general technological process in its parallel and consecutive routs, is characterized by the set of the own entrance and exit parameters. The subsystem entrance parameters are the probabilities of the defect omission of the *n* step routs 1, 2, … , m, i.e. the probabilities $P^{(1)}_{aft.contr.n,1}$, $P^{(1)}_{aft.contr.n,2}$, …, $P^{(1)}_{aft.contr.n,n}$, $P^{(2)}_{aft.contr.n,1}$, $P^{(2)}_{aft.contr.n,2}$, …, $P^{(2)}_{aft.contr.n,n}$, . . . , $P^{(m)}_{aft.contr.n,1}$, $P^{(m)}_{aft.contr.n,2}$, …, $P^{(m)}_{aft.contr.n,n}$, as well as the defect entrance probability during the performance of the assembly procedure – $P_{pr.sav}$. The exit parameters are the probability of the defect detection during the quality control of the assembly procedure $P_{det.sav}$ and the defect omission probability during the quality control.

Generally, the mathematical defectiveness model of the products, which are produced during the technological process of the composite structure, is the additive function of the total defectiveness $P_{aft.contr.Ssav}$, which arguments are the partial indices of the defectiveness, made along the rout:

$$P_{aft.contr.Ssav} = P^{(1)}_{aft.contr.n,1} \oplus P^{(1)}_{aft.contr.n,2} \oplus \dots \oplus P^{(1)}_{aft.contr.n,n} \oplus$$
$$\oplus P^{(2)}_{aft.contr.n,1} \oplus P^{(2)}_{aft.contr.n,2} \oplus \dots \oplus P^{(2)}_{aft.contr.n,n} \oplus \dots \quad (40)$$
$$\dots \oplus P^{(m)}_{aft.contr.n,1} \oplus P^{(m)}_{aft.contr.n,2} \oplus \dots \oplus P^{(m)}_{aft.contr.n,n} \oplus P_{pr.sav},$$

where $P_{aft.contr.sav}=P(P^*_{pr.sav}, P_{aft.contr.n.sav}, P_{sav}, P^{(1)}_{aft.conr.n,1}, P^{(1)}_{aft.contr.n,2}, \dots, P^{(1)}_{aft.contr.n,n}, P^{(2)}_{aft.contr.n,1}, P^{(2)}_{aft.contr.n,2}, P^{(2)}_{aft.contr.n,n}, \dots, P^{(m)}_{aft.contr.n,2}, \dots, P^{(m)}_{aft.contr.n,n})$.

The symbol $\oplus$ is for the probable summing up.

The exemplified probability models of the defectiveness formation processes during the serial technical system production are the adaptive models, which assembly is based upon the application of stochastic quality index dependencies on the parameters of technological and control procedures at the main production stages. They are characterized by the possibility of the consecutive approximation of their parameters to the object parameters without using such classic methods as the method of the set correlation, dispersion analysis, etc. For the real multi-step and multi-parameter processes the usage of such methods is practically impossible. The matrix structure of the given models provides a possibility to work with them in the dialog mode, what enhances the correction operability, the choice of the necessary resources and solution of the optimization tasks.

6. Conclusions

During sophisticated technical system production, many technological operations and quality parameters require a new approach for the development of optimization models and programs, which should be suitable for a technologist's computer work dialog mode. The developed variant of through modeling of matrix stream formation and transformation processes with the use of universal probability quality criterion meets these requirements. The variant has no significant structural and parameter limits. Experimental-statistical research of the suggested modeling methods proved their efficiency.

References

[1] S.H. Radchenko, Mathematical modeling of engineering technological processes, Ukrspetsmontazhproekt (1998) 274.

[2] V.I. Borschm, V.A Doniets, V.V. Koval, A.Y. Leybnoz, N.P. Liesovoy, Structure optimization of large systems, Naukova Dumka (2000) 191

[3] Y. Bobalo, M. Kiselychnyk, L. Nedostup. Analysis of quality of radio electronic devices in multistage production system, Electrical Review 86/1 (2010) 124-126.