Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 43
ISSUE 2
December
2010

The application of Zurek's rheological model for description of mechanical behaviour of textiles subjected to different state of loads

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Received 27.10.2010; published in revised form 01.12.2010

Analysis and modelling

ABSTRACT

Purpose: In order to describe the rheological properties of textile products there have been used various models but none of them delivers the complementary solution for textiles subjected to different fields of loads. Therefore the idea presented by Hasley in 1945 was an inspiration for us to propose the new rheological model based on theory of plastic-elastic solids.

Design/methodology/approach: It was assumed that the modified rheological model would consist of two parallel parts: I - Hooke's spring with rigidity C_1 and II - Hooke's spring with rigidity C_2, connected in series with a frictional element with a constant resistance, T and additional force $K\varepsilon_{22}$, and a piston with a weight m displacing in a liquid with a viscosity η, where ε_{22} is a shift of the piston from its initial position.

Findings: The proposed model represents adequately stress – strain relationships of polypropylene monofilaments subjected to tensile test. The results indicate that for each investigated type of nonwovens there is no significant difference between the shape of the theoretical and experimental elastic recovery curve during the recovery test.

Research limitations/implications: The application of presented model was used for illustration of the description of relaxation of polypropylene monofilament subjected to tensile load and rheological properties of non-woven fabrics made also from polypropylene fibres subjected to the compression loads.

Originality/value: The new rheological model was proposed. It can be universal for description of mechanical behaviour of textiles subjected to the tension or compression loads.

Keywords: Rheological model; Monofilament; Stress-strain curve; Relaxation; Non-woven fabrics

Reference to this paper should be given in the following way:
W. Żurek, M. Chrzanowski, W. Sybilska, I. Jałmużna, The application of Zurek's rheological model for description of mechanical behaviour of textiles subjected to different state of loads, Journal of Achievements in Materials and Manufacturing Engineering 43/2 (2010) 702-710.

1. Introduction

In order to describe the rheological properties of textile products, there have been used various models [1– 22]. One of the firsts was Voigt-Kelvin's model [1] that fails however to show relaxation at a constant strain. Another one was Zener's model [4, 5] describing the relaxation phenomenon, but based on the relaxation curve, constant values of the model rheological parameters could not be determined for linear textile products. The next modification was Vangheluwe's model [14–16], in which one of Hooke's elements with a linear character was replaced by a spring with non-linear characteristics. This model is capable of describing the stress-strain curve and the relaxation curve, but it still does not contain Saint-Venant's element being responsible for plastic strains after exceeding the limiting force. This lack is made up in Hoffman's model [8] where Zener's model is combined with Bingham's element. Zener's model is responsible for visco-elastic strains, while Bingham's element is responsible for the representation of plastic strains. In further investigations, this model was modified by replacing Newton's dumper in Binghman's element with Eyring's dumper [17]. Thus, the studies headed for the introduction of a successive element with non-linear characteristics. In this case, there was obtained a non-linear differential equation with no analytical solution. It seems that the lack of complementary description of the rheological properties of linear textile products resulted from the fact that no inertial member was taken into account. Already in 1945, Halsey wrote [2]: "In general, in the theory of plastic-elastic solids it has been customary to consider these solids as being dependent on linear differential equations of type:

$$a\ddot{q}_i + b\dot{q}_i + cq_i = Q_i$$

In this equation q is a generalized displacement from equilibrium, Q represents external forces, a is an inertial term (…), c is a potential term determining the action of the spring, while b is a viscosity..."

The idea of Hasley was an inspiration for us to propose the new rheological model, which can be universal for description of mechanical behaviour of textiles subjected to the tension or compression loads.

2. The idea of model

It was assumed that the modified rheological model would consist of two parallel parts:

I - Hooke's spring with rigidity C_1,

II - Hooke's spring with rigidity C_2, connected in series with a frictional element with a constant resistance, T and additional force $K\varepsilon_{22}$, and a piston with a weight m displacing in a liquid with a viscosity η, where ε_{22} is a shift of the piston from its initial position.

The use of springs with rigidities C_1 and C_2 in the initial phase should reflect the linear-elastic behaviour of the object under testing. Once the critical value of tensile stress is exceeded, a further part of the model connected with the inertial-frictional element is actuated. The function of this system is to reflect visco-elastic strains through the use of Newton dumper in the arrangement with springs C_1 and C_2, and the plastic strains realized through Saint-Venant's element. The parallel connection of dumper and Saint-Venant's element, and the mass allows an approximate reflection of the phenomena described in the work by Eyring where a dumper with non-linear characteristics was used. The scheme of the model is presented in Fig.1.

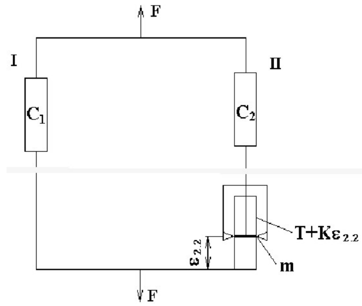

Fig. 1. Żurek's rheological model

The application of presented model was used for illustration of the description of relaxation of polypropylene monofilament subjected to tensile load and rheological properties of non-woven fabrics made also from polypropylene fibres subjected to the compression loads.

3. The description of the relaxation of PP monofilaments according to Żurek's rheological model

3.1. The analytical solution of relaxation equation

At the strain ε of model the force in the link I is:

$$F_1 = C_1\varepsilon = C_1ut \tag{1}$$

Where u is the rate of straining of model and in the link II:

$$F_2 = C_2\varepsilon = C_2ut \tag{2}$$

provided the force $F_2<T$ (or $C_2\varepsilon<T$).

After the force acting in the link II surpasses the value T, the force acting in this link can be expressed as:

$$F_2 = C_2(\varepsilon - \varepsilon_{2.2}) = C_2ut - C_2\varepsilon_{2.2} \tag{3}$$

Where $\varepsilon_{2.2}$ is intrinsic strain in the link II.

After the force in the link II reaches value T, piston begins to move with the rate:

$$\frac{d\varepsilon_{2.2}}{dt} = \frac{F_2}{\eta} \tag{4}$$

but the force F_2 is now diminished by $(T+K\varepsilon_{2.2})$ and inertial force:

$$m\frac{d^2\varepsilon_{2.2}}{dt^2} \tag{5}$$

Therefore we have:

$$F_2 = C_2(\varepsilon - \varepsilon_{2.2}) - T - K\varepsilon_{2.2} - m\frac{d^2\varepsilon_{2.2}}{dt^2} \tag{6}$$

Putting this value into previous equation we receive:

$$\frac{d\varepsilon_{2.2}}{dt} = \frac{1}{\eta}\left[C_2(\varepsilon-\varepsilon_{2.2}) - T - K\varepsilon_{2.2} - m\frac{d^2\varepsilon_{2.2}}{dt^2}\right] \quad (7)$$

and after simple transformation:

$$\frac{d^2\varepsilon_{2.2}}{dt^2} + \frac{\eta}{m}\cdot\frac{d\varepsilon_{2.2}}{dt} + \frac{C_2+K}{m}\varepsilon_{2.2} = \frac{C_2}{m}\varepsilon - \frac{T}{m} \quad (8)$$

According to present standards the load – elongation curves are determined with constant rate of elongation (or strain):

$$\frac{d\varepsilon}{dt} = u = const \quad (9)$$

To simplify previous equation we can introduce:

$$\frac{\eta}{m} = \alpha; \frac{C_2+K}{m} = \beta; \frac{C_2 u}{m} = \gamma; \frac{T}{m} = \varphi \quad (10)$$

and we receive equation:

$$\frac{d^2\varepsilon_{2.2}}{dt^2} + \alpha\frac{d\varepsilon_{2.2}}{dt} + \beta\varepsilon_{2.2} = \gamma t - \varphi \quad (11)$$

The resolution of this equation depends on the value:

$$\Delta = \alpha^2 - 4\beta \quad (12)$$

If $\Delta = 0$ we have:

$$\varepsilon_{2.2} = (At+B)e^{-\frac{\alpha}{2}t} + \frac{\gamma}{\beta}t - \left(\frac{\alpha\gamma}{\beta}+\varphi\right)\frac{1}{\beta} \quad (13)$$

The total force loading the monofilament takes the form of:

$$F = F_1 + F_2 = C_1 ut + C_2 ut - C_2\varepsilon_{2.2} = Cut - C_2\varepsilon_{2.2} \quad (14)$$

and

$$F = \left(Cu - C_2\frac{\gamma}{\beta}\right)t - C_2 Ate^{-\frac{\alpha}{2}t} - C_2 Be^{-\frac{\alpha}{2}t} + C_2\left(\frac{\alpha\gamma}{\beta}+\varphi\right)\frac{1}{\beta} \quad (15)$$

To simplify equation (15) we can introduce:

$$Cu - C_2\frac{\gamma}{\beta} = y; C_2 A = a; C_2 B = b; \left(\frac{\alpha\gamma}{\beta}+\varphi\right)\frac{1}{\beta} = n \quad (16)$$

and the force during the loading phase can be calculated as:

$$F = yt - at\,e^{-\frac{\alpha}{2}t} - be^{-\frac{\alpha}{2}t} + nC_2 \quad (17)$$

At the moment t_o the straining is stopped and the changes of force acting on filament can be registered as a function of time. At the moment t_o the force acting upon the filament is equal to:

$$F = \left(Cu - C_2\frac{\gamma}{\beta}\right)t_o - C_2 At_o e^{-\frac{\alpha}{2}t_0} - C_2 Be^{-\frac{\alpha}{2}t_0} + C_2\left(\frac{\alpha\gamma}{\beta}+\varphi\right)\frac{1}{\beta} \quad (18)$$

and in a case of u=0 we have:

$$F(t > t_0) = -C_2\frac{\gamma}{\beta}(t-t_o) - C_2 A(t-t_o)e^{-\frac{\alpha}{2}(t-t_o)} - C_2 Be^{-\frac{\alpha}{2}(t-t_o)} + C_2\left(\frac{\alpha\gamma}{\beta}+\varphi\right)\frac{1}{\beta} \quad (19)$$

To simplify previous equation we can introduce:

$$C_2\frac{\gamma}{\beta} = z; C_2 A = a; C_2 B = b; \left(\frac{\alpha\gamma}{\beta}+\varphi\right)\frac{1}{\beta} = n \quad (20)$$

Therefore the force during the relaxation phase can be calculated as:

$$F(t > t_0) = -z(t-t_o) - a(t-t_o)e^{-\frac{\alpha}{2}(t-t_o)} - be^{-\frac{\alpha}{2}(t-t_o)} + nC_2 \quad (21)$$

3.2. Experiment

The verification of proposed model was completed using three type of polypropylene monofilaments of different diameter of d_N=0.15; 0.30; 0.45 mm. The loading phase and relaxation phase was completed using Instron tester series 4204. Each type of monofilaments was subjected to the tensile test with the speed of 5,50 and 500 mm/min till the strain of 15%. Next the samples were subjected to the relaxation during the 180 sec.

3.3. Test results

The results of the empirical values of tensile forces and calculated according to the equation (17) for each type of monofilaments subjected to the tensile test with the speed equal to 5,50 and 500 mm/min are given in Tables 1-3. For each variant five samples were tested. The empirical values $F_{i(.e)}$ of tensile force were determining for five points selected within interval equal to 36.36 s for speed of 5 mm/min , 4.07 s for speed of 50 mm/min and 0.45 s for 500 mm/min. Next, after solution of simultaneous equations coefficients of equation (18) were calculated and the theoretical values of $F_{i(e)}$ were determined. All calculation is given in Tables 1-3. In the second part of investigation the simultaneous equations for relaxation was solved similar like for tension curve. Empirical and theoretical value of relaxation curve was determined. Example of calculations is given in table 4 the result are illustrated at Figs. 2-3. The empirical and theoretical forces calculated for relaxation phase according to equation (21) for the monofilaments of diameter of 0.15 mm are given in Table 4.

Example of relaxation graph of PP filament about diameter d_N = 0.45 mm, using different speed of travel cross-beam V = 5, 50, 500 mm/min show Figs. 2-3.

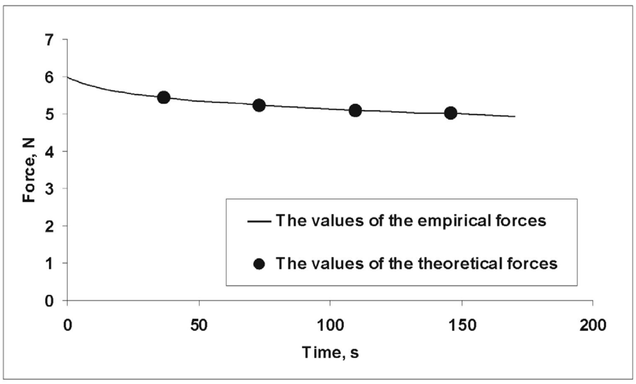

Fig. 2. The values of the empirical and theoretical forces and the values of equation coefficients of the filament PP, d_N=0.15 mm loaded with the rate of V=5 mm/min at the strain to 15 % and then subjected to relaxation

Table 1.
The values of the empirical and theoretical forces and the values of equation coefficients of the filament PP, d_N=0.15 mm loading with the rate of V=5, 50, 500 mm/min at the strain to 15 %

Speed, mm/min	Force, N										Coefficients of Equation			
	$F_{1(.e)}$	$F_{1(.t)}$	$F_{2(.e)}$	$F_{2(.t)}$	$F_{3(.e)}$	$F_{3(.t)}$	$F_{4(.e)}$	$F_{4(.t)}$	$F_{5(.e)}$	$F_{5(.t)}$	y	a	b	C_2n
5	1.82	1.82	3.64	3.63	5.27	5.29	6.36	6.43	7.18	7.29	0.018	1.21	-57.54	3.05
	1.91	1.91	3.64	3.65	5.09	5.05	6.18	6.10	7.09	6.98	0.019	0.64	-24.11	2.48
	2.00	1.96	3.64	3.62	5.09	5.07	6.18	6.16	7.09	7.06	0.020	0.76	-33.20	2.44
	2.00	2.00	3.64	3.60	5.09	5.07	6.00	5.96	6.73	6.64	0.016	4.01	-245.61	2.94
	1.91	1.53	3.64	3.27	5.09	4.68	6.18	5.83	6.91	6.75	-0.018	0.13	17.11	15.90
50	2.27	2.44	4.18	4.20	5.91	5.92	7.18	7.18	8.09	8.08	0.12	4.15	-5.22	5,89
	2.18	2.04	3.91	3.81	5.45	4.96	6.64	6.34	7.45	7.09	-0.25	1.92	17.47	17.93
	2.18	2.13	4.00	3.98	5.82	5.83	7.00	7.04	8.00	7.98	0.22	30.15	-164.59	3.14
	2.18	2.10	4.00	3.92	5.73	5.62	6.91	6.74	7.91	7.68	0.24	51.84	-305.04	2.37
	2.18	1.98	4.18	4.01	5.91	5.68	7.09	6.83	8.09	7.81	0.25	39.00	-214.47	2.28
500	1.64	1.91	3.18	3.15	5.45	5.42	7.09	7.08	8.18	8.23	1.34	45.76	-53.88	5.42
	1.80	1.90	3.60	3.42	5.67	5.57	7.38	7.34	8.55	8.48	-2.63	15.88	12.90	20.58
	1.89	1.93	3.69	3.68	5.67	5.64	7,11	7.04	8.37	8.26	2.83	202.34	-336.59	1.31
	1.71	1.40	3.33	3.18	5.49	5.14	7.38	6.99	8.64	8.50	-8.19	21.00	38.47	46.74
	1.44	1.44	3.06	3.09	5.22	5.23	6.84	6.90	8.10	8.20	2.56	55.53	-78.78	2.05

Table 2.
The values of the empirical and theoretical forces and the values of equation coefficients of the filament PP, d_N=0.30 mm loading with the rate of V=5; 50; 500 mm/min at the strain to 15 %

Speed, mm/min	Force, N										Coefficients equation			
	$F_{1(.e)}$	$F_{1(.t)}$	$F_{2(.e)}$	$F_{2(.t)}$	$F_{3(.e)}$	$F_{3(.t)}$	$F_{4(.e)}$	$F_{4(.t)}$	$F_{5(.e)}$	$F_{5(.t)}$	y	a	b	C_2n
5	8.46	8.55	16.15	15.98	22.31	22.19	26.92	28.84	30.38	30.42	0.046	25.61	22.58	22.84
	7.69	7.78	15.38	15.21	21.54	21.42	26.15	26.07	29.61	29.65	0.046	25.61	22.58	22.07
	7.69	7.78	15.38	15.21	21.54	21.42	26.15	26.07	29.61	29.65	0.046	25.61	22.58	22.07
	8.08	8.51	16.15	16.79	22.31	22.29	26.92	27.50	30.00	30.42	-0.0052	9.11	55.21	40.45
	7.69	7.55	15.38	14.90	21.92	21.43	26.54	26.08	30.00	29.66	0.068	94.31	-70.98	15.52
50	8.08	7.87	16.54	16.42	23.85	23.83	28.46	28.63	32.31	32.73	1.06	1178.06	-1895.67	9.24
	8.46	8.23	16.92	16.72	24.23	24.14	28.85	28.84	32.69	32.72	0.99	1018.22	-1606.15	10.80
	8.46	8.39	16.92	16.96	23.85	23.69	28.46	28.18	32.31	31.97	0.96	594.12	-862.92	10.72
	8.85	8.79	16.92	16.99	23.85	23.73	28.46	28.25	32.31	32.04	0.96	614.19	-914.65	10.79
	8.46	8.52	16.15	16.38	23.46	23.41	28.46	28.39	32.31	32.23	0.87	252.48	-325.69	13.20
500	5.00	5.97	13.86	13.68	25.02	25.12	33.11	32.93	38.11	38.12	0.98	156.84	-115.52	40.76
	6.93	6.93	15.78	15.78	26.56	26.32	33.88	33.95	38.11	37.69	-0.04	188.81	-159.37	41.89
	4.62	4.16	10.39	11.49	17.71	17.08	24.25	35.66	28.87	28.30	-29.05	61.09	156.73	173.03
	11.55	11.74	23.10	23.52	33.49	33.44	39.27	29.44	42.73	43.33	4.98	426.03	-521.37	29.92
	12.32	11.57	23.48	24.87	33.11	34.28	39.65	39.68	43.12	43.37	-11.42	66.05	74.48	92.09

Table 3.
The values of the empirical and theoretical forces and the values of equation coefficients of the filament PP, d_N = 0.45 mm loading with the rate of V = 5; 50; 500 mm/min at the strain to 15 %

Speed, mm/min	Force, N											Coefficients equation		
	$F_{1(.e)}$	$F_{1(.t)}$	$F_{2(.e)}$	$F_{2(.t)}$	$F_{3(.e)}$	$F_{3(.t)}$	$F_{4(.e)}$	$F_{4(.t)}$	$F_{5(.e)}$	$F_{5(.t)}$	y	a	b	C_2n
5	15.38	14.78	32.31	32.10	44.61	44.45	53.08	52.83	60.00	59.97	0.17	337.33	-322.21	23.22
	15.38	16.98	33.85	35.73	47.69	48.11	56.92	56.66	63.08	62.71	0.063	108.11	20.22	52.51
	15.38	13.38	32.31	31.21	45.38	45.39	53.85	53.87	61.54	61.31	0.20	7858.99	-13573.92	17.70
	15.38	14.92	32.31	32.24	44.61	44.59	53.08	52.97	60.00	60.11	0.17	337.33	-322.21	23.36
	13.85	14.21	30.00	30.03	43.08	42.66	52.31	51.58	59.23	58.27	0.12	162.12	-90.23	33.58
50	15.26	15.53	33.68	33.93	48.42	48.61	58.42	58.50	66.84	66.86	1.96	1049.38	-1469.77	19.98
	16.31	16.35	34.21	34.10	48.95	48.72	57.89	57.20	64.74	63.54	1.38	1200.28	-1696.0	30.68
	15.79	15.82	33.68	33.59	48.42	48.59	57.89	58.06	65.26	65.55	1.65	804.08	-1070.75	26.27
	15.12	15.00	32.56	32.53	46.51	46.56	55.81	55.59	63.95	63.36	2.02	2761.54	-4433.37	18.58
	14.74	15.24	32.10	32.24	45.79	45.98	54.74	54.87	62.63	62.68	2.05	3318.05	-5411.89	17.21
500	9.23	9.64	23.08	22.95	47.69	48.64	63.08	65.37	75.38	79.13	26.33	4375.55	-7929.26	1.92
	6.15	8.25	22.31	22.51	47.69	48.02	64.61	64.79	72.31	71.08	-15.18	461.93	-343.25	133.44
	12.31	12.68	33.85	36.23	53.85	53.69	66.92	66.79	73.85	73.32	-8.4	216.25	-24.49	110.00
	9.23	7.94	25.38	25.07	49.23	49.03	66.15	66.79	76.92	79.22	10.00	476.26	-537.77	57.01
	10.77	11.76	32.31	30.07	53.85	53.85	66.92	66.39	70.77	69.35	-44.69	264.06	92.38	231.93

Table 4.
The values of the empirical and theoretical forces and the values of equation coefficients of the filament PP, d_N=0.15 mm loaded with the rate of V=5; 50 mm/min at the strain to 15 % and then subjected to relaxation

Speed, mm/min	Force, N											Coefficients equation		
	$F_{1(.e)}$	$F_{1(.t)}$	$F_{2(.e)}$	$F_{2(.t)}$	$F_{3(.e)}$	$F_{3(.t)}$	$F_{4(.e)}$	$F_{4(.t)}$	$F_{5(.e)}$	$F_{5(.t)}$	z	a	b	C_2n
5	5.09	5.09	3.64	3.64	2.36	2.37	1.45	1.46	0.73	0.74	0.023	-30.90	34.55	4.32
	4.91	4.91	3.45	3.45	2.27	2.28	1.45	1.47	0.73	0.76	0.026	-136.60	207.47	4.85
	4.73	4.70	3.27	3.27	2.18	2.20	1.45	1.50	0.91	0.98	0.015	-16.60	9.99	3.23
	5.00	5.00	3.55	3.56	2.45	2.46	1.73	1.75	1.09	1.11	0.024	-324.03	531.49	4.83
	4.73	4.70	3.27	3.27	2.18	2.20	1.45	1.50	0.91	0.98	0.015	-16.60	9.99	3.23
50	5.82	5.81	3.91	3.90	2.45	2.45	1.45	1.45	0.73	0.73	0.15	-14.44	3.53	3.21
	5.45	5.50	3.82	3.86	2.55	2.57	1.64	1.65	0.91	0.84	0.24	-20.22	15.28	4.51
	5.91	5.91	4.09	4.09	2.64	2.64	1.73	1.73	1.09	1.09	0.18	-39.64	39.82	3.86
	5.91	5.91	4.18	4.18	2.73	2.73	1.82	1.82	1.09	1.08	0.26	-145.50	217.34	5.14
	6.18	6.19	4.36	4.37	2.91	2.92	1.91	1.91	1.18	1.18	0.19	-17.69	9.18	3.96

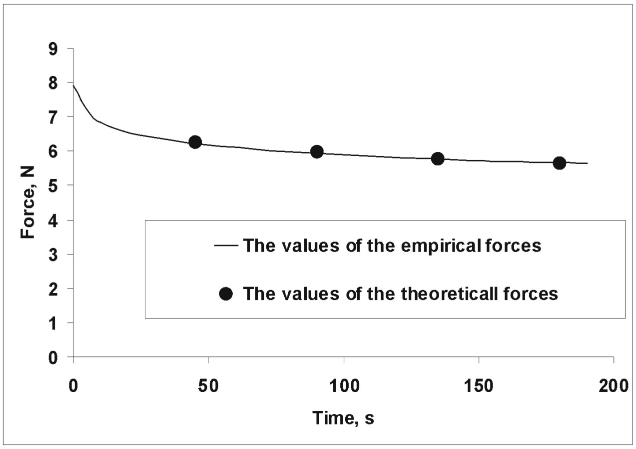

Fig. 3. The values of the empirical and theoretical forces and the values of equation coefficients of the filament PP, d_N=0.15 mm loaded with the rate of V=50 mm/min at the strain to 15 % and then subjected to relaxation

3.4. Conclusions

On the basis of conducted examinations the following conclusions can be drawn:

1. The proposed model represents adequately stress – strain relationships of polypropylene monofilaments subjected to tensile test.
2. Derived model precisely describes the relaxation phenomenon of investigated monofilaments conducted during 180 sec.

4. Description of the rheological properties of PP nonwovens subjected to the compression loads

To describe the rheological properties of polypropylene nonwovens the rheological model presented in Fig. 1. was modified assuming the weight of piston as equal to zero.

4.1. The analytical solution of rheological equations

During the model straining the compression force F is resolved into force F_1 acting in the element I and into the force F_2 acting in the element II. It is assumed that force F acting upon samples is proportional to the strain ε till the limit value ε_0 (limit of proportionality) is reached. The measurements show, that after straining in the interval $0<\varepsilon<\varepsilon_0$ residual strains of sample are not observed. In this phase of compression the force in the element I equals to:

$$F_1 = C_1\varepsilon \qquad (22)$$

and in the element II equals to:

$$F_2 = C_2\varepsilon \qquad (23)$$

As long as $F_2 < T$, where ε is the strain of model, the total force loading the model is:

$$F = F_1 + F_2 = (C_1 + C_2)\varepsilon = C\varepsilon \qquad (24)$$

where C is the total stiffness of model equals to C1 + C_2. When the load in segment II reaches the value $T=C_2ut_0$, the piston begins to move downwards in viscous liquid with the velocity

$$-\frac{d\varepsilon_{2.2}}{dt} = \frac{F_2 - (T + K\varepsilon_{2.2})}{\eta} = \frac{C_2\varepsilon - C_2\varepsilon_{2.2} - T - K\varepsilon_{2.2}}{\eta} = \frac{C_2\varepsilon - T - (C_2 + K)\varepsilon_{2.2}}{\eta} \qquad (25)$$

The rate of increase of model strain depends on the used tester. According to present standards, the stress-strain testers should work with constant rate of deformation i.e.:

$$\varepsilon = ut \qquad (26)$$

where u- is speed of motion of a compression food, and t- is time of motion of a foot. If we introduce this value to equation (25) we receive

$$-\frac{d\varepsilon_{2.2}}{dt} = \frac{C_2ut - T - (C_2 + K)\varepsilon_{2.2}}{\eta} \qquad (27)$$

The integral of this equation is

$$\varepsilon_{2.2} = \frac{C_2u}{C_2 + K}\left[t - t_0 + \tau\left(1 - e^{\frac{t-t_0}{\tau}}\right)\right] \qquad (28)$$

where time of relaxation is expressed by equation

$$\tau = \frac{\eta}{C_2 + K} \qquad (29)$$

and t_o is time at the limit of proportionality.

The load acting upon the model can be expressed as:

$$F = F_1 + F_2 = C_1ut + C_2ut - C_2\varepsilon_{22} \qquad (30)$$

and substituting to equation (30) the formula (28) we have:

$$F = Cut - \frac{C_2^2u}{C_2 + K}\left[t - t_0 + \tau(1 - e^{\frac{t-t_0}{\tau}})\right] \qquad (31)$$

For practical use the equation (31) can be presented in simpler form of:

$$F = Cut - Au\left[t - t_0 + \tau(1 - e^{\frac{t-t_0}{\tau}})\right] \qquad (32)$$

where:

$$A = \frac{C_2}{C_2 + K} \qquad (33)$$

or

$$Y = A_IX + B_I + De^{\frac{X-X_0}{R}} \qquad (34)$$

where:

$$A_I = u(C - A) \qquad (35)$$

$$B_I = Au(t_0 - \tau) \qquad (36)$$

$$D_I = Au\tau \qquad (37)$$

$$Y = F \qquad (38)$$

$$X = t \quad (39)$$

$$R = \tau \quad (40)$$

Both simpler forms of the equation (31) can be used for verification of theoretical considerations concerning the prediction of the load-deformation curves during the compression of non-woven fabrics. In presented paper the method of non-linear least square method was used to find the coefficients of the equation (34).

The model describing the relaxation phenomena

Stopping the instrument at the moment t_1 ,we allowed the material to undergo stress relaxation. At this moment the deformation of the sample was equal to deformation of spring in element I. The force in element I was not changing in a period of time of relaxation, while the force in element II was diminishing. The decrease of the value of force F_2 results from the recovery of the compressed spring in element II due to the downward movement of piston joined with spring II. The recovery of spring II can be realised till the moment of equilibrium of the value of force F_2 with the value of friction force T. The displacement of the piston in viscous fluid can be described by the equation:

$$\frac{d\varepsilon_{2.2}}{dt} = \frac{F_2}{\eta} \quad (41)$$

At the moment of t_1 the shift of a piston was equal to:

$$\varepsilon_{2.2} = \frac{1}{C_2 + K}\left\{ \begin{matrix} [T + C_2 u(t_0 - \tau)]e^{\frac{1}{\tau}(t_1 - t_0)} \\ - C_2 u(t_1 - \tau) - T \end{matrix} \right\} \quad (42)$$

The force resulting from the piston movement F_{2vp} can be determined by equation (43):

$$F_2 = -F_2(t_1) + (C_2 + K)\varepsilon_{2.2} + T \quad (43)$$

Introducing the equation (43) to formula (41) we have:

$$-\frac{d\varepsilon_{2.2}}{dt} = -\frac{C_2\varepsilon(t_1) - T}{\eta} + \frac{C_2 + K}{\eta}\varepsilon_{2.2} \quad (44)$$

Integral of this equation is:

$$\varepsilon_{2.2} = \left(\varepsilon_{2.2.1} - \frac{C_2\varepsilon(t_1) - T}{C_2 + K}\right)e^{\frac{(t_1 - t)}{\tau}} + \frac{C_2\varepsilon(t_1) - T}{C_2 + K} \quad (45)$$

Determination of equation (45) allows for the definition of the force F during the relaxation in a form of equation (46):

$$F = -C\varepsilon(t_1) + C_2\varepsilon_{2.2} = -Cut_1 + C_2\left[\left(\varepsilon_{2.2.1} - \frac{C_2\varepsilon(t_1) - T}{C_2 + K}\right)e^{\frac{t_1 - t}{\tau}} + \frac{C_2\varepsilon(t_1) - T}{C_2 + K}\right] \quad (46)$$

Introducing to the equation (46) the following formulae:

$$B_2 = C_2\frac{C_2\varepsilon(t_1) - T}{C_2 + K} - Cut_1 \quad (47)$$

$$D_1 = C_2\left(\varepsilon_{2.2.1} - \frac{C_2\varepsilon(t_1) - T}{C_2 + K}\right) \quad (48)$$

we can reduce the equation (46) to a form of:

$$F = B_2 + D_2 e^{\frac{t_1 - t}{\tau}} \quad (49)$$

Simplified form of equation (46) was used for verification of theoretical consideration on the relaxation of non-woven fabric after compression. The non-linear least square method was used to fit the parameters of model equation (50) using computer NCSS 6.0

$$Y = B_2 + D_2 e^{\frac{X_1 - X}{\tau}} \quad (50)$$

Recovery after compression

At the moment t_r the compression food changes its direction. At that moment the recovery process after compression begins. In both elements of model the linear recovery of springs I and II is observed and the elastic deformations resulting from compression of tested non-woven fabrics are reversed. When the force in element II equals to friction force T the upward movement of piston begins what results in non-linear recovery of force. In element I the force is proportional to the deformation of model, therefore we have:

$$\varepsilon_c = ut_2 - u[t - t_2] = 2t_2 u - tu \quad (51)$$

The force in element I can be defined by the formula (31)

$$F_1 = C_1\varepsilon_c = C_1[2t_2 u - tu] \quad (52)$$

and the force in element II is equal to:

$$F_2 = C_2\varepsilon_c - C_2\varepsilon_{2.2} = C_2[2t_2 u - tu] - C_2\varepsilon_{2.2} \quad (53)$$

The displacement of piston $\varepsilon_{2.2}$ can be expressed by the following equation:

$$\frac{d\varepsilon_{2.2}}{dt} = \frac{F_2 - T - K\varepsilon_{2.2}}{\eta} \quad (54)$$

The solution of differential equation (54) is given in a form of

$$\varepsilon_{2.2} = \frac{1}{1 + \kappa}\left[\begin{matrix} 2t_2 u - \vartheta - tu + u\tau + \\ [\varepsilon_{2.2.2}[1 + \kappa] - t_2 u + \vartheta - u\tau]e^{\frac{t_2 - t}{\tau}} \end{matrix}\right] \quad (55)$$

The total force acting on a model during the recovery process can be defined from the following formulae:

$$F = F_1 + F_2 \quad (56)$$

$$F_1 = C_1[2t_2 u - tu] \quad (57$$

$$F_2 = C_2[2t_2 u - tu] - C_2\varepsilon_{2.2} \quad (58)$$

$$F = C2t_2 u - Ctu - C\frac{z}{1 + \kappa}\left[\begin{matrix} 2t_2 u - (\vartheta - u\tau) - tu \\ + [\varepsilon_{2.2.2}[1 + \kappa] - t_2 u + \vartheta - u\tau]e^{\frac{t_2 - t}{\tau}} \end{matrix}\right] \quad (59)$$

Substituting to equation (59) the following of expressions:

$$P = t_2 u \quad (60)$$

$$W = [\varepsilon_{2.2.2}[1 + \kappa] - P + \vartheta - u\tau] \quad (61)$$

$$Q = C\left(2P\left(1 - \frac{z}{1 + \kappa}\right) + \frac{z(\vartheta - u\tau)}{1 + \kappa}\right) \quad (62)$$

$$S = \frac{Cz}{1 + \kappa}G \quad (63)$$

$$R = u\tau \quad (64)$$

we have the final equation describing the recovery process of tested non-woven fabrics in a form of :

$$F = Q - Stu - W\exp\left(\frac{P - ut}{R}\right) \quad (65)$$

The equation (65) was introduced to computer program NCSS6.0 in a form of :

$$Y = A_3 - B_3 X - D_3\exp\left(\frac{t_2 - X}{\tau}\right) \quad (66)$$

where:

$$A_3 = Q \tag{67}$$

$$B_3 = Su \tag{68}$$

$$D_3 = W \tag{69}$$

and the values of parameters of model equation were fitted using the non-linear least square method.

4.2. Procedure of model verification

Based on the above equations describing the shape of the elastic recovery curve it is possible to calculate the rheological properties of investigated type of nonwovens under the stress-recovery test.

Using the non-linear least squares method it is necessary to calculate the value of τ fitting the equation of the relaxation curve to experimental data. Basing on the value of τ which was found from the equation describing the relaxation curve and experimental points of the elastic recovery curve it is necessary to fit the equation 5 finding the value of A, B and C.

4.3. Experiment

The verification of proposed model was completed using three various types of nonwoven samples. All samples were manufactured using needle-punching technology. The nonwovens were prepared from polypropylene fibres of linear density equals to 6.7, 12, 18 dtex and length equals to 60 mm. The area mass of nonwovens was approximately equals to 400 g/m^2. The prepared samples were investigated under compression load using Instron tester. All samples were compressed to the pressure equals to 0.0247 N/mm^2 with the speed of 5 mm/min. The characteristics of test material are presented in Table 5.

Table 5.
Design of experiment

Type of fibre	Linear density [dtex]	Length of fibre [mm]	Depth of needle punching [mm]	Number of needle punching [1/cm^2]
PP	6.7 12 18	60	14	100 130 160

In order to carry out the verification the value of t_0, t_1, t_2 was calculated for each set of investigations. Based on the set of experimental data the constant values of equations of compression, relaxation, and recovery: A_1, A_3, B_1, B_2, B_3, D_1, D_2, D_3 and τ were evaluated using method of Levenberg-Marquardt. The example of the shape of theoretical and experimental curves for all types of designs is presented in Figs. 4 a-c. Next the correlation between the experimental data and data from the theoretical model were evaluated. The values of correlation coefficients are given in Table 6.

4.4. Conclusions

The results presented in Table 6 indicate that for each investigated type of nonwovens there is no significant difference between the shape of the theoretical and experimental elastic recovery curve during the recovery test (as shown in Fig. 4). For all variants of nonwovens the correlation coefficient between experimental and theoretical data is between 0.883 to 0.947.

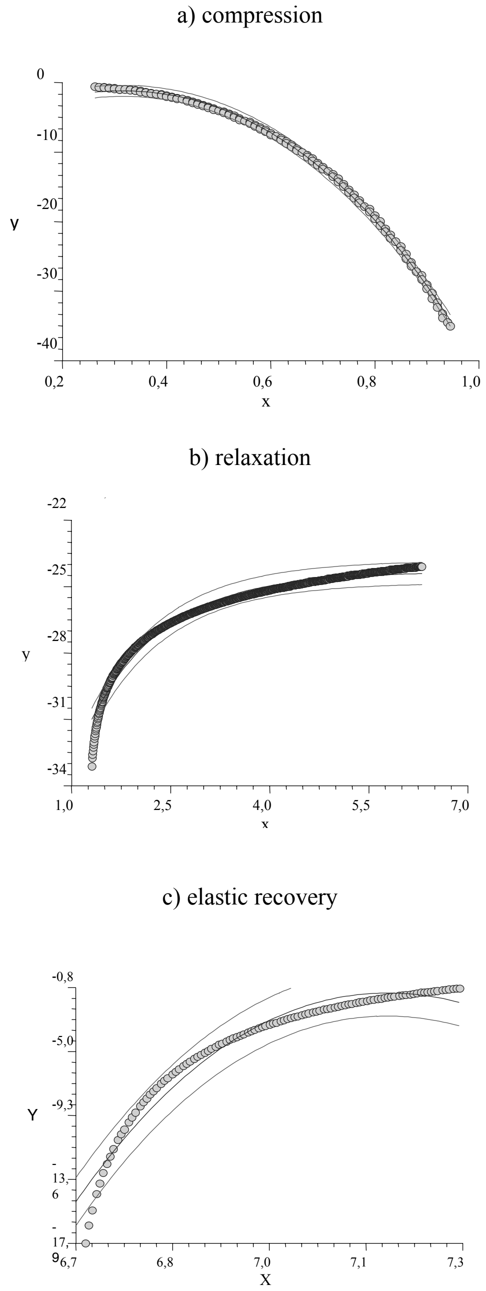

Fig. 4. Example of the verification of the compression, the relaxation and the elastic recovery curves for PP nonwovens

Table 6.
The list of correlation coefficients

Type of fibres	Number of needle punching [l/cm^2]	Correlation coefficient of compression	Correlation coefficient of relaxation	Correlation coefficient of recovery
PP6,7/60	100	0.981	0.988	0.898
	130	0.922	0.986	0.906
	160	0.922	0.987	0.924
PP12/60	100	0.987	0.986	0.903
	130	0.995	0.984	0.899
	160	0.992	0.985	0.883
PP18/60	100	0.988	0.987	0.928
	130	0.997	0.983	0.947
	160	0.999	0.984	0.941

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