Optimisation of cutting velocity of bundles of various metal sheets on a guillotine with respect to heating process

J. Kaczmarczyk*
Department of Applied Mechanics, Faculty of Mechanical Engineering, Silesian University of Technology, ul. Konarskiego 1Ba, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: jaroslaw.kaczmarczyk@polsl.pl
Received 15.10.2010; published in revised form 01.12.2010

ABSTRACT

Purpose: The work was aimed at elaboration of methods and algorithms for thermal process optimisation during cutting the bundles of various types of metal sheets made of copper, brass and bronze interleaved with paper on a guillotine with respect to the heating process.

Design/methodology/approach: The numerical simulations were conducted using an author’s computer program prepared in the object oriented language C++ using the finite difference method. The optimisation has been conducted employing the genetic algorithms which were also implemented in C++ language and elaborated as an author’s computer program. As the objective function the maximum values of temperatures occurring in the direct cutting zone of the sheet bundles were assumed and the cutting velocity was established as a design variable.

Findings: Possibilities of designation of the optimum cutting parameters on account of maximum admissible temperature of the bundles of various metals have been indicated. The constraints imposed on the temperature were needed to avoid defects which might occur in the direct cutting zone as the result of progressing heating during cutting.

Research limitations/implications: The elaborated methods and algorithms for optimisation of the chosen cutting parameters may be generalized for the wide gamut of materials and for changeable cutting conditions; however, the obtained numerical results are specific and related to the chosen types of materials and fixed cutting conditions.

Practical implications: Optimisation of the cutting parameters is essential in terms of industrial economy. It allows to reduce the amount of waste caused by defects in cutting bundles of sheets and decrease wear of the cutting tool.

Originality/value: Proposed elaborated methods and algorithms for optimisation of cutting parameters are novel and original tools supporting the reduction of defects’ number occurring during cutting were designed.

Keywords: Finite difference method; Genetic algorithms; Heat flow; Defects

Reference to this paper should be given in the following way:
1. Introduction

Nowadays, guillotines are more frequently used in industry for cutting bundles of metal sheets taking into consideration their high efficiency in a cutting process in comparison with the cutting efficiency of single sheets on guillotine shears. However, it frequently happens that during cutting, random undesirable defects occur in bundles’ cross-section, and then guillotines use more energy, a cutting tool of the guillotine becomes blunt more quickly, more heat is emitted in the cutting process, consequently the machines undergo extended wear and provide a large amount of waste, which is directly linked to increased expenses incurred on production connected with cutting. Heating of the bundles during cutting is a result of friction, plastic deformation etc. Depending on temperature, the mechanical properties of copper, brass and bronze as well as the properties of coated protective layers on the sheet surfaces change in the direct cutting zone and therefore it seems necessary to model and optimise the parameters of an unsteady heat flow \[1-4, 8, 9\].

In this article, the results of a minimisation of maximum temperature values occurring in the direct cutting zone have been presented. The optimisation was conducted by using the genetic algorithm \[5\]. In \(t\) iteration, the genetic algorithm maintained the population of potential solutions. Each solution was subjected to assessment in terms of values of its “fitting”. Next, in \(t+1\) iteration a new population was formed by choosing the fittest individuals. Some members of the new population were submitted to the changes caused by: crossing, mutation and cloning, creating in this way new solutions. In order to perform the optimisation process, the physical models and corresponding to them the mathematical models taking into account a transient heat flow and anisotropic thermophysical properties of material were elaborated. The numerical calculations were conducted using an author’s computer program prepared in the object oriented language C++ based on the finite difference method and the genetic algorithm, which in opinion of many specialists are the most convenient and effective ones of approximate task solution connected with optimisation as well as with a heat flow.

2. Physical model of a cutting process

The elaborated physical model of a cutting process of bundles made of three selected different metals such as: copper, brass and bronze, all of them interleaved with paper is presented in Figs. 1-3. The bundles of sheets are arranged on the table of a guillotine and the pressure beam loads the bundle with a certain force (Fig. 2). The bundle is next being cut using a cutting tool (Fig. 1).

After the bundle has been cut, the cutting tool returns to its original position, and the pressure beam is released; next, the bundle is shifted using a feeding device to the desirable width of cutting and the process is periodically repeated until the required size of sheets is obtained.

A bundle of sheets, in which single metal sheets were separated with paper and the whole thickness of the bundle equals 1 \(cm\), was submitted to modelling. The bottom and the top surface of the bundle were separated by cardboard, which is a good insulator. According to the above-mentioned, the assumption of lack of the heat transfer between the bottom and the top surface of the bundle and the ambient medium was established (Fig. 3).

![Fig. 1. Physical model of a bundle of sheets – front view](image1)

![Fig. 2. Physical model of a bundle of sheets – side view](image2)

![Fig. 3. Physical model of a bundle of sheets being cut with imposed boundary conditions](image3)
3. Assumptions and data in numerical calculations

On the left side of the bundle of sheets, the heat transfer between the bundle and the ambient medium through convection and radiation was assumed and on the right side of the bundle, the heat flux moving with a velocity corresponding to the velocity of a cutting tool was modelled (Fig. 3). It was assumed that the heat flux is distributed symmetrically into two heat fluxes with respect to the cutting line, where one of them is transferred to the bundle placed on the left side of the cutting line, and the second one is distributed into several heat fluxes on the right side of the cutting line. According to the above-mentioned, only the half of a value of the initial heat flux was taken into consideration in the modelling process of a heat flow during cutting. Next, the value was decreased by 1% on account of friction arising between the bundle and the cutting tool. Simultaneously, on the right side of the cutting line, the heat transfer by convection and radiation with the ambient medium was assumed. The axes of the assumed coordinate system (Fig. 4) were chosen in such a way that their directions coincided with the principal axes of anisotropy of the cut bundle.

![Fig. 4. Discretisation of the modelled bundle into nodes](image)

Because of high costs linked with purchase of professional equipment for heat flow measuring, the mechanical work equal to 400 J for all three kinds of metals was assumed. The mechanical work was pertained to the resultant of cross section area of a single metal sheet interleaved with a single piece of paper. Moreover, such mechanical work can be found in an experimental way or can be calculated by preparation a mechanical model of cutting process [10, 14, 18, 19], however, such performances are quite expensive.

The width of the separated metal sheet equals 10 cm and the combined thickness of a separated piece of paper with separated piece of metal sheet equals together 0.4 mm.

<table>
<thead>
<tr>
<th>Materials of bundle</th>
<th>Percentage volume fraction [%]</th>
<th>Percentage mass fraction [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper and copper</td>
<td>25</td>
<td>3.35</td>
</tr>
<tr>
<td>Paper and brass</td>
<td>25</td>
<td>3.50</td>
</tr>
<tr>
<td>Paper and bronze</td>
<td>25</td>
<td>3.40</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>96.60</td>
</tr>
</tbody>
</table>

The coefficient of thermal conductivity for the bundle in the direction of the x axis (Fig. 2) was assumed and its value was close to the value of the coefficient of thermal conductivity for copper (0.30, 0.78 W/(m·K)), whereas in the direction of the y axis the coefficient was approximated by the harmonic mean [6] of coefficients of thermal conductivity for copper and paper (0.52 W/(m·K)) (Table 3). The thickness of the separated sheet of all three kinds of metals equal 0.3 mm and the thickness of the separated interleaved sheet of paper equals 0.1 mm. Percentage volume fractions corresponding to the assumed thicknesses equal for paper and copper adequately: \( x_y = 25\% \) and \( x_x = 75\% \). Corresponding to them percentage mass fractions equal adequately: \( x_x = 3.35\% \) and \( x_y = 96.65\% \) (Table 2). The density and specific heat for the bundle composed of paper and copper were calculated as the arithmetic mean of the percentage mass fractions \( \rho = 6932.25 \, \text{kg/m}^3, c_p = 455.85 \, \text{J/(kg·K)} \) (Table 3).
### 4. Mathematical model

The considered area of thermal interaction (Fig. 4) was divided into a sufficiently large number of small parts (900 difference elements which corresponds to 961 nodes) and the equation of energy balance was made for each finite difference element. It led to formulation of the differential equation (1), with the aid of which the nodal temperature was calculated:

\[ \rho \cdot c_p \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_x \cdot \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_y \cdot \frac{\partial T}{\partial y} \right) + \dot{q} \]  

(1)

where:
- \( \rho \) - density [kg/m³],
- \( c_p \) - specific heat [J/(kg·K)],
- \( \lambda_x, \lambda_y \) - coefficients of heat conductivity correspondingly in the x and y axes [W/(m·K)],
- \( T \) - temperature [K] or [°C],
- \( t \) - time [s],
- \( x, y \) - coordinates [m],
- \( \dot{q} \) - unitary power of volumetric heat sources [W/m³].

The linear temperature distribution between adjacent nodes and the assumption of incompressibility of the bundle were established in the numerical calculations. The above-formulated equation was completed with the following initially-boundary conditions:

a) initial condition called Cauchy’s condition:

\[ T(x,y,t)\big|_{t=0} = T_0(x,y) \]  

(2)

b) boundary condition of I type (Dirichlet’s condition):

\[ T(x,y,t)\big|_{x=x_\text{lim}} = T_\text{lim}(x,y,t) \]  

(3)

where: \( T_\text{lim} \) – surface temperature [K].

c) boundary condition of II type (von Neumann’s condition):

\[ -\left( \lambda_x \cdot \frac{\partial T}{\partial x} \cdot n_x + \lambda_y \cdot \frac{\partial T}{\partial y} \cdot n_y \right) = g(x,y,t), \]  

(4)

where:
- \( n_x = \cos(n_x,x) \), \( n_y = \cos(n_y,y) \) - are the direction cosines of the normal to a surface.

For thermal insulated surfaces, the following condition was assumed:

\[ -\left( \lambda_x \cdot \frac{\partial T}{\partial x} \cdot n_x + \lambda_y \cdot \frac{\partial T}{\partial y} \cdot n_y \right) = 0, \]  

(5)

d) boundary condition of III type (Robin’s or Newton’s condition) with nonlinear boundary condition (radiation) in order to model a combined heat flow by convection and radiation:

\[ -\left( \lambda_x \cdot \frac{\partial T}{\partial x} \cdot n_x + \lambda_y \cdot \frac{\partial T}{\partial y} \cdot n_y \right) = a(x,y,t,T_x) \cdot [T(x,y,t) - T_\text{lim}] + \varepsilon \cdot \sigma \cdot \kappa \cdot [T(x,y,t)^4 - T_\text{lim}^4], \]  

(6)

where:
- \( a \) - coefficient of convective heat transfer [W/(m²·K)],
- \( \sigma \) - Stefan-Boltzmann’s constant,
- \( \varepsilon \) - coefficient of emission,
- \( T_\text{lim} \) - temperature of the surrounding walls, which “see” the analysed body [K].

The partial differential equation (1) with conditions from (2) to (6) was digitized according to explicit finite difference method (central). In order to ensure the solution stability, the assumption was made that the coefficients in the difference equation, which are next to the temperatures, should be non-negative. From that condition, the maximum time step was designated and expressed in the following form:

\[ \Delta t \leq \frac{\rho \cdot c_p \cdot h_t \cdot k^2}{2 \cdot (\lambda_x \cdot k_t^2 + \lambda_y \cdot h_t^2)}, \]  

(7)

where:
- \( \Delta t \) – maximum time step [s],
- \( h, k \) –distance between two adjacent nodes correspondingly in horizontal and vertical level [m].

### 5. Optimisation

The optimisation process was carried out by using the genetic algorithms which structure is the same as structure of any evolutionary program. Such genetic operators as: selection, crossing, mutation and cloning have been used.

Selection allows choosing the best fitted individuals (chromosomes) in the next generation.

Crossing leads to coupling of the features of two parental chromosomes in the chromosomes of two offspring by interchanging the segments of the parental chromosomes.
Crossing may be interpreted as the exchange of genetic information between potential solutions. Mutation consists in the random exchange of one or more genes in a selected chromosome (from zero to one or from one to zero), with probability equal to the mutation frequency. Mutation operator means introduction of a certain additional variability in a population. Cloning is used in order to prevent a possible loss of the best genetic material.

As an objective function, the maximum values of temperatures occurring in the direct cutting zone have been assumed according to the equation:

$$\psi = \max \{t(\theta)\}$$

and for a design variable the cutting velocity of the sheet bundle $\theta$ corresponding to the cutting tool velocity of a guillotine has been assumed. The optimisation problem has been formulated as a minimisation of the maximum values of temperatures:

$$\min \{\psi(\theta)\} = \min \left\{ \max \{t(\theta)\} \right\}$$

with the following constraints imposed on the design variable and objective function:

$$\vartheta_{\min} \leq \theta \leq \vartheta_{\max}$$
$$T_{\min} \leq \psi(\theta) \leq T_{\max}$$

On account of the constraint condition (11) imposed on the assumed objective function (8), its modification was introduced in the following form:

$$\psi^{\prime}(\theta) = \begin{cases} \psi(\theta), & \text{when } \theta \in \mathbb{A}, \\ \psi(\theta)\cdot \Psi(\theta), & \text{when } \theta \in \mathbb{N}, \end{cases}$$

where:

$$\Psi(\theta)$$ - penalty function; equals one in the case when the constraints imposed on the objective function are not violated and in the opposite case it is within the range $0 \leq \psi(\theta) \leq 1$.

$\mathbb{A}$ - domain of permissible solutions,

$\mathbb{N}$ - domain of impermissible solutions.

The genetic algorithms are generally employed for a maximisation of the objective function. The task formulated earlier concerns a minimisation (9), and to move from a minimisation task to a maximisation task, the following transformation was applied [5]:

$$\psi^{\prime\prime}(\theta) = \begin{cases} C_{\max} - \psi^{\prime}(\theta), & \text{for } \psi^{\prime}(\theta) < C_{\max}, \\ 0, & \text{for } \psi^{\prime}(\theta) \geq C_{\max}, \end{cases}$$

where: $C_{\max}$ – certain coefficient (constant).

The optimisation task therefore may be presented in the following form:

$$\min \{\psi(\theta)\} = \max \{\psi^{\prime\prime}(\theta)\}.$$
to the cutting process on guillotines. The carried out numerical algorithms allow to determine the maximum values of temperatures in the direct cutting zone and also to designate the desired parameters of a cutting process, at which no damage will occur in coated protective layers.

Thus, two variants of the optimisation process were considered. In the first variant, the bundle of all three kinds of pairs of materials was treated as a monolith (Figs.: 6, 8, 10) and in the second variant; the bundle was treated as a set of separated sheets (Figs.: 7, 9, 11). For both variants, the considered bundle consisted of twenty-five metal sheets interleaved with twenty-five paper sheets. The friction between the cutting tool and the cutting surface of the bundle was taken into consideration for all analysed cases.

In the direct cutting zone, local, of short duration and rapidly varying heating along a cutting line occurs during the cutting process [10, 11, 13, 15-17, 20]. If the metal sheets are painted or coated by powder spraying etc., therefore, it may happen that the degradation of painted surfaces close to the cutting line appears. Changes in temperature occur in each node of a meshed model of the bundle (Fig. 4). Three nodes from all applied 961 nodes were selected to present the largest variation in temperature before and after optimisation (Figs. 6b - 11b). The selected nodes are located on the right edge of the model on the cutting line and they are as follows:

- first (node no. 31) corresponds to the beginning of the cutting process in the instant when the cutting tool cuts the first sheet in a bundle,
- second (node no. 496) corresponds to the intermediate position in the instant when the cutting tool passes through the middle of the height of a bundle,
- third (node no. 961) corresponds to the end of the cutting process in the instant when the cutting tool cuts the last sheet in a bundle.

### 7.1. Results for a bundle composed of copper and paper

According to the first variant, the bundle was treated as a monolith. The coefficients of thermal conductivity ($\lambda_c = \lambda_w = 300.780 \ W/(m \cdot K)$) were assumed. In Fig. 6a, the course of the best modified objective function versus the number of populations is presented.

The courses of temperature versus time, before and after optimisation for the best solution found in the twenty-fourth population, are presented in Fig. 6b. The optimum solution for velocity ($\vartheta = 0.315 \ m/s$) and for maximum temperature ($\max[T(\vartheta)]=150 ^\circ C$) was obtained.

In the second variant, the bundle consisted of twenty-five separated copper sheets interleaved with paper. The coefficients of thermal conductivity ($\lambda_c = 300.780 \ W/(m \cdot K)$, $\lambda_w = 0.520 \ W/(m \cdot K)$) were assumed. In Fig. 7a, the course of the best modified objective function versus the number of populations is presented. The courses of temperature versus time, before and after optimisation for the best solution found in the twenty-fourth population, are presented in Fig. 7b. The optimum solution for velocity ($\vartheta = 0.804 \ m/s$) and for maximum temperature ($\max[T(\vartheta)]=150 ^\circ C$) was obtained.
7.2. Results for a bundle composed of brass and paper

According to the first variant, the bundle was treated as a monolith. The coefficients of thermal conductivity ($\lambda_a = \lambda_w = 83.280 \text{ W/(m K)}$) were assumed. In Fig. 8a, the course of the best modified objective function versus the number of populations is presented. The courses of temperature versus time, before and after optimisation for the best solution found in the twenty-ninth population, are presented in Fig. 8b. The optimum solution for velocity ($v = 0.079 \text{ m/s}$) and for maximum temperature ($\max[T(\varphi)] = 150 \text{ C}$) was obtained.

In the second variant, the bundle consisted of twenty-five separated brass sheets interleaved with paper. The coefficients of thermal conductivity ($\lambda_a = 83.280 \text{ W/(m K)}, \lambda_w = 0.518 \text{ W/(m K)}$) were assumed. In Fig. 9a, the course of the best modified objective function versus the number of populations is presented. The courses of temperature versus time, before and after optimisation for the best solution found in the twenty-first population, are presented in Fig. 9b. The optimum solution for velocity ($v = 0.165 \text{ m/s}$) and for maximum temperature ($\max[T(\varphi)] = 150 \text{ C}$) was obtained.
7.3. Results for a bundle composed of bronze and paper

According to the first variant, the bundle was treated as a monolith. The coefficients of thermal conductivity ($\lambda_m = \lambda_w = 39,000 \, W/(m \cdot K)$) were assumed. In Fig. 10a, the course of the best modified objective function versus the number of populations is presented. The courses of temperature versus time, before and after optimisation for the best solution found in the twenty-fifth population, are presented in Fig. 10b. The optimum solution for velocity ($\vartheta = 0.0477 \, m/s$) and for maximum temperature ($\max[T(\vartheta)] = 150 \, ^\circ C$) was obtained.

Fig. 10a. Course of the optimisation process for a bundle (bronze and paper) treated as a monolith

Fig. 10b. Courses of temperature versus relative time for a bundle treated as a monolith

In the second variant, the bundle consisted of twenty-five separated bronze sheets interleaved with paper. The coefficients of thermal conductivity ($\lambda_m = 39,000 \, W/(m \cdot K)$, $\lambda_w = 0.516 \, W/(m \cdot K)$) were assumed. In Fig. 11a, the course of the best modified objective function versus the number of populations is presented. The courses of temperature versus time, before and after optimisation for the best solution found in the sixteenth population, are presented in Fig. 11b. The optimum solution for velocity ($\vartheta = 0.171 \, m/s$) and for maximum temperature ($\max[T(\vartheta)] = 150 \, ^\circ C$) was obtained.

Fig. 11a. Course of the optimisation process for a bundle (bronze and paper) treated as a set of sheets

Fig. 11b. Courses of temperature versus relative time for a bundle treated as a set of separated sheets

8. Description of achieved results

Two variants of a bundle were elaborated. The first one was modelled as a monolith and the second was treated as a set of separated metal sheets interleaved with paper. For each variant, three pairs of materials such as: copper and paper, brass and paper, bronze and paper were considered.

Optimum cutting velocities for monoliths are significantly smaller in comparison with optimum cutting velocities for sets of separated metal sheets under the same cutting conditions and with the same delivered heat flux. The maximum temperature arising in a bundle was limited and did not exceed $150^\circ C$ in order to avoid damages in the thin coated surfaces of metal sheets located...
close to the cutting line. In the case such defects occur, they can be detected e.g. using ultrasonic waves [12]. The curves of heating for all cases (before and after optimisation) have similar courses (Figs. 6b - 11b).

Three following positions of the cutting tool at work were considered:
- the top one (first) – cutting the first sheet in a bundle,
- the middle (second) – cutting the middle sheet in a bundle,
- the bottom (third) – cutting the last sheet in a bundle.

The above mentioned positions of the cutting tool correspond to the consecutive temperature peaks (Figs. 6b - 11b). The consecutive peaks in the courses of temperature versus time after optimisation for a bundle modelled as a monolith are characterised by higher and higher values of temperatures. Hence, the differences in temperatures between the consecutive peaks are roughly equal to:
- 15% - between the first and the second peak,
- 28% - between the second and the third peak.

In the case of a bundle modelled as a set of separated sheets, one can notice the rise in temperature between the consecutive peaks are insignificant and with comparison to the differences between peaks obtained for the monolith are negligible.

The genetic algorithm parameters such as: probability of crossing, probability of mutation, number of chromosomes, and number of populations employed for optimisation of the cutting velocity of a bundle with respect to the limited temperature imposed on the objective function have been chosen properly, therefore, for all optimised cases of the cutting processes, the satisfied convergence was achieved (Figs. 6a - 11a).

From the presented graphs, one can observe that an arbitrary chosen particle lying on the cutting line to which the blade of a cutting tool moves with high speed is rapidly heated and attains maximum value of a temperature in an instant, in which its position overlaps with the position of the blade of a cutting tool.

9. Conclusions

The optimisations consisted in designation of cutting velocities of different materials with taking into account penalty function concerning limitation imposed on heating temperature. The found velocities depend on the kind of cut materials and as well as on the type of a considered model (e.g. a monolith or a set of separated sheets in a bundle). A monolith heats much faster than a set of separated sheets in a bundle and therefore the cutting velocities are much smaller for a monolith.

Under the same cutting conditions and as well as with the same delivered heat flux, the maximum cutting velocity was obtained for a bundle composed of copper interleaved with paper and treated as a monolith and minimum cutting velocity was attained for a bundle composed of bronze interleaved with paper. In the case of material treated as a set of separated sheets, the maximum velocity was reached by a bundle consisted of copper and paper and minimum velocity was found for a bundle consisted of brass and paper. In the second case, the least cutting speed found for a bundle of brass and paper was probably caused by the least value of an equivalent specific heat.

In the direct cutting zone increasing temperature values might influence in a significant way on local changes of mechanical properties of the cutting material.

The elaborated methods and algorithms for modelling and optimisation of heating of sheets bundle in the direct cutting zone might be applied in dynamic coupled thermo–mechanical simulations of a cutting process.

The elaborated author’s computer program allows for optimisation of an anisotropic transient heat flow during cutting on a guillotine and might be used for designation of the desired parameters in a cutting process reducing to the minimum an amount of waste, with respect to not exceeding permissible values of temperatures in protective layers, by which the metal sheets surfaces are coated.

The introduced optimisation algorithm enables to control the parameters of a cutting process depending on a kind of cut material, geometrical dimensions of metal sheets, types and dimensions of the applied interlayer.

Acknowledgements

This work was financially supported by Polish Minister of Science and Higher Education as a part of project N N503 326435.

References


