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Application of the homotopy perturbation method for calculation of the temperature distribution in the cast-mould heterogeneous domain

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Analysis and modelling

ABSTRACT

Purpose of this paper: In this paper an application of the new method for solving the heat conduction equation in the heterogeneous cast-mould system, with an assumption of the ideal contact at the cast-mould contact point, is introduced. An example illustrating the discussed approach and confirming its usefulness for solving problems of that kind is also presented in the paper.

Design/methodology/approach: For solving the discussed problem the homotopy perturbation method is used, which consists in determining the series convergent to the exact solution or enabling to built the approximate solution of the problem.

Findings: The paper shows that the homotopy perturbation method, effective in solving many technical problems, is successful also for examining the considered problem.

Research limitations/implications: Solution of the problem is provided with the assumption of an ideal contact between the cast and the mould. In further, research of the discussed method shall be employed to solve problems involving the presence of thermal resistance at the cast-mould contact

Practical implications: The method allows to determine the solution in form of the continuous function, which is significant for the analysis of the cast cooling in the mould, in order to avoid the defects formation in the cast.

Originality/value: Application of the new method for solving the considered problem.

Keywords: Numerical techniques; Heat transfer; Cooling of the cast; Homotopy perturbation method

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1. Introduction

In the recent time a number of methods enabling to solve different kinds of physical and technical problems have found an application. Group of those methods include, among others, the Adomian decomposition method [1-4], the variational iteration method [5-11] and the homotopy perturbation method [11-19]. General mathematical formulation of the above mentioned methods allows to find a solution of the wide class of nonlinear operator equations. The idea of all of those methods consists in constructing the functional sequence, or series, which limit (or sum) represents the solution of the considered problem (under the proper assumptions). In general, the speed of convergence of the received sequences or series is quite good, thanks to which calculating only a few first terms ensures usually the satisfying approximation of the sought solution. The evidence of popularity of those three methods gives the fact that the special issues of different journals are sacrificed exactly to them (among others also the journals from ISI Master Journal List, like Computers & Mathematics with Applications and Topological Methods in Nonlinear Analysis).

The Adomian decomposition method (ADM) is named after its inventor, George Adomian, and is used for solving the different kind of problems described, for example, with the aid of partial and ordinary differential equations, integral equations and so on. In papers [20-28], ADM is used for solving the linear and nonlinear heat conduction equation. The wave equation is examined in papers [29-31], while in papers [32,33] the inverse problem for differential equations is considered. In paper [34], the authors solve, by using ADM, the fuzzy differential equations and in [35] the boundary problem for the differential equations of the higher order is considered. Another applications of ADM in examining the mathematical models describing different kinds of technical problems can be found in papers [36-38]. Whereas, convergence of the Adomian method is discussed in [20,39,40].

The other mentioned methods, variational iteration method (VIM) and homotopy perturbation method (HPM), was created by Ji-Huan He. VIM is useful for solving many different kinds of nonlinear problems. Momani and his colleagues [41] have applied VIM for finding the solution of ordinary differential equations with boundary conditions. Similarly, Dehghan and Shakeri [42] have used the described method for determining the approximate solution of some differential equation arising in astrophysics. There are also available papers in which VIM is applied for finding the exact or approximate solution of partial differential equations. For example, Momani and Abuasad [43] have used VIM for solving the Helmholtz equation and Wazwaz [44,45] has applied the method for determining the exact solutions of Laplace and wave equations. In papers [46,47], the heat-like and wave-like equations are solved, while the heat transfer and diffusion equations are examined, by means of VIM, in [48,51]. Solution of the systems of partial differential equations with the aid of VIM is presented in [52], whereas Tatari and Dehghan in [53] have used this method for computing a parameter in semi-linear inverse parabolic equation. Convergence of VIM is discussed by Tatari and Dehghan in [54]. Some new interpretations and applications of the variational iteration method are proposed by He in papers [55-57].

Homotopy perturbation method arised as a combination of elements of two other methods: the homotopy analysis method [14,58-62] and the perturbation method [16,63,64]. HPM appeared as an effective and powerful method for solving the wide class of problems. For example, Ramos [65] has applied HPM for solving the nonlinear second-order ordinary differential equations with boundary conditions. Similar application is presented in paper [66]. Solution of boundary value problems for integro-differential equations by using the homotopy perturbation method is described in [67], whereas Shakeri and Dehghan [68] have used the described method for solving the delay differential equation arising in biology and engineering. There can be also found some papers in which HPM is applied for determining the exact and approximate solutions of partial differential equations, like, for example, the nonlinear wave equations [69], the wave and the nonlinear diffusion equations [70] and the fractional wave-like equation [71]. Furthermore, Li and his colleagues in [72] have used HPM for examining the time-fractional diffusion equation with the moving boundary condition, Shakeri and Dehghan in [73] have applied the method for solving the inverse problem of diffusion equation, Sadighi and Ganji in [74] have found the exact solutions of Laplace equation and Biazar and Ghazvini in [75] have solved the hyperbolic partial differential equation by means of HPM. Finally, Ganji and his colleagues in the series of papers [48,49,63,76,77] have considered the application of HPM for solving different problems concerning the heat transfer processes. Some information about the convergence of the homotopy perturbation method can be found in papers [12,78].

Employees of the Faculty of Mathematics and Physics at the Silesian University of Technology, especially researchers of the Department of Applied Mathematics, from many years deal in their work with applying the above described methods for solving various problems concerning the heat conduction. First effect of this research become the chapter in volume [4], in which the Adomian decomposition method is presented. Application of this method for solving the heat conduction problems is also described in monograph [79]. In papers [80,81], ADM combined with some optimization procedures is used for solving the inverse one-phase Stefan problem with the boundary condition of the first and second kind. In the presented approach, the distribution of temperature in the considered domain is calculated in the ground of ADM. The received temperature distribution depends on some coefficients, values of which are determined with the aid of the mean square method. Accuracy of the procedure is verified on the basis of the exact solution. The same approach for the direct Stefan problem is presented in papers [82,83]. In the further works another approach is proposed, in which the Stefan problem is first approximated by the system of ordinary differential equations, and next, the obtained system is solved with the aid of ADM. In this way, the need of constructing and minimizing some functional, which was necessary in previous approach, can be omitted. The same way of using ADM is showed in paper [84], for the case of one-phase Stefan problem, and in [85], for the case of two-phase Stefan problem. Comparison of precision of the one-phase Stefan problem solution, received with the aid of Adomian decomposition method and Runge-Kutty method of the fourth order, is presented in [86]. In the both approach the Stefan problem is first approximated by the system of ordinary differential equations, solved afterwards by means of ADM and Runge-Kutty method. Received results demonstrate better precision of Adomian decomposition method, also the time of calculations is shorter for ADM. In paper [87], application of ADM for finding the exact solution of heat conduction equation in the cast-mould heterogeneous domain is described. Adaptation of the variational iteration method for solving that kind of problem is proposed in [88]. In works [89-93], VIM is used for solving the one-phase direct and inverse Stefan problem, whereas paper [94] presents an application of the homotopy perturbation method for determining the exact (or approximated) solution of the one-phase Stefan problem. Moreover, researchers of the *Department of Applied Mathematics* have prepared many other works concerning the direct and inverse heat conduction problems, like for example [95-107]. In part of those papers, verification of the developed methods is executed by using the experimental data received with the aid of UMSA equipment (Universal Metallurgical Simulator and Analyzer), designed for analysis of the heat processes occurring in metals [108-110]. Experimental results were received thanks to the collaboration with the employees of the *Institute of Engineering Materials and Biomaterials* in the *Faculty of Mechanical Engineering* of the *Silesian University of Technology*.

In the current paper, an application of the homotopy perturbation method for solving the heat conduction equation in the heterogeneous cast-mould system, with an assumption of the ideal contact at the cast-mould contact point is presented. An example illustrating the discussed approach and confirming usefulness of the proposed procedure for solving problems of that kind is also showed.

2. Homotopy perturbation method

By using the homotopy perturbation method, solution of the following nonlinear operator equations can be found:

$$A(u) = f(z), \qquad z \in \Omega, \tag{1}$$

where A denotes the operator, f is the given function and u is the sought function. Let us assume that the operator A can be presented as the sum:

$$A(u) = L(u) + N(u), \tag{2}$$

where L represents the linear operator and N denotes the nonlinear operator. From this, equation (1) can be written in the form:

$$L(u) + N(u) = f(z), \qquad z \in \Omega. \tag{3}$$

Let us define a new operator, named the homotopy operator, in the following way:

$$H(v,p) \equiv (1-p)(L(v) - L(u_0)) + p(A(v) - f(z)), \tag{4}$$

where $p \in [0,1]$ is the, so called, homotopy parameter, $v(z,p) : \Omega \times [0,1] \to R$, and u_0 denotes the initial approximation of solution of the equation (1). By using the relation (2) we obtain:

$$H(v,p) \equiv L(v) - L(u_0) + p\,L(u_0) + p(N(v) - f(z)). \tag{5}$$

Since $H(v,0) = L(v) - L(u_0)$, then for $p = 0$ solution of the operator equation $H(v,0) = 0$ is equivalent to solution of the trivial problem $L(v) - L(u_0) = 0$. Whereas, for $p = 1$ solution of the operator equation $H(v,1) = 0$ is equivalent to solution of the input equation. In this way, the monotonic change of parameter p, from 0 to 1, corresponds with the monotonic change of the equation, from the trivial one: $L(v) - L(u_0) = 0$ to the input form of the considered equation (and with the monotonic change of the solution v, from u_0 to u).

Now, let us assume that the solution of equation $H(v,p) = 0$ can be written in the form of the power series:

$$v = \sum_{j=0}^{\infty} p^j u_j . \tag{6}$$

If the above series is convergent, then, by substituting $p = 1$, we receive the solution of equation (1):

$$u = \lim_{p \to 1} v = \sum_{j=0}^{\infty} u_j . \tag{7}$$

Information about convergence of the series (6) is included in papers [12,78]. In many cases the speed of convergence of series (7) is great, thanks to which the sum composed from only few first terms gives already a very good approximation of the sought solution. By confining to the first n+1 elements, we get the, so called, n-order approximate solution:

$$\hat{u}_n = \sum_{j=0}^{n} u_j . \tag{8}$$

For finding the form of functions u_j we substitute the relation (6) into the equation $H(v,p) = 0$ and we compare the elements appearing by the same powers of parameter p. In this way, we receive the sequence of operator equations, which allows to determine the successive functions u_j. In this manner, solving of the input problem can be reduced to the task of solving the sequence of problems, simple to analyse.

3. Mathematical model of the problem

In the current paper we consider the problem of determining distribution of temperature in the heterogeneous cast-mould system, with an assumption of the ideal contact at the cast-mould contact point. Let us start with formulation of the mathematical model of the problem.

Let us have two regions: $D_1 = \{(x,t) : x \in [x_1,0],\, t \in [0,t^*)\}$ and $D_2 = \{(x,t) : x \in [0,x_2],\, t \in [0,t^*)\}$ (see Fig. 1).

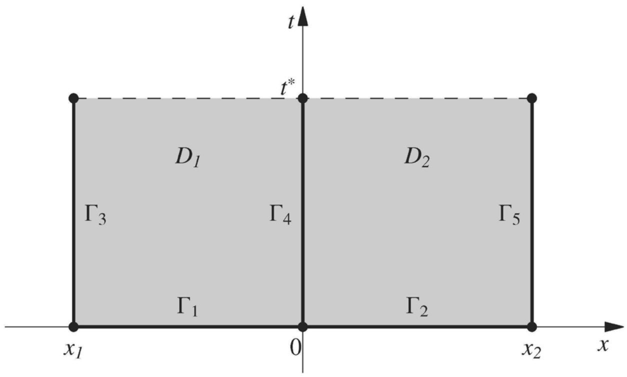

Fig.1. Domain of the considered problem

On the boundary of these domains five components are distributed:

$$\Gamma_1 = \{(x,0) : x \in [x_1,0]\},$$
$$\Gamma_2 = \{(x,0) : x \in [0,x_2]\},$$
$$\Gamma_3 = \{(x_1,t) : t \in [0,t^*)\},$$
$$\Gamma_4 = \{(0,t) : t \in [0,t^*)\},$$
$$\Gamma_5 = \{(x_2,t) : t \in [0,t^*)\},$$

where the initial and boundary conditions are given.

In the cast (region D_1) and mould (region D_2) we consider the heat conduction equations:

$$\frac{\partial u_1}{\partial t}(x,t) = a_1 \frac{\partial^2 u_1(x,t)}{\partial x^2}, \qquad (x,t) \in D_1, \tag{9}$$
$$\frac{\partial u_2}{\partial t}(x,t) = a_2 \frac{\partial^2 u_2(x,t)}{\partial x^2}, \qquad (x,t) \in D_2, \tag{10}$$

where a_i, $i = 1,2$, are the thermal diffusivity, u_i, $i = 1,2$, denote the temperature, and t and x refer the time and spatial location, respectively. On boundaries Γ_1 and Γ_2 the initial conditions are given:

$$u_1(x,0) = \varphi_1(x), \qquad x \in [x_1,0], \tag{11}$$
$$u_2(x,0) = \varphi_2(x), \qquad x \in [x_2,0]. \tag{12}$$

On boundaries Γ_3 and Γ_5 the Dirichlet conditions are determined:

$$u_1(x_1,t) = \psi_1(t), \qquad t \in [0,t^*), \tag{13}$$
$$u_2(x_2,t) = \psi_2(t), \qquad t \in [0,t^*). \tag{14}$$

And finally, on the cast-mould contact boundary (boundary Γ_4) the boundary conditions of the fourth kind are known (condition of temperature continuity and condition of heat flux continuity):

$$u_1(0,t) = u_2(0,t), \qquad t \in [0,t^*), \tag{15}$$
$$-\lambda_1 \left.\frac{\partial u_1(x,t)}{\partial x}\right|_{x=0} = -\lambda_2 \left.\frac{\partial u_2(x,t)}{\partial x}\right|_{x=0}, \qquad t \in [0,t^*), \tag{16}$$

where λ_i, $i = 1,2$, denote the thermal conductivity. Additionally, we assume that functions describing the considered problem satisfy the consistency conditions:

$$\varphi_1(x_1) = \psi_1(0), \qquad \varphi_1(0) = \varphi_2(0),$$
$$\varphi_2(x_2) = \psi_2(0), \qquad -\lambda_1 \frac{\partial \varphi_1}{\partial x}(0,0) = -\lambda_2 \frac{\partial \varphi_2}{\partial x}(0,0).$$

We seek the functions $u_1(x,t)$ and $u_2(x,t)$ defined in domains D_1 and D_2, respectively, which satisfy the heat conduction equations together with the above presented conditions.

4. Solution of the problem

Let us start with defining the homotopy operators for equations (9) and (10). The proper operators have the form (for $i = 1,2,$):

$$H_i(v_i, p) \equiv \frac{\partial^2 v_i}{\partial x^2} - \frac{\partial^2 u_{i,0}}{\partial x^2} + p\left(\frac{\partial^2 u_{i,0}}{\partial x^2} - \frac{1}{a_i}\frac{\partial v_i}{\partial t}\right). \tag{17}$$

Solutions of the equation (for $i = 1,2,$):

$$H_i(v_i, p) = 0 \tag{18}$$

will be sought in the form of the power series of variable p:

$$v_i = \sum_{j=0}^{\infty} p^j u_{i,j}. \tag{19}$$

By substituting the relation (19) into the equation (18) (and by using definition (17)) we receive (for $i = 1,2,$):

$$\sum_{j=0}^{\infty} p^j \frac{\partial^2 u_{i,j}}{\partial x^2} - \frac{\partial^2 u_{i,0}}{\partial x^2} + p\left(\frac{\partial^2 u_{i,0}}{\partial x^2} - \frac{1}{a_i}\sum_{j=0}^{\infty} p^j \frac{\partial u_{i,j}}{\partial t}\right) = 0, \tag{20}$$

or, equivalently:

$$\sum_{j=0}^{\infty} p^j \frac{\partial^2 u_{i,j}}{\partial x^2} = \frac{\partial^2 u_{i,0}}{\partial x^2} - p\frac{\partial^2 u_{i,0}}{\partial x^2} + \frac{1}{a_i}\sum_{j=1}^{\infty} p^j \frac{\partial u_{i,j-1}}{\partial t}. \tag{21}$$

By comparing the elements occurring by the same powers of parameter p we obtain the following systems of equations:

$$\begin{cases} \dfrac{\partial^2 u_{1,1}}{\partial x^2} = \dfrac{1}{a_1}\dfrac{\partial u_{1,0}}{\partial t} - \dfrac{\partial^2 u_{1,0}}{\partial x^2}, \\ \dfrac{\partial^2 u_{2,1}}{\partial x^2} = \dfrac{1}{a_2}\dfrac{\partial u_{2,0}}{\partial t} - \dfrac{\partial^2 u_{2,0}}{\partial x^2}, \end{cases} \tag{22}$$

and for $j \geq 2$:

$$\begin{cases} \dfrac{\partial^2 u_{1,j}}{\partial x^2} = \dfrac{1}{a_1}\dfrac{\partial u_{1,j-1}}{\partial t}, \\ \dfrac{\partial^2 u_{2,j}}{\partial x^2} = \dfrac{1}{a_2}\dfrac{\partial u_{2,j-1}}{\partial t}. \end{cases} \tag{23}$$

Systems of partial differential equations (22) and (23) must be additionally completed with the boundary conditions, which ensure the uniqueness of solution. So, for the system (22) we put the conditions:

$$\begin{cases} u_{1,0}(x_1,t) + u_{1,1}(x_1,t) = \psi_1(t), \\ u_{2,0}(x_2,t) + u_{2,1}(x_2,t) = \psi_2(t), \\ u_{1,0}(0,t) + u_{1,1}(0,t) = u_{2,0}(0,t) + u_{2,1}(0,t), \\ -\lambda_1\left(\dfrac{\partial u_{1,0}}{\partial x}(0,t) + \dfrac{\partial u_{1,1}}{\partial x}(0,t)\right) = -\lambda_2\left(\dfrac{\partial u_{2,0}}{\partial x}(0,t) + \dfrac{\partial u_{2,1}}{\partial x}(0,t)\right), \end{cases} \tag{24}$$

while for the systems (23) we give the boundary conditions of the form (for $j \geq 2$):

$$\begin{cases} u_{1,j}(x_1,t) \ = 0, \\ u_{2,j}(x_2,t) = 0, \\ u_{1,j}(0,t) \ = u_{2,j}(0,t), \\ -\lambda_1 \dfrac{\partial u_{1,j}}{\partial x}(0,t) = -\lambda_2 \dfrac{\partial u_{2,j}}{\partial x}(0,t). \end{cases} \tag{25}$$

In this way, instead of solving the input problem we will consider the sequence of systems of partial differential equations, which are simple to solve. Before starting the calculations we need to determine the initial approximations of functions $u_{i,0}(x,t)$. As the initial approximations we will take the functions describing initial conditions (for $i = 1,2,$):

$$u_{i,0}(x,t) = \varphi_i(x). \tag{26}$$

5. Computing example

Application of the presented procedure will be tested with the aid of an example, in which $x_1 = -1$, $x_2 = 1$, $a_1 = 1/4$, $a_2 = 1$, $\lambda_1 = 1$ and $\lambda_2 = 2$. The initial conditions have the form:

$$\varphi_1(x) = e^{2x},$$
$$\varphi_2(x) = e^{x},$$

while the boundary conditions of the first kind are the following:

$$\psi_1(x) = e^{t-2},$$
$$\psi_2(x) = e^{t+1}.$$

The exact solution of such formulated problem give the functions:

$$u_1(x,t) = e^{2x+t},$$
$$u_2(x,t) = e^{x+t}.$$

As the initial approximations $u_{i,0}(x,t)$, $i = 1,2$, of the sought functions we put the functions satisfying initial conditions:

$$u_{1,0}(x,t) = e^{2x},$$
$$u_{2,0}(x,t) = e^{x}.$$

By solving the system (22) with boundary conditions (24) we find:

$$u_{1,1}(x,t) = -e^{2x} + \frac{1}{3}e^{-2+t}\left(1 - 2x + 2e^3(1+x)\right),$$
$$u_{2,1}(x,t) = -e^{x} + \frac{1}{3}e^{-2+t}\left(1 - 2x + 2e^3(2+x)\right).$$

The next functions $u_{i,j}(x,t)$, $i = 1,2$, $j \geq 2$, are calculated recurrently by solving the systems (23) with boundary conditions (25). For example, for $j = 2$ and $j = 3$ we receive:

$$u_{1,2}(x,t) = \frac{1}{9}e^{t-2}(2x^2 + x - 1)\left(4 - 2x + e^3(2x+5)\right),$$
$$u_{2,2}(x,t) = \frac{1}{18}e^{t-2}(x^2 + x - 2)\left(4 - x + e^3(x+5)\right),$$
$$u_{1,3}(x,t) = \frac{1}{180}e^{t-2}(2x^2 + x - 1) \times$$
$$\times\left(8(2-x)(x^2 - x - 5) + e^3(2x+5)(4x^2 + 8x - 17)\right),$$
$$u_{2,3}(x,t) = \frac{1}{360}e^{t-2}(x^2 + x - 2) \times$$
$$\times\left((4-x)(x^2 - 2x - 20) + e^3(x+5)(x^2 + 4x - 17)\right).$$

In Table 1, the errors of reconstruction of the functions describing temperature distribution in domains D_1 and D_2 are compiled. Whereas, Table 2 presents the errors, with which the approximate functions $\hat{u}_{1,n}$ and $\hat{u}_{2,n}$ fulfil the initial conditions for different number of iterations n (see the relation (8)). The other boundary conditions on boundaries Γ_3, Γ_4 and Γ_5 are satisfied exactly. Presented results show that the errors are getting smaller with the growing number of terms in the sum (8).

Table 1.
Errors of reconstruction of temperature distribution (Δ - absolute error, δ - relative error)

n	Δ_{u_1}	δ_{u_1}	Δ_{u_2}	δ_{u_2}
1	29.3327	126.16 %	29.1426	34.74 %
10	13.7720	54.02 %	8.7552	10.44 %
20	5.4755	21.48 %	3.4809	4.15 %
30	2.1770	8.54 %	1.3840	1.65 %
40	0.8655	3.39 %	0.5502	0.66 %
50	0.3441	1.35 %	0.2188	0.26 %
60	0.1368	0.59 %	0.0954	0.11 %

Table 2.
Errors of reconstruction of initial conditions (Δ - absolute error, δ - relative error)

n	Δ_{φ_1}	δ_{φ_1}	Δ_{φ_2}	δ_{φ_2}
1	0.6250	126.16 %	0.6210	34.74 %
10	0.2676	54.02 %	0.1866	10.44 %
20	0.1064	21.48 %	0.0742	4.15 %
30	0.0423	8.54 %	0.0295	1.65 %
40	0.0168	3.39 %	0.0117	0.66 %
50	0.0067	1.35 %	0.0047	0.26 %
60	0.0029	0.59 %	0.0020	0.11 %

The exact and reconstructed functions, describing the initial conditions, are also presented in Figs. 2-7. The figures display approximations of the 30, 45 and 60-order. Additionally, the errors of reconstructed initial conditions are drawn in the Figures.

a)

b)

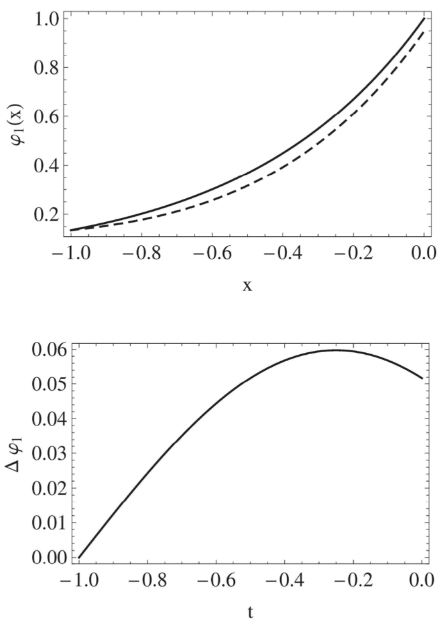

Fig. 2. Temperature on the boundary Γ_1 (a) and error of its reconstruction (b) for $n = 30$ (solid line - exact values, dashed line - reconstructed values)

a)

b)

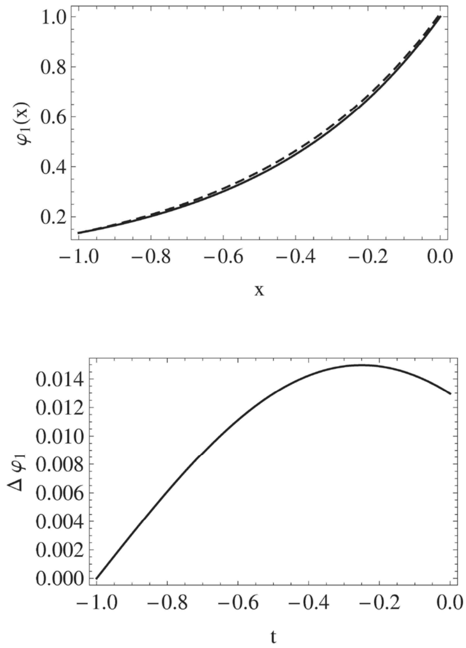

Fig. 3. Temperature on the boundary Γ_1 (a) and error of its reconstruction (b) for $n = 45$ (solid line - exact values, dashed line - reconstructed values)

a)

b)

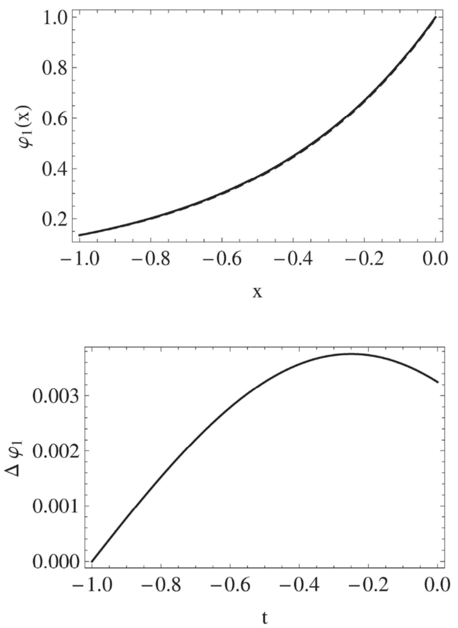

Fig. 4. Temperature on the boundary Γ_1 (a) and error of its reconstruction (b) for $n = 60$ (solid line - exact values, dashed line - reconstructed values)

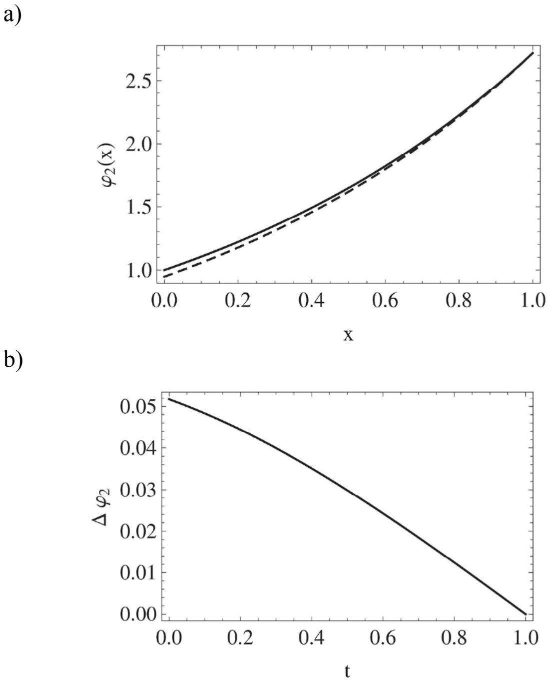

Fig. 5. Temperature on the boundary Γ_2 (a) and error of its reconstruction (b) for $n = 30$ (solid line - exact values, dashed line - reconstructed values)

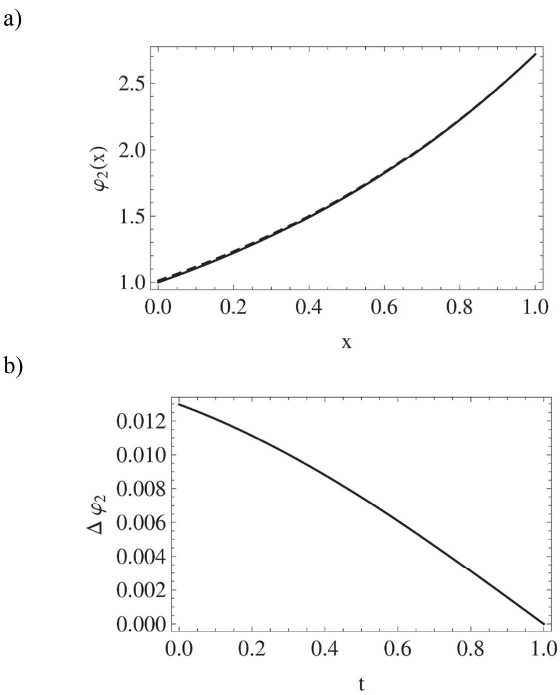

Fig. 6. Temperature on the boundary Γ_2 (a) and error of its reconstruction (b) for $n = 45$ (solid line - exact values, dashed line - reconstructed values)

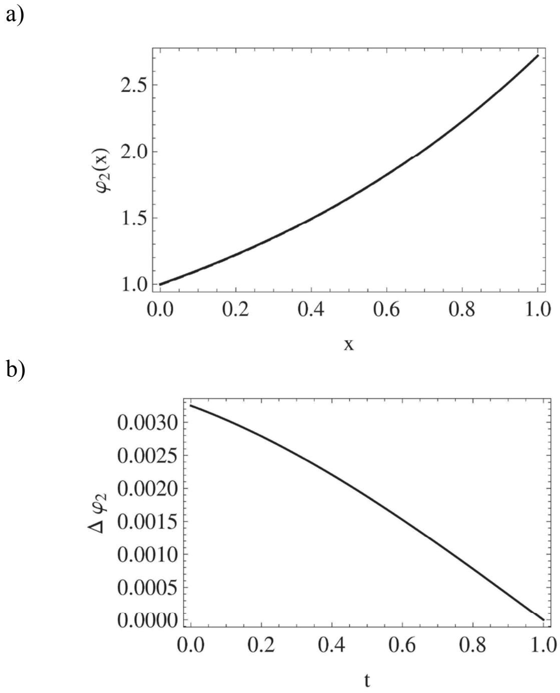

Fig. 7. Temperature on the boundary Γ_2 (a) and error of its reconstruction (b) for $n = 60$ (solid line - exact values, dashed line - reconstructed values)

6. Conclusions

By using the homotopy perturbation method we receive the function series, convergent to the solution of considered problem (under the proper assumptions). In many cases we are able to determine the sum of obtained series analytically, which means that we can calculate the exact solution of the examined problem. In case when analytic calculation of the sum of series is not possible, we still can use few of the first terms for building the approximate solution. Series received in the presented example is not convergent very fast. However, reviewing literature concerning the application of HPM one can notice that series obtained in this method is usually convergent much more fast, thanks to which taking only few first terms ensure a very good approximation of the exact solution. For example, in paper [94] homotopy perturbation method is used for solving the one-phase inverse Stefan problem and, in that case, calculating only five first terms of the series (it means, reduction to $\hat{u}_5$) gives the approximation of sought functions with the error less than 0.1%, calculating one more term reduces the error to 0.016% and another one - to 0.0022%.

In the current paper solution of the problem is provided with the assumption of an ideal contact between the cast and the mould.. In future, the authors plan to consider an application of the described procedure for problems involving the presence of thermal resistance at the cast-mould contact.

References

[1] G. Adomian, Stochastic Systems, Academic Press, New York,, 1983.
[2] G. Adomian, A review of the decomposition method in applied mathematics, Journal of Mathematical Analysis and Applications 135 (1988) 501-544.
[3] G. Adomian, Solving frontier problems of physics: the decomposition method, Kluwer, Dordrecht, 1994.
[4] R. Grzymkowski, E. Hetmaniok, D. Słota, Selected computational methods in calculus of variation and differential and integral equations, WPKJS, Gliwice, 2002 (in Polish).
[5] J.-H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media, Computer Methods in Applied Mechanics and Engineering 167 (1998) 57-68.
[6] J.H. He, Approximate solution of nonlinear differential equations with convolution product nonlinearities, Computer Methods in Applied Mechanics and Engineering 167 (1998) 69-73.
[7] J.H. He, Variational iteration method - a kind of non-linear analytical technique: some examples, International Journal of Non-Linear Mechanics 34 (1999) 699-708.
[8] J.H. He, Variational iteration method for autonomous ordinary differential systems, Applied Mathematics and Computation 114 (2000) 115-123.
[9] J.H. He, X.-H. Wu, Construction of solitary solution and compacton-like solution by variational iteration method, Chaos, Solitons & Fractals 29 (2006) 108-113.
[10] J.H. He, Some asymptotic methods for strongly nonlinear equations, International Journal of Modern Physics B 20 (2006) 1141-1199.
[11] J.-H. He, Homotopy perturbation method for solving boundary value problems, Physics Letters A 350 (2006) 87-88.
[12] J.H. He, Homotopy perturbation technique, Computer Methods in Applied Mechanics and Engineering 178 (1999) 257-262.
[13] J.H. He, A coupling method of a homotopy technique and a perturbation technique for non-linear problems, International Journal of Non-Linear Mechanics 35 (2000) 37-43.
[14] J.H. He, Comparision of homotopy perturbation method and homotopym analysis method, Applied Mathematics and Computation 156 (2004) 527-539.
[15] J.H. He, Homotopy perturbation method for bifurcation of nonlinear problems, International Journal of Nonlinear Sciences and Numerical Simulation 6 (2005) 207-208.
[16] J.-H. He, Non-Perturbative Methods for Strongly Nonlinear Problems, Dissertation.de-Verlag, Berlin, 2006.
[17] J.H. He, New interpretation of homotopy perturbation method, International Journal of Modern Physics B 20 (2006) 2561-2568.
[18] J.H. He, Recent development of the homotopy perturbation method, Topological Methods in Nonlinear Analysis 31 (2008) 205-209.
[19] J.H. He, An elementary introduction to the homotopy perturbation method, Computers & Mathematics with Applications 57 (2009) 410-412.
[20] D. Lesnic, Convergence of Adomian's decomposition metod: periodic temperatures, Computers & Mathematics with Applications 44 (2002) 13-24.
[21] D. Lesnic, Decomposition method for non-linear, non-characteristic Cauchy heat problems, Communications in Nonlinear Science and Numerical Simulation 10 (2005) 581-596.
[22] D. Lesnic, The decomposition method for Cauchy reaction-diffusion problems, Applied Mathematics Letters 20 (2007) 412-418.
[23] D. Lesnic, A nonlinear reaction-diffusion process using the Adomian decomposition method, International Communications in Heat and Mass Transfer 34 (2007) 129-135.
[24] S. Pamuk, An application for linear and nonlinear heat equations by Adomian's decomposition method, Applied Mathematics and Computation 163 (2005) 89-96.
[25] A. Soufyane, M. Boulmalf, Solution of linear and nonlinear parabolic equations by the decomposition method, Applied Mathematics and Computation 162 (2005) 687-693.
[26] A.M. Wazwaz, Exact solutions to nonlinear diffusion equations obtained by the decomposition method, Applied Mathematics and Computation 123 (2001) 109-122.
[27] A.M. Wazwaz, A. Gorguis, Exact solutions for heat-like and wave-like equations with variable coefficients, Applied Mathematics and Computation 149 (2004) 15-29.
[28] D.N.K. Marwat, S. Asghar, Solution of the heat equation with variable properties by two-step Adomian decomposition method, Mathematical and Computer Modelling 48 (2008) 83-90.
[29] D. Lesnic, The Cauchy problem for the wave equation using the decomposition method, Applied Mathematics Letters 15 (2002) 697-701.
[30] A.M. Wazwaz, A reliable technique for solving the wave equation in an infinite one-dimensional medium, Applied Mathematics and Computation 92 (1998) 1-7.
[31] A.M. Wazwaz, Blow-up for solutions of some linear wave equations with mixed nonlinear boundary conditions, Applied Mathematics and Computation 123 (2001) 133-140.
[32] S. Guellal, Y. Cherruault, Application of the decomposition method to identify the distributed parameters of an elliptical equation, Mathematical and Computer Modelling 21 (1995) 51-55.
[33] D. Lesnic, L. Elliott, The decomposition approach to inverse heat conduction, Journal of Mathematical Analysis and Applications 232 (1999) 82-98.
[34] E. Babolian, H. Sadeghi, S. Javadi, Numerically solution of fuzzy differential equations by Adomian method, Applied Mathematics and Computation 149 (2004) 547-557.
[35] A.M. Wazwaz, Approximate solutions of boundary value problems of higher order by the modified decomposition method, Computers & Mathematics with Applications 40 (2000) 679-691.
[36] M.H. Chang, A decomposition solution for fins with temperature dependent surface heat flux, International Journal of Heat and Mass Transfer 48 (2005) 1819-1824.
[37] C.H. Chiu, C.K. Chen, A decomposition method for solving the convective longitudinal fins with variable thermal

conductivity, International Journal of Heat and Mass Transfer 45 (2002) 2067-2075.
[38] D.J. Evans, H. Bulut, A new approach to the gas dynamics equation: an application of the decomposition method, International Journal of Computer Mathematics 79 (2002) 817-822.
[39] E. Babolian, J. Biazar, On the order of convergence of Adomian method, Applied Mathematics and Computation 130 (2002) 383-387.
[40] D.K.R. Babajee, M.Z. Dauhoo, M.T. Darvishi, A. Barati, A note on the local convergence of iterative methods based on Adomian decomposition method and 3-node quadrature rule, Applied Mathematics and Computation 200 (2008) 452-458.
[41] S. Momani, S. Abuasad, Z. Odibat, Variational iteration method for solving nonlinear boundary value problems, Applied Mathematics and Computation 183 (2006) 1351-1358.
[42] M. Dehghan, F. Shakeri, Approximate solution of a differential equation arising in astrophysics using the variational iteration method, New Astronomy 13 (2008) 53-59.
[43] S. Momani, S. Abuasad, Application of He's variational iteration method to Helmholtz equation, Chaos, Solitons & Fractals 27 (2006) 1119-1123.
[44] A.M. Wazwaz, The variational iteration method for exact solutions of Laplace equation, Physics Letters A 363 (2007) 260-262.
[45] A.M. Wazwaz, The variational iteration method: A reliable analytic tool for solving linear and nonlinear wave equations, Computers & Mathematics with Applications 54 (2007) 926-932.
[46] B. Batiha, M.S.M. Noorani, I. Hashim, Application of variational iteration method to heat- and wave-like equations, Physics Letters A 369 (2007) 55-61.
[47] D.H. Shou, J.-H. He, Beyond Adomian method: The variational iteration method for solving heat-like and wave-like equations with variable coefficients, Physics Letters A 372 (2008) 233-237.
[48] D.D. Ganji, M.J. Hosseini, J. Shayegh, Some nonlinear heat transfer equations solved by three approximate methods, International Communications in Heat and Mass Transfer 34 (2007) 1003-1016.
[49] H. Khaleghi, D.D. Ganji, A. Sadighi, Application of variational iteration and homotopy-perturbation methods to nonlinear heat transfer equations with variable coefficients, Numerical Heat Transfer Part A 52 (2007) 25-42.
[50] A. Sadighi, D.D. Ganji, Exact solution of nonlinear diffusion equations by variational iteration method, Computers & Mathematics with Applications 54 (2007) 1112-1121.
[51] A.M. Wazwaz, The variational iteration method: A powerful scheme for handling linear and nonlinear diffusion equations, Computers & Mathematics with Applications 54 (2007) 933-939.
[52] A.M. Wazwaz, The variational iteration method for solving linear and nonlinear systems of PDEs, Computers & Mathematics with Applications 54 (2007) 895-902.
[53] M. Tatari, M. Dehghan, He's variational iteration method for computing a control parameter in a semi-linear inverse parabolic equation, Chaos, Solitons & Fractals 33 (2007) 671-677.
[54] M. Tatari, M. Dehghan, On the convergence of He's variational iteration method, Journal of Computational and Applied Mathematics 207 (2007) 121-128.
[55] J.H. He, Variational iteration method - some recent results and new interpretations, Journal of Computational and Applied Mathematics 207 (2007) 3-17.
[56] J.H. He, X.-H. Wu, Variational iteration method: New development and applications, Computers & Mathematics with Applications 54 (2007) 881-894.
[57] J.H. He, G.-C. Wu, F. Austin, The variational iteration method which should be followed, Nonlinear Science Letters A 1 (2010) 1-30.
[58] S. Abbasbandy, The application of homotopy analysis method to nonlinear equations arising in heat transfer, Physics Letters A 360 (2006) 109-113.
[59] S. Abbasbandy, Homotopy analysis method for heat radiation equations, International Communications in Heat and Mass Transfer 34 (2007) 380-387.
[60] G. Domairry, N. Nadim, Assessment of homotopy analysis method and homotopy perturbation method in non-linear heat transfer equation, International Communications in Heat and Mass Transfer 35 (2008) 93-102.
[61] B. Han, H.S. Fu, Z. Li, A homotopy method for the inversion of a two-dimensional acoustic wave equation, Inverse Problems in Science and Engineering 13 (2005) 411-431.
[62] S.J. Liao, Beyond Perturbation: Introduction to the Homotopy Analysis Method, Chapman and Hall/CRC Press, Boca Raton, 2003.
[63] D.D. Ganji, A. Rajabi, Assessment of homotopy-perturbation and perturbation methods in heat radiation equations, International Communications in Heat and Mass Transfer 33 (2006) 391-400.
[64] A.H. Nayfeh, Perturbation Method, Wiley, New York, 1973.
[65] J.I. Ramos, Piecewise homotopy methods for nonlinear ordinary differential equations, Applied Mathematics and Computation 198 (2008) 92-116.
[66] L. Cveticanin, Homotopy-perturbation method for pure nonlinear differential equation, Chaos, Solitons & Fractals 30 (2006) 1221-1230.
[67] A. Yildirim, Solution of BVPs for fourth-order integro-differential equations by using homotopy perturbation method, Computers & Mathematics with Applications 56 (2008) 3175-3180.
[68] F. Shakeri, M. Dehghan, Solution of delay differential equations via a homotopy perturbation method, Mathematical and Computer Modelling 48 (2008) 486-498.
[69] J.H. He, Application of homotopy perturbation method to nonlinear wave equations, Chaos, Solitons & Fractals 26 (2005) 695-700.
[70] C. Chun, H. Jafari, Y.-I. Kim, Numerical method for the wave and nonlinear diffusion equations with the homotopy perturbation method, Computers & Mathematics with Applications 57 (2009) 1226-1231.

[71] H. Xu, J. Cang, Analysis of a time fractional wave-like equation with the homotopy analysis method, Physics Letters A 372 (2008) 1250-1255.
[72] X. Li, M. Xu, X. Jiang, Homotopy perturbation method to time-fractional diffusion equation with a moving boundary condition, Applied Mathematics and Computation 208 (2009) 434-439.
[73] F. Shakeri, M. Dehghan, Inverse problem of diffusion equation by He's homotopy perturbation method, Physica Scripta 75 (2007) 551-556.
[74] A. Sadighi, D.D. Ganji, Exact solutions of Laplace equation by homotopy-perturbation and Adomian decomposition methods, Physics Letters A 367 (2007) 83-87.
[75] J. Biazar, H. Ghazvini, Homotopy perturbation method for solving hyperbolic partial differential equations, Computers & Mathematics with Applications 56 (2008) 453-458.
[76] D.D. Ganji, G.A. Afrouzi, R.A. Talarposhti, Application of variational iteration method and homotopy-perturbation method for nonlinear heat diffusion and heat transfer equations, Physics Letters A 368 (2007) 450-457.
[77] A. Rajabi, D.D. Ganji, H. Taherian, Application of homotopy perturbation method in nonlinear heat conduction and convection equations, Physics Letters A 360 (2007) 570-573.
[78] J. Biazar, H. Ghazvini, Convergence of the homotopy perturbation method for partial differential equations, Nonlinear Analysis: Real World Application 10 (2009) 2633-2640.
[79] R. Grzymkowski, Non-classical methods of solving the heat conduction problems, Silesian University of Technology Press, Gliwice (in press, in Polish).
[80] R. Grzymkowski, D. Słota, An application of the Adomian decomposition method for inverse Stefan problem with Neumann's boundary condition, Lecture Notes In Computer Science 3516 (2005) 895-898.
[81] R. Grzymkowski, D. Słota, One-phase inverse Stefan problems solved by Adomian decomposition method, Computers & Mathematics with Applications 51 (2006) 33-40.
[82] R. Grzymkowski, D. Słota, Stefan problem solved by Adomian decomposition method, International Journal of Computer Mathematics 82 (2005) 851-856.
[83] R. Grzymkowski, D. Słota, Moving boundary problem solved by Adomian decomposition method, Fluid Structure Interaction and Moving Boundary Problems, Wit Press, Southampton, 2005, 653-660.
[84] R. Grzymkowski, M. Pleszczyński, D. Słota, Application of the Adomian decomposition method for solving the Stefan problem, Proceedings of the Numerical Heat Transfer 2005, EUROTHERM Seminar 82, Silesian University of Technology, Gliwice, 2005, 249-258.
[85] R. Grzymkowski, M. Pleszczyński, D. Słota, The two-phase Stefan problem solved by the Adomian decomposition method, Proceedings of the 15th IASTED International Conference Applied Simulation and Modelling, ACTA Press, Rhodos, 2006, 511-516.
[86] R. Grzymkowski, M. Pleszczyński, D. Słota, Comparing the Adomian decomposition method and Runge-Kutta method for the solutions of the Stefan problem, International Journal of Computer Mathematics 83 (2006) 409-417.
[87] R. Grzymkowski, M. Pleszczyński, D. Słota, Application of the Adomian decomposition method for solving the heat equation in the cast-mould heterogeneous domain, Archives of Foundry Engineering 9/4 (2009) 57-62.
[88] D. Słota, Exact solution of the heat equation with boundary condition of the fourth kind by He's variational iteration method, Computers & Mathematics with Applications 58 (2009) 2495-2503.
[89] R. Grzymkowski, D. Słota, One-phase Stefan problem with Neumann boundary condition solved by variational iteration method, Proceedings of the 2007 International Conference Scientific Computing (CSC'07), CSREA Press, Las Vegas, 2007, 1-6.
[90] D. Słota, Direct and inverse one-phase Stefan problem solved by variational iteration method, Computers & Mathematics with Applications 54 (2007) 1139-1146.
[91] D. Słota, Application of the variational iteration method for inverse Stefan problem with Neumann's boundary condition, Lecture Notes In Computer Science 5/101 (2008) 1005-1012.
[92] E. Hetmaniok, D. Słota, A. Zielonka, Solution of the solidification problem by using the variational iteration method, Archives of Foundry Engineering 9 /4 (2009) 63-68.
[93] D. Słota, A. Zielonka, A new application of He's variational iteration method for the solution of the one-phase Stefan problem, Computers & Mathematics with Applications 58 (2009) 2489-2494.
[94] D. Słota, The application of the homotopy perturbation method to one-phase inverse Stefan problem, International Communications in Heat and Mass Transfer 37 (2010) 587-592.
[95] I. Nowak, A.J. Nowak, L.C. Wrobel, Identification of phase change fronts by Bezier splines and BEM, International Journal of Thermal Sciences 41 (2002) 492-499.
[96] I. Nowak, A.J. Nowak, L.C. Wrobel, Inverse analysis of continuous casting processes, International Journal of Numerical Methods for Heat and Fluid Flow 13 (2003) 547-564.
[97] R. Grzymkowski, D. Słota, Multi-phase inverse Stefan problems solved by approximation method, Lecture Notes In Computer Science 2328 (2002) 679-686.
[98] R. Grzymkowski, D. Słota, Numerical method for multi-phase inverse Stefan design problems, Archives of Metallurgy and Materials 51 (2006) 161-172.
[99] D. Słota, Using genetic algorithms for the determination of an heat transfer coefficient in three-phase inverse Stefan problem, International Communications in Heat and Mass Transfer 35 (2008) 149-156.
[100] D. Słota, Determination of the boundary conditions in two-dimensional solidification of pure metals, Journal of Achievements in Materials and Manufacturing Engineering 28 (2008) 59-62.
[101] R. Grzymkowski, D. Słota, Determination of the continuous casting cross-section with prescribed average temperature, Archives of Foundry Engineering 8 (4) (2008) 51-54.
[102] D. Słota, Solving the inverse Stefan design problem using genetic algorithms, Inverse Problems in Science and Engineering 16 (2008) 829-846.

[103] D. Słota, Identification of the cooling condition in 2-D and 3-D continuous casting processes, Numerical Heat Transfer Part B 55 (2009) 155-176.
[104] D. Słota, Modelling of the optimum cooling condition in two-dimensional solidification processes, Archives of Computational Materials Science and Surface Engineering 1/1 (2009) 45-52.
[105] D. Słota, Calculation of the cooling condition in the phase change problem, Journal of Achievements in Materials and Manufacturing Engineering 33/1 (2009) 70-77.
[106] D. Słota, Reconstruction of the heat transfer coefficient on the grounds of experimental data, Journal of Achievements in Materials and Manufacturing Engineering 34/1 (2009) 63-70.
[107] E. Hetmaniok, D. Słota, A. Zielonka, Solution of the inverse heat conduction problem by using the ABC algorithm, Lecture Notes in Artificial Intelligence 6086 (2010) 659-668.
[108] L.A. Dobrzański, R. Maniara, J. H. Sokolowski, The effect of cast Al-Si-Cu alloy solidification rate on alloy thermal characteristics, Journal of Achievements in Materials and Manufacturing Engineering 17 (2006) 217-220.
[109] L.A. Dobrzański, W. Kasprzak, M. Kasprzak, J.H. Sokolowski, A novel approach to the design and optimization of aluminum cast component heat treatment processes using advanced UMSA physical simulations, Journal of Achievements in Materials and Manufacturing Engineering 24/1 (2007) 139-142.
[110] L.A. Dobrzański, R. Maniara, J. Sokolowski, W. Kasprzak, Effect of cooling rate on the solidification behaviour of AC AlSi7Cu2 alloy, Journal of Materials Processing Technology 191 (2007) 317-320.