Application of the homotopy perturbation method for calculation of the temperature distribution in the cast-mould heterogeneous domain

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ABSTRACT

Purpose of this paper: In this paper an application of the new method for solving the heat conduction equation in the heterogeneous cast-mould system, with an assumption of the ideal contact at the cast-mould contact point, is introduced. An example illustrating the discussed approach and confirming its usefulness for solving problems of that kind is also presented in the paper.

Design/methodology/approach: For solving the discussed problem the homotopy perturbation method is used, which consists in determining the series convergent to the exact solution or enabling to build the approximate solution of the problem.

Findings: The paper shows that the homotopy perturbation method, effective in solving many technical problems, is successful also for examining the considered problem.

Research limitations/implications: Solution of the problem is provided with the assumption of an ideal contact between the cast and the mould. In further, research of the discussed method shall be employed to solve problems involving the presence of thermal resistance at the cast-mould contact

Practical implications: The method allows to determine the solution in form of the continuous function, which is significant for the analysis of the cast cooling in the mould, in order to avoid the defects formation in the cast.

Originality/value: Application of the new method for solving the considered problem.

Keywords: Numerical techniques; Heat transfer; Cooling of the cast; Homotopy perturbation method

Reference to this paper should be given in the following way:
1. Introduction

In the recent time a number of methods enabling to solve different kinds of physical and technical problems have found an application. Group of these methods include, among others, the Adomian decomposition method [1-4], the variational iteration method [5-11] and the homotopy perturbation method [11-19]. General mathematical formulation of the above mentioned methods allows to find a solution of the wide class of nonlinear operator equations. The idea of all of those methods consists in constructing the functional sequence, or series, which limit (or sum) represents the solution of the considered problem (under the proper assumptions). In general, the speed of convergence of the received sequences or series is quite good, thanks to which calculating only a few first terms ensures usually the satisfying approximation of the sought solution. The evidence of popularity of those three methods gives the fact that the special issues of different journals are sacrificed exactly to them (among others also the journals from ISI Master Journal List, like Computers & Mathematics with Applications and Topological Methods in Nonlinear Analysis).

The Adomian decomposition method (ADM) is named after its inventor, George Adomian, and is used for solving the different kind of problems described, for example, with the aid of partial and ordinary differential equations, integral equations and so on. In papers [20-28], ADM is used for solving the linear and nonlinear heat conduction equation. The wave equation is examined in papers [29-31], while in papers [32,33] the inverse problem for differential equations is considered. In paper [34], the authors solve, by using ADM, the fuzzy differential equations and in [35] the boundary problem for the differential equations of the higher order is considered. Another applications of ADM in examining the mathematical models describing different kinds of technical problems can be found in papers [36-38]. Whereas, convergence of the Adomian method is discussed in [20,39,40].

The other mentioned methods, variational iteration method (VIM) and homotopy perturbation method (HPM), was created by Ji-Huan He. VIM is useful for solving many different kinds of nonlinear problems. Momani and his colleagues [41] have applied VIM for finding the solution of ordinary differential equations with boundary conditions. Similarly, Dehghan and Shakeri [42] have used the described method for determining the approximate solution of some differential equation arising in astrophysics. There are also available papers in which VIM is applied for finding the exact or approximate solution of partial differential equations. For example, Momani and Abuasad [43] have used VIM for solving the Helmholtz equation and Wazwaz [44,45] has applied the method for determining the exact solutions of Laplace and wave equations. In papers [46,47], the heat-like and wave-like equations are solved, while the heat transfer and diffusion equations are examined, by means of VIM, in [48,51]. Solution of the systems of partial differential equations with the aid of VIM is presented in [52], whereas Tataru and Dehghan in [53] have used this method for computing a parameter in semi-linear inverse parabolic equation. Convergence of VIM is discussed by Tataru and Dehghan in [54]. Some new interpretations and applications of the variational iteration method are proposed by He in papers [55-57].

Homotopy perturbation method arised as a combination of elements of two other methods: the homotopy analysis method [14,58-62] and the perturbation method [16,63,64]. HPM appeared as an effective and powerful method for solving the wide class of problems. For example, Ramos [65] has applied HPM for solving the nonlinear second-order ordinary differential equations with boundary conditions. Similar application is presented in paper [66]. Solution of boundary value problems for integro-differential equations by using the homotopy perturbation method is described in [67], whereas Shakeri and Dehghan [68] have used the described method for solving the delay differential equation arising in biology and engineering. There can be also found some papers in which HPM is applied for determining the exact and approximate solutions of partial differential equations, like, for example, the nonlinear wave equations [69], the wave and the nonlinear diffusion equations [70] and the fractional wave-like equation [71]. Furthermore, Li and his colleagues in [72] have used HPM for examining the time-fractional diffusion equation with the moving boundary condition, Shakeri and Dehghan in [73] have applied the method for solving the inverse problem of diffusion equation, Sadighi and Ganji in [74] have found the exact solutions of Laplace equation and Biazar and Ghazvini in [75] have solved the hyperbolic partial differential equation by means of HPM. Finally, Ganji and his colleagues in the series of papers [48,49,63,76,77] have considered the application of HPM for solving different problems concerning the heat transfer processes. Some information about the convergence of the homotopy perturbation method can be found in papers [12,78].

Employees of the Faculty of Mathematics and Physics at the Silesian University of Technology, especially researchers of the Department of Applied Mathematics, from many years deal in their work with applying the above described methods for solving various problems concerning the heat conduction. First effect of this research become the chapter in volume [4], in which the Adomian decomposition method is presented. Application of this method for solving the heat conduction problems is also described in monograph [79]. In papers [80,81], ADM combined with some optimization procedures is used for solving the inverse one-phase Stefan problem with the boundary condition of the first and second kind. In the presented approach, the distribution of temperature in the considered domain is calculated in the ground of ADM. The received temperature distribution depends on some coefficients, values of which are determined with the aid of the mean square method. Accuracy of the procedure is verified on the basis of the exact solution. The same approach for the direct Stefan problem is presented in papers [82,83]. In the further works another approach is proposed, in which the Stefan problem is first approximated by the system of ordinary differential equations, and next, the obtained system is solved with the aid of ADM. In this way, the need of constructing and minimizing some functional, which was necessary in previous approach, can be omitted. The same way of using ADM is showed in paper [84], for the case of one-phase Stefan problem, and in [85], for the case of two-phase Stefan problem. Comparison of precision of the one-phase Stefan problem solution, received with the aid of Adomian decomposition method and Runge-Kutty method of the fourth order, is presented in [86]. In the both approach the Stefan problem is first approximated by the system of ordinary
differential equations, solved afterwards by means of ADM and Runge-Kutta method. Received results demonstrate better precision of Adomian decomposition method, also the time of calculations is shorter for ADM. In paper [87], application of ADM for finding the exact solution of heat conduction equation in the cast-mould heterogeneous domain is described. Adaptation of the variational iteration method for solving that kind of problem is proposed in [88]. In works [89-93], VIM is used for solving the one-phase direct and inverse Stefan problem, whereas paper [94] presents an application of the homotopy perturbation method for determining the exact (or approximated) solution of the one-phase Stefan problem. Moreover, researchers of the Department of Applied Mathematics have prepared many other works concerning the direct and inverse heat conduction problems, like for example [95-107]. In part of those papers, verification of the developed methods is executed by using the experimental data received with the aid of UMSA equipment (Universal Metallurgical Simulator and Analyzer), designed for analysis of the heat processes occurring in metals [108-110]. Experimental results were received thanks to the collaboration with the employees of the Institute of Engineering Materials and Biomaterials in the Faculty of Mechanical Engineering of the Silesian University of Technology.

In the current paper, an application of the homotopy perturbation method for solving the heat conduction equation in the heterogeneous cast-mould system, with an assumption of the ideal contact at the cast-mould contact point is presented. An example illustrating the discussed approach and confirming usefulness of the proposed procedure for solving problems of that kind is also showed.

2. Homotopy perturbation method

By using the homotopy perturbation method, solution of the following nonlinear operator equations can be found:

\[ A(u) = f(z), \quad z \in \Omega, \]  

(1)

where \( A \) denotes the operator, \( f \) is the given function and \( u \) is the sought function. Let us assume that the operator \( A \) can be presented as the sum:

\[ A(u) = L(u) + N(u), \]  

(2)

where \( L \) represents the linear operator and \( N \) denotes the nonlinear operator. From this, equation (1) can be written in the form:

\[ L(u) + N(u) = f(z), \quad z \in \Omega. \]  

(3)

Let us define a new operator, named the homotopy operator, in the following way:

\[ H(v, p) = (1 - p)(L(v) - L(u_0)) + p(A(v) - f(z)), \]  

(4)

where \( p \in [0,1] \) is the, so called, homotopy parameter, \( v(z, p) : \Omega \times [0,1] \to \mathbb{R} \), and \( u_0 \) denotes the initial approximation of solution of the equation (1). By using the relation (2) we obtain:

\[ H(v, p) = L(v) - L(u_0) + pL(u_0) + p(N(v) - f(z)). \]  

(5)

Since \( H(v, 0) = L(v) - L(u_0) \), then for \( p = 0 \) solution of the operator equation \( H(v, 0) = 0 \) is equivalent to solution of the trivial problem \( L(v) - L(u_0) = 0 \). Whereas, for \( p = 1 \) solution of the operator equation \( H(v, 1) = 0 \) is equivalent to solution of the input equation. In this way, the monotonic change of parameter \( p \), from 0 to 1, corresponds with the monotonic change of the equation, from the trivial one: \( L(v) - L(u_0) = 0 \) to the input form of the considered equation (and with the monotonic change of the solution \( v \), from \( u_0 \) to \( u \)).

Now, let us assume that the solution of equation \( H(v, p) = 0 \) can be written in the form of the power series:

\[ v = \sum_{j=0}^{\infty} p^j u_j. \]  

(6)

If the above series is convergent, then, by substituting \( p = 1 \), we receive the solution of equation (1):

\[ u = \lim_{p \to 1} v = \sum_{j=0}^{\infty} u_j. \]  

(7)

Information about convergence of the series (6) is included in papers [12,78]. In many cases the speed of convergence of series (7) is great, thanks to which the sum composed from only few first terms gives already a very good approximation of the sought solution. By confining to the first \( n+1 \) elements, we get the, so called, n-order approximate solution:

\[ \hat{u}_n = \sum_{j=0}^{n} u_j. \]  

(8)

For finding the form of functions \( u_j \), we substitute the relation (6) into the equation \( H(v, p) = 0 \) and we compare the elements appearing by the same powers of parameter \( p \). In this way, we receive the sequence of operator equations, which allows to determine the successive functions \( u_j \). In this manner, solving of the input problem can be reduced to the task of solving the sequence of problems, simple to analyse.

3. Mathematical model of the problem

In the current paper we consider the problem of determining distribution of temperature in the heterogeneous cast-mould system, with an assumption of the ideal contact at the cast-mould contact point. Let us start with formulation of the mathematical model of the problem.
Let us have two regions: \( D_1 = \{ (x,t) : x \in [x_1,0], t \in [0,t') \} \) and \( D_2 = \{ (x,t) : x \in [0,x_2], t \in [0,t') \} \) (see Fig. 1).

![Fig. 1. Domain of the considered problem](image)

On the boundary of these domains five components are distributed:

\[ \Gamma_1 = \{ (x,0) : x \in [x_1,0] \}, \]
\[ \Gamma_2 = \{ (x,0) : x \in [0,x_2] \}, \]
\[ \Gamma_3 = \{ (x_1,t) : t \in [0,t') \}, \]
\[ \Gamma_4 = \{ (0,t) : t \in [0,t') \}, \]
\[ \Gamma_5 = \{ (x_2,t) : t \in [0,t') \}, \]

where the initial and boundary conditions are given.

In the cast (region \( D_1 \)) and mould (region \( D_2 \)) we consider the heat conduction equations:

\[
\begin{align*}
\frac{\partial u_1(x,t)}{\partial t} &= a_1 \frac{\partial^2 u_1(x,t)}{\partial x^2}, & (x,t) \in D_1, \\
\frac{\partial u_2(x,t)}{\partial t} &= a_2 \frac{\partial^2 u_2(x,t)}{\partial x^2}, & (x,t) \in D_2,
\end{align*}
\]

where \( a_i, i = 1,2 \), are the thermal diffusivity, \( u_i, i = 1,2 \), denote the temperature, and \( t \) and \( x \) refer the time and spatial location, respectively. On boundaries \( \Gamma_1 \) and \( \Gamma_2 \) the initial conditions are given:

\[
\begin{align*}
u_i(0,t) &= u_i(0,t), & t \in [0,t'), \\
u_i(x_0) &= u_i(x_0), & x \in [x_1,0], \\
u_i(x_0) &= u_i(x_0), & x \in [x_2,0].
\end{align*}
\]

On boundaries \( \Gamma_1 \) and \( \Gamma_2 \) the Dirichlet conditions are determined:

\[
\begin{align*}
u_i(x_1,t) &= \psi_i(t), & t \in [0,t'), \\
u_i(x_2,t) &= \psi_i(t), & t \in [0,t').
\end{align*}
\]

And finally, on the cast-mould contact boundary (boundary \( \Gamma_4 \)) the boundary conditions of the fourth kind are known (condition of temperature continuity and condition of heat flux continuity):

\[
\begin{align*}
u_1(0,t) &= u_2(0,t), & t \in [0,t'), \\
-\lambda_i \frac{\partial \nu_i(x,t)}{\partial x} &= -\lambda_i \frac{\partial \nu_i(x,t)}{\partial x}, & t \in [0,t'),
\end{align*}
\]

where \( \lambda_i, i = 1,2 \), denote the thermal conductivity. Additionally, we assume that functions describing the considered problem satisfy the consistency conditions:

\[
\begin{align*}
\phi_i(x_1) &= \psi_i(0), \\
\phi_i(x_2) &= \psi_i(0), \\
-\lambda_i \phi_i(x_1) &= -\lambda_i \phi_i(x_2).
\end{align*}
\]

We seek the functions \( u_i(x,t) \) and \( u_i(x,t) \) defined in domains \( D_1 \) and \( D_2 \), respectively, which satisfy the heat conduction equations together with the above presented conditions.

### 4. Solution of the problem

Let us start with defining the homotopy operators for equations (9) and (10). The proper operators have the form (for \( i = 1,2 \)):

\[
H_i(v_i, p) = \frac{\partial^2 v_i}{\partial x^2} - \frac{\partial^2 u_{i0}}{\partial x^2} + p \left( \frac{\partial^2 u_{i0}}{\partial x^2} - \frac{1}{a_i} \frac{\partial v_i}{\partial t} \right).
\]

Solutions of the equation (for \( i = 1,2 \)):

\[
H_i(v_i, p) = 0
\]

will be sought in the form of the power series of variable \( p \):

\[
v_i = \sum_{j=0}^{\infty} p^j u_{i,j}.
\]

By substituting the relation (19) into the equation (18) (and by using definition (17)) we receive (for \( i = 1,2 \)):

\[
\sum_{j=0}^{\infty} p^{j+1} \frac{\partial^2 u_{i,j+1}}{\partial x^2} - p \frac{\partial^2 u_{i0}}{\partial x^2} + \frac{1}{a_i} \sum_{j=0}^{\infty} p^j \frac{\partial u_{i,j}}{\partial t} = 0,
\]

or, equivalently:

\[
\sum_{j=0}^{\infty} p^{j+1} \frac{\partial^2 u_{i,j+1}}{\partial x^2} = -p \frac{\partial^2 u_{i0}}{\partial x^2} + \frac{1}{a_i} \sum_{j=0}^{\infty} p^j \frac{\partial u_{i,j+1}}{\partial t}.
\]
By comparing the elements occurring by the same powers of parameter $p$ we obtain the following systems of equations:

\[
\begin{align*}
\frac{\partial^2 u_{1,i}}{\partial x^2} + \frac{1}{a_i} \frac{\partial u_{1,i}}{\partial t} & = \frac{\partial^2 u_{1,i}}{\partial x^2}, \\
\frac{\partial^2 u_{2,i}}{\partial x^2} & = \frac{1}{a_2} \frac{\partial u_{2,i}}{\partial t} = \frac{\partial^2 u_{2,i}}{\partial x^2},
\end{align*}
\]

and for $j \geq 2$:

\[
\begin{align*}
\frac{\partial^2 u_{1,i,j}}{\partial x^2} & = \frac{1}{a_i} \frac{\partial u_{1,i,j}}{\partial t}, \\
\frac{\partial^2 u_{2,i,j}}{\partial x^2} & = \frac{1}{a_2} \frac{\partial u_{2,i,j}}{\partial t} = \frac{\partial^2 u_{2,i,j}}{\partial x^2}.
\end{align*}
\]

Systems of partial differential equations (22) and (23) must be additionally completed with the boundary conditions, which ensure the uniqueness of solution. So, for the system (22) we put the conditions:

\[
\begin{align*}
&u_{1,0}(x_1,t) + u_{1,2}(x_1,t) = \psi_1(t), \\
&u_{2,0}(x_2,t) + u_{2,2}(x_2,t) = \psi_2(t), \\
&u_{1,0}(0,t) + u_{1,1}(0,t) + u_{1,2}(0,t), \\
&\lambda_1 \frac{\partial u_{1,0}}{\partial x}(0,t) + \lambda_1 \frac{\partial u_{1,1}}{\partial x}(0,t) = \lambda_2 \left( \frac{\partial u_{2,0}}{\partial x}(0,t) + \frac{\partial u_{2,1}}{\partial x}(0,t) \right).
\end{align*}
\]

while for the systems (23) we give the boundary conditions of the form (for $j \geq 2$):

\[
\begin{align*}
u_{1,i}(x_1,t) & = 0, \\
u_{2,i}(x_2,t) & = 0, \\
u_{1,i}(0,t) & = u_{1,j}(0,t), \\
\lambda_1 \frac{\partial u_{1,i}}{\partial x}(0,t) & = \lambda_2 \frac{\partial u_{1,j}}{\partial x}(0,t).
\end{align*}
\]

In this way, instead of solving the input problem we will consider the sequence of systems of partial differential equations, which are simple to solve. Before starting the calculations we need to determine the initial approximations of functions $u_{1,i}(x,t)$. As the initial approximations we will take the functions describing initial conditions (for $i = 1, 2$):

\[
u_{1,0}(x,t) = \phi_1(x), \quad \nu_{2,0}(x,t) = \phi_2(x).
\]

5. Computing example

Application of the presented procedure will be tested with the aid of an example, in which $x_1 = -1, x_2 = 1, a_1 = 1/4, a_2 = 1, \lambda_1 = 1$ and $\lambda_2 = 2$. The initial conditions have the form:

$\phi_1(x) = e^{2x}$,  
$\phi_2(x) = e^x$,

while the boundary conditions of the first kind are the following:

$\psi_1(x) = e^{x^2}$,  
$\psi_2(x) = e^{x^4}$.

The exact solution of such formulated problem gives the functions:

$\nu_{1,0}(x,t) = e^{2x^2}$,  
$\nu_{2,0}(x,t) = e^{x^4}$.

As the initial approximations $\nu_{1,i}(x,t), i = 1, 2$, of the sought functions we put the functions satisfying initial conditions:

$\nu_{1,0}(x,t) = e^{2x}$,  
$\nu_{2,0}(x,t) = e^x$.

By solving the system (22) with boundary conditions (24) we find:

$\nu_{1,1}(x,t) = -e^{x^2} + \frac{1}{3} e^{2x} \left( 1 - 2x + 2e^3(1 + x) \right)$,  
$\nu_{2,1}(x,t) = -e^x + \frac{1}{3} e^{2x} \left( 1 - 2x + 2e^3(2 + x) \right)$.

The next functions $\nu_{1,i,j}(x,t), i = 1, 2, j \geq 2$, are calculated recurrently by solving the systems (23) with boundary conditions (25). For example, for $j = 2$ and $j = 3$ we receive:

$\nu_{1,2}(x,t) = \frac{1}{9} e^{x^2} (2x^2 + x - 1) \left( 4 - 2x + e^3 (2x + 5) \right)$,  
$\nu_{2,2}(x,t) = \frac{1}{18} e^{x^2} (4 - x + e^3 (x + 5))$,

$\nu_{1,3}(x,t) = \frac{1}{180} e^{x^2} (2x^2 + x - 1) \times \left( 8(2 - x)(x^2 - x - 5) + e^3 (2x + 5)(4x^2 + 8x - 17) \right)$,  
$\nu_{2,3}(x,t) = \frac{1}{360} e^{x^2} (x^2 + x - 2) \times \left( (4 - x)(x^2 - 2x - 20) + e^3 (x + 5)(x^2 + 4x - 17) \right)$.

In Table 1, the errors of reconstruction of the functions describing temperature distribution in domains $D_1$ and $D_2$ are compiled. Whereas, Table 2 presents the errors, with which the approximate functions $\hat{u}_{1,n}$ and $\hat{u}_{2,n}$ fulfill the initial conditions for different number of iterations $n$ (see the relation (8)). The other boundary conditions on boundaries $\Gamma_1, \Gamma_2$ and $\Gamma_3$ are satisfied exactly. Presented results show that the errors are getting smaller with the growing number of terms in the sum (8).
Table 1.
Errors of reconstruction of temperature distribution (Δ - absolute error, δ - relative error)

<table>
<thead>
<tr>
<th>n</th>
<th>Δui</th>
<th>δui %</th>
<th>Δvi</th>
<th>δvi %</th>
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<tbody>
<tr>
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<tr>
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<tr>
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<tr>
<td>50</td>
<td>0.3441</td>
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<td>0.2188</td>
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</tr>
<tr>
<td>60</td>
<td>0.1368</td>
<td>0.59%</td>
<td>0.0954</td>
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</table>

Table 2.
Errors of reconstruction of initial conditions (Δ - absolute error, δ - relative error)

<table>
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<th>n</th>
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<th>δvi %</th>
<th>Δvi</th>
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<td>0.59%</td>
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The exact and reconstructed functions, describing the initial conditions, are also presented in Figs. 2-7. The figures display approximations of the 30, 45 and 60-order. Additionally, the errors of reconstructed initial conditions are drawn in the Figures.

Fig. 2. Temperature on the boundary Γ1 (a) and error of its reconstruction (b) for n = 30 (solid line - exact values, dashed line - reconstructed values)

Fig. 3. Temperature on the boundary Γ1 (a) and error of its reconstruction (b) for n = 45 (solid line - exact values, dashed line - reconstructed values)

Fig. 4. Temperature on the boundary Γ1 (a) and error of its reconstruction (b) for n = 60 (solid line - exact values, dashed line - reconstructed values)
Analysis and modelling

Application of the homotopy perturbation method for calculation of the temperature distribution ...

Table 1. Errors of reconstruction of temperature distribution (\( u' \)- absolute error, \( G \)- relative error)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( u' )</th>
<th>( u' )</th>
<th>( G )</th>
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<td>60</td>
<td>0.1368</td>
<td>0.59%</td>
<td>0.0954</td>
</tr>
</tbody>
</table>

Table 2. Errors of reconstruction of initial conditions (\( u' \)- absolute error, \( G \)- relative error)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( M' )</th>
<th>( M' )</th>
<th>( G )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6250</td>
<td>126.16%</td>
<td>0.6210</td>
</tr>
<tr>
<td>10</td>
<td>0.2676</td>
<td>54.02%</td>
<td>0.1866</td>
</tr>
<tr>
<td>20</td>
<td>0.1064</td>
<td>21.48%</td>
<td>0.0742</td>
</tr>
<tr>
<td>30</td>
<td>0.0423</td>
<td>8.54%</td>
<td>0.0295</td>
</tr>
<tr>
<td>40</td>
<td>0.0168</td>
<td>3.39%</td>
<td>0.0117</td>
</tr>
<tr>
<td>50</td>
<td>0.0067</td>
<td>1.35%</td>
<td>0.0047</td>
</tr>
<tr>
<td>60</td>
<td>0.0029</td>
<td>0.59%</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

The exact and reconstructed functions, describing the initial conditions, are also presented in Figs. 2-7. The figures display approximations of the 30, 45 and 60-order. Additionally, the errors of reconstructed initial conditions are drawn in the Figures.

6. Conclusions

By using the homotopy perturbation method we receive the function series, convergent to the solution of considered problem (under the proper assumptions). In many cases we are able to determine the sum of obtained series analytically, which means that we can calculate the exact solution of the examined problem. In case when analytic calculation of the sum of series is not possible, we still can use few of the first terms for building the approximate solution. Series received in the presented example is not convergent very fast. However, reviewing literature concerning the application of HPM one can notice that series obtained in this method is usually convergent much more fast, thanks to which taking only few first terms ensure a very good approximation of the exact solution. For example, in paper [94] homotopy perturbation method is used for solving the one-phase inverse Stefan problem and, in that case, calculating only five first terms of the series (it means, reduction to \( \tilde{u}_5 \)) gives the approximation of sought functions with the error less than 0.1%, calculating one more term reduces the error to 0.016% and another one - to 0.0022%.

In the current paper solution of the problem is provided with the assumption of an ideal contact between the cast and the mould. In future, the authors plan to consider an application of the described procedure for problems involving the presence of thermal resistance at the cast-mould contact.
References


