Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 49 ISSUE 2 December 2011

Design of damping systems with required frequency spectrum

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Received 22.10.2011; published in revised form 01.12.2011

Analysis and modelling

ABSTRACT

Purpose: The presented work extends the task of synthesis with new methods, with particular focus on the method of active and passive synthesis of vibrating damping mechanical systems. This paper is concerned with the formulation and formalisation of the problem of active and passive synthesis of mechanical damping systems.
Design/methodology/approach: The work proposes a combination of the synthesis method of vibrating systems with the method of determining the active force as the method for active synthesis of mechanical damping systems and the values of two-terminal damping in case passive synthesis of mechanical damping systems.
Findings: The performed active and passive synthesis of restrained systems has demonstrated an ambiguity of synthesis of damping systems. The resulting structures and set system parameters are not the only ones that can be obtained in the case of the assumed frequencies.
Research limitations/implications: The scope of discussion is the synthesis of discrete damping systems.
Practical implications: The advantage of the proposed method of determining the active force in the system is the ease of its programming. The proposed method can be included in the method of proportional control with force feedback from the state.
Originality/value: The document is also an attempt at identifying new opportunities and a direction of research in the design of machine components with a desired frequency spectrum.
Keywords: Applied mechanics; Constructional design; Rheological Voigt's-Kelvin's model; Rayleigh's model; Impedance; Characteristic functions

Reference to this paper should be given in the following way:
A. Dymarek, T. Dzitkowski, Design of damping systems with required frequency spectrum, Journal of Achievements in Materials and Manufacturing Engineering 49/2 (2011) 269-274.

1. Introduction

Vibration damping occupies a significant place among the extensive issues of machinery and apparatus dynamics. It is one of the factors of mechanical energy dissipation, inseparably connected with mechanical systems movement. The harmful effect of vibration during machine operation, especially in the area of the resonant frequencies, can lead to a loss of stability or damage. A proper selection of frequency and operation outside the resonance range solve the problem of vibration avoidance. Leaving the resonance zone is the basic condition of apparatus functioning, however, it does not eliminate the problem completely. In many machines a lot of free vibration frequencies appear. In these cases damping is of crucial importance, because it lowers vibration amplitude significantly [10, 13].

Equally important phenomenon, in which damping plays a major role, is the phenomenon of transition through the resonance zone. A number of components are adapted to operate in supercritical zone i.e. at frequencies higher than resonance frequency. In such a case transition through the resonance zone occurs during machine movement. Then poorly dampened components are becoming temporarily subjected to strong vibrations. When damping phenomenon limits vibration amplitude insufficiently, additional damping is introduced [10, 20, 21].

To prevent this, it is advantageous to design a machine to work in the desired resonant frequency range. The adoption of an appropriate model for the machine subject to optimal control depends on the experience and intuition of the designer. The adopted model may prove to be incorrect due to resonant frequencies, which have a significant impact on the controlled object. It is necessary to clearly delineate the criterion for structure and parameter identification of the model, as well as for controlling that model, which will meet the dynamic properties of the machine under optimal control. One of the conditions for the adoption of such a machine model is a synthesis of mechanical systems with the desired dynamic properties [1-9, 22, 23]. However, the resulting model does not always prove correct due to the resonant frequencies obtained that are the basis for adoption of an appropriate control system.

The presented work extends the task of synthesis with new methods, with particular focus on the method of active and passive synthesis of vibrating damping mechanical systems.

Passive synthesis was done, using well-known methods of discrete vibration systems synthesis and by introduction of an assumption concerning damping type element, existing in the sought after system, and taking rheological Voigt's-Kelvin's model, Rayleigh's model and damping assumption as a mass damping model [5, 8].

Active synthesis is understood as a search for parameters and structure of dynamic systems in conjunction with the value of the adjusting force based on the requirements put forward. These requirements apply to obtaining the set dynamic properties of systems with control as characteristic functions (impedance, mobility) [14-19].

2. The synthesis of mechanical vibrating systems

This chapter will discuss methods for synthesis of vibrating systems with a cascade structure used in active and passive synthesis of mechanical damping systems. The presented methods are used for distribution of characteristic functions in the form of impedance describing restrained systems. The dynamic characteristic subjected to synthesis is a rational function, namely:

$$U_U(s) = H\frac{\prod_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} = H\frac{s^l + d_l s^{l-2} + \ldots + d_1}{s^k + c_k s^{k-2} + \ldots + c_0 s}, \tag{1}$$

where: l – odd degree of the numerator for $l-k=1$, k – degree of denominator, ω_{2n-1} – resonant frequency, ω_{2n} – anti-resonant frequency, H – any positive real number.

2.1. Method of characteristic function distribution into a continued fraction

The impedance under consideration is presented as a quotient of two polynomials:

$$U_U(s) = \frac{L_l(s)}{M_{l-1}(s)} \tag{2}$$

If the numerator is divided by the denominator (2), the result is a continued fraction in the form:

$$U_U(s) = m_1 s + \cfrac{1}{\cfrac{s}{c_1} + \cfrac{1}{m_2 s + \ldots + \cfrac{1}{\cfrac{s}{c_{l-1}} + \cfrac{1}{m_{l-1}s + \cfrac{1}{\cfrac{s}{c_l} + \cfrac{1}{m_l s}}}}}} \tag{3}$$

As a result of the distribution of the characteristic function (1) into a continuous fraction (3), the discrete mechanical system (Fig. 1) and values of elastic $c_1,\ldots,c_{\frac{l+1}{2}-2},c_{\frac{l+1}{2}-1}$ and inertial type $m_1,m_2,\ldots,m_{\frac{l+1}{2}-1},m_{\frac{l+1}{2}}$ elements are obtained.

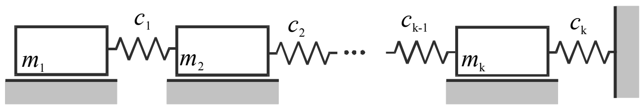

Fig. 1. System obtained through distribution of the characteristic into a continuous fraction

2.2. Algorithm for determining characteristics of elastic one-ports

The method uses the properties of rational functions such as impedance and mobility. Restrained systems described through dynamic characteristics have an anti-resonant value for zero, regardless of the number of restraints imposed on the system. To perform the synthesis of such systems, the author has proposed a transformation of the present characteristic function. The transformation consists of multiplying the denominator and the numerator of impedance by the Lapace operator raised to the appropriate power, according to the quantity of restraints desired.

The starting point for the presented method is a characteristic function (1) describing the dynamic properties in the form of resonant and anti-resonant frequencies of the desired system.

To designate values of stiffness for the restraint, the coefficient standing closest to the lowest numerator power is to be divided by the coefficient of the lowest denominator power $U_U(s)$ (1) yielding:

$$H\frac{d_0}{c_1 s}=\frac{c}{s}, \tag{4}$$

where: $\frac{c}{s}$ corresponds to a flexible element in the impedance set.

The resulting value is the sought stiffness of the restraint ascertained from the dynamic characteristic in the form of impedance (1).

This method brings the characteristic function to a form, which permits the use of known methods of synthesis. In this case, the denominator of the impedance in question must be written in the following form:

$$s\left(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c_1\right). \tag{5}$$

The polynomial (5) is multiplied by the value of the ascertained stiffness (4) as follows:

$$H\frac{d_0}{c_1}\left(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c_1\right). \tag{6}$$

Function (6) is subtracted from the numerator of the considered impedance (1) as follows:

$$H\left(d_l s^l+d_{l-1}s^{l-1}+\ldots+d_0\right)-H\frac{d_0}{c_1}\left(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c_1\right), \tag{7}$$

as a result, the following expression is obtained:

$$H\left(d_l s^l+d^1_{l-1}s^{l-2}+\ldots+d^1_1 s\right). \tag{8}$$

After performing the calculations (4-8), the impedance $U_U(s)$ (1) can be written as:

$$U_U(s)=H\frac{d_0}{c_1 s}+H\frac{s(d_l s^{l-1}+d^1_{l-1}s^{l-2}+\ldots+d^1_1)}{s(c_k s^{k-1}+c_{k-1}s^{k-2}+\ldots+c)_1}=\frac{c}{s}+U'_U(s), \tag{9}$$

where: $U'_U(s)$ - resulting impedance, which is subjected to further synthesis through a method consisting of its distribution into a continued fraction or simple fractions.

The numerical value $\frac{d_0}{c_1}$ is the upper limit for the range of numbers greater than zero, from which the sought stiffness is determined. The stiffness depends on the number of restraints on the system. In the present case the author has assumed that the system has one restraint. The number of restraints imposed does not affect the algorithm, but only the value of sought elements.

In the case of determining p elements of the $\frac{c}{s}$ type, the considered characteristic in the form of (1) should be multiplied by $\frac{s^{p-1}}{s^{p-1}}$. This procedure is there to obtain additional zeroes within the inverse impedance function, which signify the number p-1 of restraints on the synthesised system. In addition to this, figures should be specified for sequentially designated p elastic elements, resulting from the synthesis of the impedance function $U_U(s)$ for the interval $\left(0, H\frac{d_0}{c_1}\right\rangle$. When the value of elasticity does not belong to the interval $\left(0, H\frac{d_0}{c_1}\right\rangle$, then the continued use of this method is impossible because the inertial elements obtained in subsequent steps of the synthesis take negative values.

2.3. Synthesis of restrained systems using the method of proportional distribution

The presented method, as opposed to the ones described above, does not necessitate the value of the anti-resonant frequency to be specified.

Dynamic properties to be satisfied by the system, with a cascade structure and restrained on both sides, are given in the form of the resonant frequencies only.

In case of application of such properties, the dynamic characteristic is built in the form of (1) assuming that every second resonant frequency becomes an anti-resonant frequency and obtaining the appropriate form of impedance:

$$U_U(s)=H\frac{\prod_{n=1}^{i}\left(s^2+\omega^2_{2n-1}\right)}{s\prod_{n=1}^{j}\left(s^2+\omega^2_{2n-3}\right)}=H\frac{s^l+d_l s^{l-2}+\ldots+d_1}{s^k+c_k s^{k-2}+\ldots+c_0 s}, \tag{10}$$

where: $\omega_{2n-1}, \omega_{2n-3}$ – resonant frequency H -any positive real number.

Such a form of the dynamic characteristic is subjected to synthesis using the method of its distribution into a continuous fraction. The result of the synthesis carried out at this stage is a cascade system as presented in Figure 1.

The resulting mechanical system is subjected to modification. The modification consists of proportional distribution of inertial and elastic parameters of the system. This results in a system with two restraints (Fig. 2) that meets the desired requirements in the form of resonant frequencies, which are the zeros and poles of the dynamic characteristic considered (10).

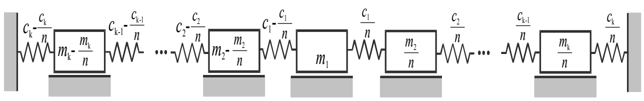

Fig. 2. Discrete system obtained through synthesis using the method of proportional distribution

3. Passive synthesis of mechanical damping systems

In order to start the synthesis of damping systems [9, 11, 14] we should specify the properties of the sought-after system and the damping model, in accordance with the presented course of action.

1. Specify the values of resonance and counter-resonance frequencies in case of free vibration, i.e.:

$$\begin{cases} \omega_{b1}, \omega_{b2}, \dots, \omega_{bn}, \\ \omega_{z1}, \omega_{z2}, \dots, \omega_{zn}. \end{cases} \tag{11}$$

2. Determine the value of dimensional vibration factor λ, in case of V-K model:

$$b_{ci} = \lambda\, c_i \ , \tag{12}$$

$$\lambda = \frac{2h_n}{\omega_{bn}^2} \ , \tag{13}$$

$$0 < \lambda < \frac{2}{\omega_n} , \tag{14}$$

where:
b_i – damping, c_i – stiffness resulting from the synthesis carried out, $\lambda = \text{idem}$ – dimensional vibration factor, h_n – parameter corresponding to damping in the system for the individual resonance frequencies and counter-resonance frequencies.

3. Determine the parameter h_n from the equation (13) of in the following way:

$$h_n = \frac{\lambda\, \omega_{bn}^2}{2} \ , \tag{15}$$

or by providing damping decrement δ_n for the individual resonance frequencies and counter-resonance frequencies:

$$\delta_n \approx \frac{2\pi\, h_n}{\omega_{(z)bn}} . \tag{16}$$

4. Determine the value of dimensional vibration factor h in case of mass model of damping:

$$b_{mi} = 2h \cdot m_i \ , \tag{17}$$

where:
b_i – damping, h, – parameter corresponding to damping in the system, having frequency dimension, m_i – the value of inertia element determined as a result of synthesis.
The h parameter in the discussed case is constant, i.e.:

$$h = \text{idem} , \tag{18}$$

and its value should be chosen from the following bracket

$$0 < h < |\omega_{\min}| , \tag{19}$$

where: $|\omega_{\min}| \neq 0$ – the smallest value of resonance frequency, when the system restrained is synthesized or the smallest value which is equal to counter-resonance frequency when the half-defined system is synthesized.

5. Determine the value of dimensional vibration factor h in case of Rayleigh model

$$b_{ci} \frac{\text{Ns}}{\text{m}}, b_{mi} \frac{\text{Ns}}{\text{m}}, h_{Rn} = h_n + h \ \ \frac{1}{s} \tag{20}$$

The examined characteristics are immobility U(s) and mobility V(s) built on the basis of assumed dynamic properties as described in paragraphs 1-5.
The immobility function U(s)

$$U(s) = H\frac{d_l s^l + d_{l-1} s^{l-1} + \ldots + d_0}{c_k s^k + c_{k-1} s^{k-1} + \ldots + c_1 s} , \tag{21}$$

The immobility function V(s)

$$V(s) = H\frac{c_k s^k + c_{k-1} s^{k-1} + \ldots + c_1 s}{d_l s^l + d_{l-1} s^{l-1} + \ldots + d_0} \tag{22}$$

where: l – odd or even numerator order with l-k=1, k – denominator order, H – any real positive number.
In order to determine elastic and inertial values, characteristic functions describing free vibrations in the form of slowness U'(s) (23) and mobility V'(s) (24) should be subject to the synthesis.
The immobility function U'(s)

$$U'(s) = H\frac{d_l s^l + d_{l-2} s^{l-2} + \ldots + d_0}{c_k s^k + c_{k-2} s^{k-2} + \ldots + c_1 s} , \tag{23}$$

The mobility function V'(s)

$$V'(s) = H\frac{c_k s^k + c_{k-2} s^{k-2} + \ldots + c_1 s}{d_l s^l + d_{l-2} s^{l-2} + \ldots + d_0} , \tag{24}$$

where: l – odd or even numerator order with l-k=1, k – denominator order, H – any real positive number.
Then, the values of two-terminal damping type are to be determined on the basis of the relationship between (12) and (17), using inertial and elastic elements obtained in the first step of the synthesis

4. Active synthesis of mechanical damping systems

The active synthesis presented in this work is a combination of passive synthesis methods with the direct method of determination of active force on the basis of its dynamic characteristic. As a result of a distribution of the characteristic function in the form of impedance or mobility, the structure of the discrete mechanical system and the value of inertial and elastic parameters, together with the control force, is obtained. The researched system should meet the desired dynamic properties. On the basis of the adopted dynamic properties (25) in the form of a series of resonant and anti-resonant frequencies and the adopted resonance frequency drop value, also called in literature the stability reserve, the characteristic function is built. The resultant characteristic is a real rational function and the difference between the degree of the polynomial in the denominator and numerator should equal 1.

$$\begin{aligned}
&\omega_1, \omega_3, \ldots, \omega_{2n-1} \ \text{– resonant frequencies,} \\
&\omega_0, \omega_2, \ldots, \omega_{2n} \ \text{– anti-resonant frequencies,} \\
&h_1, h_3, \ldots, h_{(2n-1)} \ \text{– decline in natural vibration angular frequency.}
\end{aligned} \tag{25}$$

To determine the dynamic characteristics in the form of mechanical impedance, the adopted values of angular frequency (25) should be included in the relation (26) in the form:

$$\begin{aligned}
U(s) &= H\frac{L(s)}{M(s)} = H\frac{\prod_{n=1}^{i}\left(s^2+\left(\omega_{2n-1}+h_{(2n-1)}\right)^2\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} = \\
&= H\frac{\prod_{n=1}^{i}\left(s^2+2h_{(2n-1)}s+\left(\omega_{2n-1}^2+2\omega_{2n-1}h_{(2n-1)}+h_{(2n-1)}^2\right)\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} = \\
&= H\frac{\prod_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} + \\
&H\frac{\sum_{n=1}^{i}\prod_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)\left(2h_{(2n-1)}s+\left(\omega_{2n-1}^2+2\omega_{2n-1}h_{(2n-1)}+h_{(2n-1)}^2\right)\right)-}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} \\
&\frac{-\sum_{n=1}^{i}\left(s^2+\omega_{2n-1}^2\right)\left(2h_{(2n-1)}s+\left(\omega_{2n-1}^2+2\omega_{2n-1}h_{(2n-1)}+h_{(2n-1)}^2\right)\right)}{s\prod_{n=0}^{j}\left(s^2+\omega_{2n}^2\right)} \\
&= U_U(s) + U_F(s),
\end{aligned} \tag{26}$$

where: H – constant of proportionality, s – Laplace operator, $\omega_{2n-1}, \omega_{2n}$ – values of desired resonant and anti-resonant frequencies, $U_U(s)$ – dynamic characteristic of the desired system, $U_F(s)$ – dynamic characteristic of control force sought.

A characteristic function (26) obtained in this way is the mechanical impedance of the sought system. Mechanical impedance as a sum of two functions is subjected to distribution using the methods of passive synthesis and the method of direct determination of the value of the active force.

Method of Determining the Active Force

The starting point for the presented method is the system obtained by applying the method of passive synthesis in the case of impedance $U_U(s)$ and the characteristic $U_F(s)$.

On the basis of the system obtained through passive synthesis, the characteristic functions are determined in the form of stiffness, if the active force is exerted on a material point, corresponding to the impedance (1).

$$\frac{F(s)}{X_1(s)} = \frac{L(s)}{M_1(s)}, \frac{F(s)}{X_2(s)} = \frac{L(s)}{M_2(s)}, \ldots, \frac{F(s)}{X_k(s)} = \frac{L(s)}{M_k(s)}. \tag{27}$$

The values of polynomials $M_1(s), M_2(s), \ldots, M_k(s)$ from equation 27 will be used to determine the value of the force in the following way:

$$\begin{aligned}
&U_F(s) = \frac{L_1(s)}{M\ (s)} \\
&\qquad \frac{L_1(s)}{M_1(s)} = a_1 s + b_1 + \frac{L_2(s)}{M_1(s)}, \\
&\qquad \frac{L_2(s)}{M_2(s)} = a_2 s + b_2 + \frac{L_3(s)}{M_2(s)}, \\
&\qquad \vdots \\
&\qquad \frac{L_{k-1}(s)}{M_{k-1}(s)} = a_{k-1} s + b_{k-1} + \frac{L_k(s)}{M_{k-1}(s)}, \\
&\qquad \frac{L_k(s)}{M_k(s)} = a_k s + b_k, \\
&F(s) = a_1 s + b_1 + a_2 s + b_2 + \ldots + a_k s + b_k, \qquad (2) \\
&F(t) = a_1 \dot{x}_1 + b_1 x_1 + a_2 \dot{x}_2 + b_2 x_2 + \ldots + a_k \dot{x}_k + b_k x_k.
\end{aligned} \tag{28}$$

Finally, as the result of the active synthesis performed, that is, a combination of passive synthesis and determining active force, the system shown in Figure 3 is obtained, with a dynamic characteristic consistent with the impedance (26) subject to synthesis.

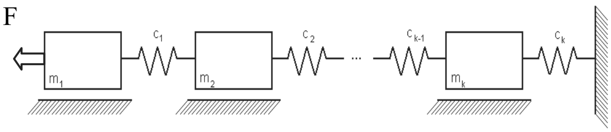

Fig. 3. System obtained through synthesis

5. Conclusions

This paper is concerned with the formulation and formalisation of the problem of active and passive synthesis of mechanical damping systems. The work proposes a combination of the synthesis method of vibrating systems with the method of determining the active force as the method for active synthesis of mechanical damping systems and the values of two-terminal damping in case passive synthesis of mechanical damping systems. The performed active and passive synthesis of restrained systems has demonstrated an ambiguity of synthesis of damping systems. The resulting structures and set system parameters are not the only ones that can be obtained in the case of the assumed frequencies. The formalised method of active synthesis is a proposal for the selection of active force and a vibrating system as a system meeting the required dynamic properties in the form of a sequence of resonant frequencies, anti-resonant frequencies and the decline in natural vibration frequencies. The advantage of the proposed method of determining the active force in the system is the ease of its programming. The proposed method can be included in the method of proportional control with force feedback from the state. The document is also an attempt at identifying new opportunities and a direction of research in the design of machine components with a desired frequency spectrum.

Acknowledgements

This work has been conducted as a part of the research project N 502 447239 supported by the Ministry of Science and Higher Education of Poland in 2010 – 2013.

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